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QUASI-THREE-DIMENSIONAL MODEL APPLIED TO GEOTHERMAL RESERVOIR SYSTEM WITH HEAT FLOW FROM DEEPER ZONES

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ABSTRACT

It is important to grasp the reservoir fluid behavior under operations in the geothermal reservoir development. The authors clarify the calculation procedure of quasi-three-dimensional model and present a flow chart of the model. As example calculations, the developed simulator is applied to simplified geothermal reservoirs which contain heat flow from the deeper zones.

INTRODUCTION

In the geothermal reservoir development, it is important to grasp the reservoir fluid behavior under operations. Generally in mathematic model, fluid behavior in the reservoir is expressed by the three dimensional finite difference equations of mass and energy.[1)-3)] Three dimensional calculations require large data preparation, computing time and storage. Considering these defects of three-dimensional model, Faust and Mercer[4)] presented quasi-three-dimensional model. The equations of the mathematical model are derived by vertical integration of three-dimensional equations. Although the vertical equilibrium in the reservoir is assumed to be achieved in order to obtain the alternative equations, the calculation procedure and the flow chart are not published in their paper. So, the authors clarify the calculation procedure of quasi-three-dimensional model and present a flow chart of the model in this paper. As example calculations, the developed simulator is applied to simplified geothermal reservoirs which contain heat flow from the deeper zones. And, the calculated results on no heat flow case are compared with those of three dimensional calculation.[5)]

THEORY

The behavior of three dimensional two phase flow in a geothermal reservoir is described by conservation equations for mass and energy. Assuming potential and kinetic energy terms and capillary pressure terms are negligible, the equations are written as,[4)]

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla\cdot(\rho_w \mathbf{v}_w + \rho_s \mathbf{v}_s) - q'_m = 0 \qquad \text{------- (1)}$$

$$\frac{\partial\left[\phi\rho h + (1-\phi)\rho_r h_r\right]}{\partial t} + \nabla\cdot(\rho_w h_w \mathbf{v}_w + \rho_s h_s \mathbf{v}_s) - \nabla\cdot(K_m \nabla T) - q'_h = 0 \qquad \text{------- (2)}$$

where

$$\mathbf{v}_w = -\frac{Kk_{rw}}{\mu_w}(\nabla p_w - \rho_w g \nabla D)$$

$$\mathbf{v}_s = -\frac{Kk_{rs}}{\mu_s}(\nabla p_s - \rho_s g \nabla D)$$

$$\rho = S_w \rho_w + S_s \rho_s$$

$$h = (S_w \rho_w h_w + S_s \rho_s h_s)/\rho$$

The equations describing quasi-three-dimensional (areal) two phase flow are obtained by partial integration of Eq.1 and Eq.2 in the vertical direction.

The equations of areal model are written as,

$$b\frac{\partial}{\partial t}(\langle\phi\rho\rangle) - \frac{\partial}{\partial x}\left[b\langle w_x\rangle\left(\frac{\partial}{\partial x}\langle p\rangle - \langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial x}\right)\right]$$
$$- \frac{\partial}{\partial y}\left[b\langle w_y\rangle\left(\frac{\partial}{\partial y}\langle p\rangle - \langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial y}\right)\right] - b\langle q'_m\rangle$$
$$- \left(w_x\frac{\partial p}{\partial x}\right)\Big|_{z_1}\frac{\partial z_1}{\partial x} - \left(w_y\frac{\partial p}{\partial y}\right)\Big|_{z_1}\frac{\partial z_1}{\partial y}$$
$$+ \left(w_z\frac{\partial p}{\partial z} + w_{gz}\right)\Big|_{z_1} + \left(w_x\frac{\partial p}{\partial x}\right)\Big|_{z_2}\frac{\partial z_2}{\partial x}$$
$$+ \left(w_y\frac{\partial p}{\partial y}\right)\Big|_{z_2}\frac{\partial z_2}{\partial y} - \left(w_z\frac{\partial p}{\partial z} + w_{gz}\right)\Big|_{z_2} = 0 \qquad \text{------- (3)}$$

$$b\frac{\partial}{\partial t}\left[(\langle\phi\rho h\rangle+\langle\rho_r h_r\rangle-\langle\phi\rho_r h_r\rangle)\right]$$

