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TOWARD A TWO-EQUATION METHOD FOR GEOTHERMAL SIMULATION OF FRACTURED POROUS RESERVOIRS

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ABSTRACT

In the "double-porosity" approach to the simulation of rigid geothermal reservoirs there are several options available in formulating the governing equations. The most basic method requires for each fluid phase the solution of four equations: the mass and energy balances for both the fractures and the porous matrix. It is customary to sum the equations for the fluid phases to eliminate condensation terms from the equations. When a double porosity approach is employed, fracture-matrix exchange terms arise; these terms require constitutive assertions to solve the resulting equations. Alternatively, one may add the fracture and porous matrix equations together. This procedure not only eliminates the troublesome fracture-matrix exchange terms from explicit consideration but also reduces the number of dependent variables and equations. The various summation procedures result in the transfer of information to terms which usually lead to a more diffuse solution.

INTRODUCTION

Equations of the motion for steam and water in a geothermal reservoir considered as a porous medium have been presented by many authors (e.g., Garg et al., 1978; Faust and Mercer, 1979). In this paper we examine the fractured porous medium case. The so-called double-porosity or double-continuum approach will be employed. Having presented the primitive equations we will examine the implications of summing certain of them. The paper extends the work of Kazemi et al. (1976), O'Neill and Pinder (1981), and Pinder and Shapiro (1982). We are particularly concerned about ways detailed information can be masked or lost through use of the summed equations.

CONCEPTUAL MODEL

In Figure 1, each of the six central blocks (1a-3a, 1b-3b) represents a set of balance equations for mass, momentum and energy. Each is associated with liquid, vapor, or solid in either fractures or matrix. There are thus 30 equations in this description of a fractured porous geothermal reservoir. Even at this level of complexity, the system has been simplified. Nevertheless let us take Figure 1 and the associated equations as our point of departure.

Although it is possible to consider the liquid, vapor, and rock phases of a porous medium separately, it is common to combine the balance equations for the 3 phases (1a-3a) to yield a single set of equations, Σa. Similarly, we combine (1b-3b) to form the set Σb. In summing across rows of the figure we have thus reduced from 30 to 10 the number of equations to be solved. The specification of fracture-matrix exchange terms is unfortunately an outstanding problem.

An alternative scheme for equation reduction is to sum columns, e.g., combine 1a and 1b to form $\Sigma 1$. In this case the condensation terms will appear in our equations although the fracture-matrix exchange terms will not.

Finally, all the exchange terms can be completely eliminated by adding all the equations together to form $\Sigma = \Sigma 1 + \Sigma 2 + \Sigma 3 = \Sigma a + \Sigma b$. This set of equations is generally used in current geothermal models.

One might actually wonder about the information costs paid for the equation simplifications achieved through this summation procedure. To determine this we must examine in more detail the strategy outlined above.

THE PRIMITIVE EQUATIONS

We begin with notation. Let the subscript α denote the alpha-th phase, and let the subscript λ denote the lambda-th topophase (fractures or porous blocks).

If λ is a property on the macroscopic, or measurable, scale then ψ_L^f and ψ_V^m are the values of ψ for the liquid (L) in the fractures (f) and for the vapor (V) in the matrix (m), respectively. This notation allows us to distinguish the type of rock comprising the porous matrix from the fill in the fractures.

The primitive equations corresponding to the entries of Figure 1 are:

(1) Mass

$$\partial_t(\rho_\alpha^\lambda) + \underline{\nabla}\cdot(\rho_\alpha^\lambda \underline{v}_\alpha^\lambda) = R_\alpha + R^\lambda$$

(2) Momentum

$$\partial_t(\rho_\alpha^\lambda \underline{v}_\alpha^\lambda) + \underline{\nabla}\cdot(\rho_\alpha^\lambda \underline{v}_\alpha^\lambda \underline{v}_\alpha^\lambda) - \underline{\nabla}\cdot\underline{\underline{t}}_\alpha^\lambda - \rho_\alpha^\lambda \underline{g} = \underline{P}_\alpha + \underline{P}^\lambda$$

(3) Energy

$$\partial_t(\rho_\alpha^\lambda e_\alpha^\lambda + \tfrac{1}{2}\rho_\alpha^\lambda|\underline{v}_\alpha^\lambda|^2) + \underline{\nabla}\cdot[(\rho_\alpha^\lambda e_\alpha^\lambda + \tfrac{1}{2}\rho_\alpha^\lambda|\underline{v}_\alpha^\lambda|^2)\underline{v}_\alpha^\lambda] - \underline{\nabla}\cdot\underline{q}_\alpha^\lambda - \underline{\nabla}\cdot\underline{\underline{t}}_\alpha^\lambda\cdot\underline{v}_\alpha^\lambda - \rho_\alpha^\lambda \underline{v}_\alpha^\lambda\cdot\underline{g} = T_\alpha + T^\lambda$$

subject to

(4) $$\sum_\alpha \sum_\lambda (R_\alpha + R^\lambda) = \sum_\alpha \sum_\lambda (\underline{P}_\alpha + \underline{P}^\lambda) = \sum_\alpha \sum_\lambda (T_\alpha + T^\lambda) = 0$$

where ρ_α^λ, $\underline{v}_\alpha^\lambda$, $\underline{\underline{t}}_\alpha^\lambda$, e_α^λ, $\underline{g}$, and $\underline{q}^\lambda$ are the partial mass density, velocity, partial stress, specific internal