$$-\frac{\partial}{\partial x}\left[b\langle w_{hx}\rangle\left(\frac{\partial}{\partial x}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial x}\right)\right]$$

$$-\frac{\partial}{\partial y}\left[b\langle w_{hy}\rangle\left(\frac{\partial}{\partial y}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial y}\right)\right]$$

$$-\frac{\partial}{\partial x}\left[b\langle w_{cp}\rangle\left(\frac{\partial}{\partial x}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial x}\right)\right]$$

$$-\frac{\partial}{\partial y}\left[b\langle w_{cp}\rangle\left(\frac{\partial}{\partial y}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial y}\right)\right]$$

$$-\frac{\partial}{\partial x}\left(b\langle w_{ch}\rangle\frac{\partial}{\partial x}\langle h\rangle\right)-\frac{\partial}{\partial y}\left(b\langle w_{ch}\rangle\frac{\partial}{\partial y}\langle h\rangle\right)$$

$$-b\langle q'_h\rangle-\left(w_{hx}\frac{\partial p}{\partial x}\right)\Big|_{z_1}\frac{\partial z_1}{\partial x}$$

$$-\left(w_{hy}\frac{\partial p}{\partial y}\right)\Big|_{z_1}\frac{\partial z_1}{\partial y}+\left(w_{hz}\frac{\partial p}{\partial z}+w_{hgz}\right)\Big|_{z_1}$$

$$+\left(w_{hx}\frac{\partial p}{\partial x}\right)\Big|_{z_2}\frac{\partial z_2}{\partial x}+\left(w_{hy}\frac{\partial p}{\partial y}\right)\Big|_{z_2}\frac{\partial z_2}{\partial y}$$

$$-\left(w_{hz}\frac{\partial p}{\partial z}+w_{hgz}\right)\Big|_{z_2}$$

$$-\left(w_{cp}\frac{\partial p}{\partial x}+w_{ch}\frac{\partial h}{\partial x}\right)\Big|_{z_1}\frac{\partial z_1}{\partial x}$$

$$-\left(w_{cp}\frac{\partial p}{\partial y}+w_{ch}\frac{\partial h}{\partial y}\right)\Big|_{z_1}\frac{\partial z_1}{\partial y}$$

$$+\left(w_{cp}\frac{\partial p}{\partial z}+w_{ch}\frac{\partial h}{\partial z}\right)\Big|_{z_1}$$

$$+\left(w_{cp}\frac{\partial p}{\partial x}+w_{ch}\frac{\partial h}{\partial x}\right)\Big|_{z_2}\frac{\partial z_2}{\partial x}$$

$$-\left(w_{cp}\frac{\partial p}{\partial y}+w_{ch}\frac{\partial h}{\partial y}\right)\Big|_{z_2}\frac{\partial z_2}{\partial y}$$

$$-\left(w_{cp}\frac{\partial p}{\partial z}+w_{ch}\frac{\partial h}{\partial z}\right)\Big|_{z_2}=0 \quad \text{-------- (4)}$$

where

$$w_x=\frac{K_x k_{rw}\rho_w}{\mu_w}+\frac{K_x k_{rs}\rho_s}{\mu_s}$$

$$w_{hx}=\frac{K_x k_{rw}\rho_w h_w}{\mu_w}+\frac{K_x k_{rs}\rho_s h_s}{\mu_s}$$

$$w_{cp}=K_m\left(\frac{\partial T}{\partial p}\right)_h$$

$$w_{ch}=K_m\left(\frac{\partial T}{\partial h}\right)_p$$

$$w_{gz}=\frac{K_z k_{rw}\rho_w^2 g}{\mu_w}+\frac{K_z k_{rs}\rho_s^2 g}{\mu_s}$$

$$w_{hgz}=\frac{K_z k_{rw}\rho_w^2 g h_w}{\mu_w}+\frac{K_z k_{rs}\rho_s^2 g h_s}{\mu_s}$$

In the above integration, Faust and Mercer's technique[4] detailed in Appendix is used.

BOUNDARY CONDITIONS

Boundary conditions are required for pressure and enthalpy. For this simulater, no flow boundary condition is applied except for top and bottom boundaries. Convective and conductive fluxes at the reservoir top and bottom are considered in Eq.3 and Eq.4. These terms include the derivatives $\partial p/\partial z$ and $\partial h/\partial z$, which are evaluated by pressure and enthalpy gradients in each grid. In addition to this, as a source term, heat flow rate q_b is specified across the reservoir bottom. Eq.4 is rewritten as,

$$b\frac{\partial}{\partial t}\left[(\langle\phi\rho h\rangle+\langle\rho_r h_r\rangle-\langle\phi\rho_r h_r\rangle)\right]$$

$$-\frac{\partial}{\partial x}\left[b\langle w_{hx}\rangle\left(\frac{\partial}{\partial x}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial x}\right)\right]$$

$$-\frac{\partial}{\partial y}\left[b\langle w_{hy}\rangle\left(\frac{\partial}{\partial y}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial y}\right)\right]$$

$$-\frac{\partial}{\partial x}\left[b\langle w_{cp}\rangle\left(\frac{\partial}{\partial x}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial x}\right)\right]$$

$$-\frac{\partial}{\partial y}\left[b\langle w_{cp}\rangle\left(\frac{\partial}{\partial y}\langle p\rangle-\langle\rho g\rangle\frac{\partial\langle D\rangle}{\partial y}\right)\right]$$

$$-\frac{\partial}{\partial x}\left(b\langle w_{ch}\rangle\frac{\partial}{\partial x}\langle h\rangle\right)-\frac{\partial}{\partial y}\left(b\langle w_{ch}\rangle\frac{\partial}{\partial y}\langle h\rangle\right)$$

$$-b\langle q_i\rangle-q_b-\left(w_{hx}\frac{\partial p}{\partial x}\right)\Big|_{z_1}\frac{\partial z_1}{\partial x}$$

$$-\left(w_{hy}\frac{\partial p}{\partial y}\right)\Big|_{z_1}\frac{\partial z_1}{\partial y}+\left(w_{hz}\frac{\partial p}{\partial z}+w_{hgz}\right)\Big|_{z_1}$$

$$+\left(w_{hx}\frac{\partial p}{\partial x}\right)\Big|_{z_2}\frac{\partial z_2}{\partial x}+\left(w_{hy}\frac{\partial p}{\partial y}\right)\Big|_{z_2}\frac{\partial z_2}{\partial y}$$

$$-\left(w_{hz}\frac{\partial p}{\partial z}+w_{hgz}\right)\Big|_{z_2}$$

$$-\left(w_{cp}\frac{\partial p}{\partial x}+w_{ch}\frac{\partial h}{\partial x}\right)\Big|_{z_1}\frac{\partial z_1}{\partial x}$$

$$- \left(w_{cp}\frac{\partial p}{\partial y} + w_{ch}\frac{\partial h}{\partial y}\right)\Big|_{z_1}\frac{\partial z_1}{\partial y}$$

$$+ \left(w_{cp}\frac{\partial p}{\partial z} + w_{ch}\frac{\partial h}{\partial z}\right)\Big|_{z_1}$$

$$+ \left(w_{cp}\frac{\partial p}{\partial x} + w_{ch}\frac{\partial h}{\partial x}\right)\Big|_{z_2}\frac{\partial z_2}{\partial x}$$

$$- \left(w_{cp}\frac{\partial p}{\partial y} + w_{ch}\frac{\partial h}{\partial y}\right)\Big|_{z_2}\frac{\partial z_2}{\partial y}$$

$$- \left(w_{cp}\frac{\partial p}{\partial z} + w_{ch}\frac{\partial h}{\partial z}\right)\Big|_{z_2} = 0 \qquad \text{------- (5)}$$

In Eqs.3 and 5, independent variables are pressure and enthalpy of the steam-water mixture. Saturated values of h_w, h_s, ρ_w, ρ_s are calculated directly from steam tables as functions of pressure and enthalpy. The density of the steam-water mixture and water saturation can be treated as functions of pressure and enthalpy. Several additional assumption are necessary to describe behavior of geothermal fluid. First, it is assumed that thermal equilibrium exist among all phases. With this assumption, temperature is treated as a function of pressure and enthalpy. The other assumptions are as follows. Viscosities are functions of pressure and temperature given from steam tables. Porosity is a function[4] of pressure, given by $\phi = \phi_i + \beta(p - p_i)$. Relative permeabilities are linear functions[6] of water saturation, given by $k_{rw} = (S_w - S_{wr})/(1.0 - S_{wr})$.

CALCULATION PROCEDURE

Eqs.3 and 5 for the areal model are expanded in finite difference formulas, and solved by the Newton-Raphson iteration. Using a direct method,[6),7)] the equations are solved for pressure and enthalpy at each iteration.

Flow chart of calculation procedure is shown in Fig.1. The followings are detail of this procedure.

Step 1. Read parameters required for the difference calculation (time step, block number, block size and convergence criterion) and reservoir data (permeability, elevations on the base and top of the reservoir, thermal conductivity, specific heat of reservoir rock and specific gravity of reservoir rock).

Step 2. Read initial condition of reservoir for each block (pressure, enthalpy, production rate and injection rate).

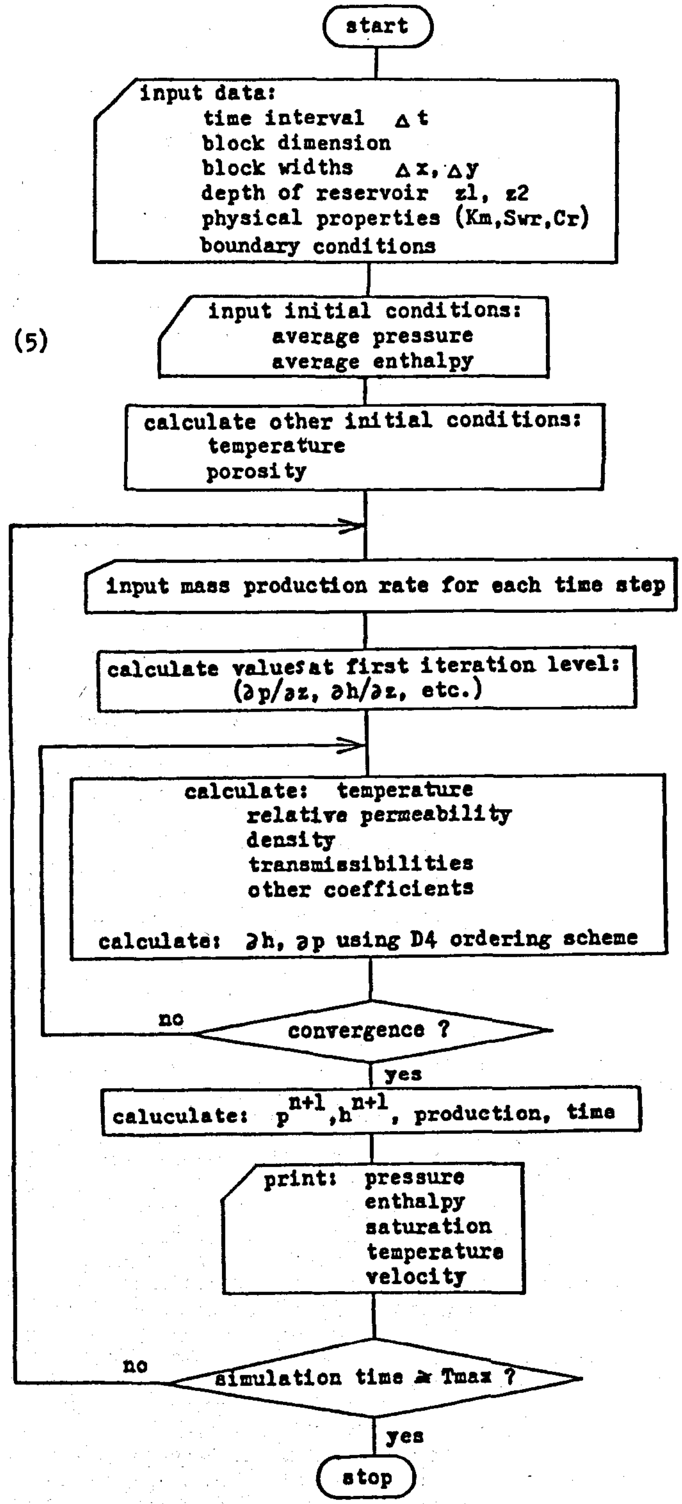

Fig. 1 Flow chart of quasi-three-dimensional model

Step 3. Calculate the other initial conditions[8),9)] (temperature and porosity).
Step 4. As precondition of iteration, calculate variables required for the first iteration level.
Step 5. Calculate variables (relative permeabilities, fluid densities, transmissibilities and so on) required for solving Eqs.3 and 5, and obtain δp and δh using D4 ordering.[6)]
Step 6. Check convergence for pressure and enthalpy.
Step 7. Calculate values of pressure and enthalpy.
Step 8. Print calculated values (pressure, enthalpy, saturation, temperature and velocity).
Step 9. Check computing period.

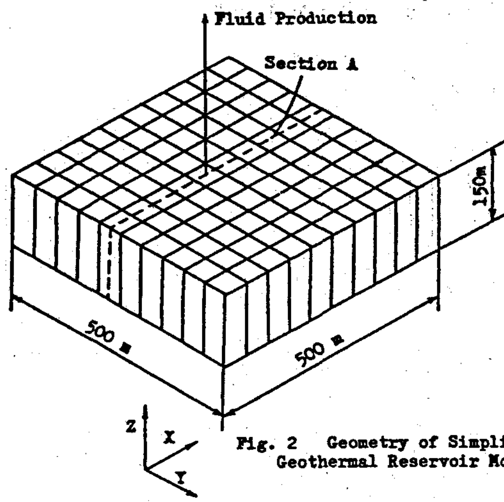

Fig. 2 Geometry of Simplified Geothermal Reservoir Model

Table 1 Initial Condition for Example Calculation

Length of reservoir, (cm)	50000
Width of reservoir, (cm)	50000
Thickness of reservoir, (cm)	15000
Grid size: Δx, Δy, (cm)	5000
Porosity	0.380
Horizontal permeability, (darcy)	0.100
Vertical permeability, (darcy)	0.0100
Thermal conductivity, (cal/cm-sec-°C)	0.450×10^{-2}
Specific heat of reservoir rock, (cal/g-°C)	0.170
Specific gravity of reservoir rock, (g-cm^{-3})	2.65
Formation compressibility, (atm^{-1})	1.67×10^{-5}
Vertical averaged pressure, (atm)	40.0
Vertical averaged enthalpy, (cal/g)	260
Vertical averaged temperature, (°C)	248
Residual water saturation	0.200

SAMPLE CALCULATION

This sample calculation illustrate the application of the quasi-three-dimensional model to a simplified geothermal reservoir. The grid for the reservoir model is shown in Fig. 2. The quasi-three-dimensional calculation is performed over 10x10 grids. The reservoir fluid is assumed to be pure water and to be initially in the two phase zone. Parameters and initial conditions used in this sample calculation are given in Table 1. For this calculation, location of assumed production well is the grid point (5,5). Initial production rate is 0.683×10^4 g/sec. Time step is 1.44×10^5s. After comfirming the stability and the convergence of the model, the simulater is applied to the geothermal systems. Assuming heat flow of 4.0×10^{-5} cal/sec-cm^2 and no mass flow from bottom of the reservoir, simulations of the thermal conduction and dispersion are carried out. Fig.3 shows the assumed heat flow is not large enough to maintain the isoenthalpy production.

Shown in Figs.4 and 5, the calculated results in the author's another paper [10)] are

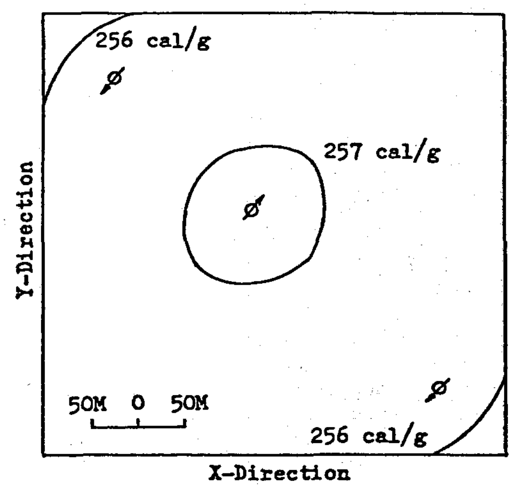

Fig. 3 Depth-averaged enthalpy map for a two-phase geothermal reservoir after 300days. This geothermal system involves heat flow (4.0×10^{-5} cal/sec·cm^2) from exterior of the reservoir.

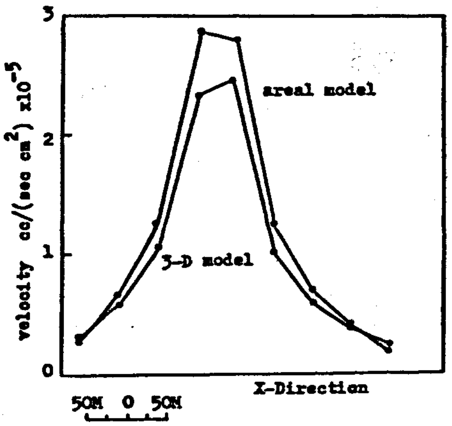

Fig. 4 Depth-averaged fluid velocity along section A (Fig.2) after 100days areal and 3-D model.

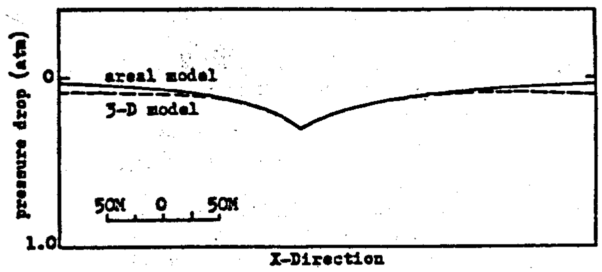

Fig. 5 Depth averaged pressure drop along section A (Fig.2) after 100days for areal and 3-D model.

compared with those of three dimensional calculation in case of no heat flow at the boundaries. HITAC M200-H was used for this computation. As to computing time, quasi-three-dimensional calculation required 1.57s per iteration step and three-dimensional calculation required 10.38s per iteration step. This indicates that reduction of computing time is performed by the quasi-three-dimensional model. These values mainly depend on the grid number, so that, if the grid number become increase, the reduction of computing time will become larger. If the convergence conditions are equal, whole computing time will be more small for the quasi-three-dimensional model, which needs less iteration steps because of its smaller grid number.

NOMENCLATURE

b = reservoir thickness, cm
C_r = specific heat of reservoir rock, cal/g-°C
D = depth, cm
g = gravitational acceleration, cm/sec^2
h = specific enthalpy, cal/g
K = absolute permeability, darcy
k_{rl} = relative permeability of the phase l, fraction
K_m = reservoir thermal conductivity, cal/cm-sec-°C
p = reservoir pressure, atm
p_i = initial reservoir pressure, atm
q_b = heat flow rate across boundary, cal/sec-cm^2
q'_h = heat source term, cal/sec-cm^3
q'_m = mass source term, g/sec-cm^3
S = saturation, fraction
S_{wr} = residual water saturation, fraction
T = reservoir temperature, °C
T_{max} = period of simulation, sec
t = time, sec
Δt = time step size, sec
v = averaged phase velocity, cm/sec
z_1 = elevation of the bottom of the reservoir, cm
z_2 = elevation of the top of the reservoir, cm
β = formation compressibility, atm^{-1}
μ = viscosity, cp
ϕ = porosity, fraction
ϕ_i = initial porosity, fraction
ρ = density, g/cm^3

Subscripts

h = enthalpy, m = mass,
n = iteration level, r = reservoir,
s = steam, w = water,
x,y,z = directions in the Cartesian coordinate system

Operators

∇ = divergence operator

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APPENDIX

Derivation of quasi-three-dimensional equation by Faust and Mercer's technique.
In the equations of the areal model, parameters are expressed as depth averaged value given by

$$<\phi> = \frac{1}{b}\int_{z_1}^{z_2}\phi \, dz$$

where z_1, z_2 and b are function of x-y space and time. For the following equation should hold.

$$\int_{z_1}^{z_2}\frac{\partial \phi}{\partial x}\, dz = \frac{\partial}{\partial x}\int_{z_1}^{z_2}\phi \, dz$$

$$+ \phi(x, y, z_1, t)\frac{\partial z_1}{\partial x}$$

$$- \phi(x, y, z_2, t)\frac{\partial z_2}{\partial x}$$

The diviation of ϕ from $<\phi>$ is defined as

$$\hat{\phi} = \phi - <\phi>$$

The above two equations are applied in the integration of Eq.1 and Eq.2. In the areal equations, the deviation terms higher than second order can be omitted, and the discontinuities in the quantities on the water-steam interface are negligible. With those assumptions and assuming gravity segregation, the equations of quasi-three-dimensional are derived.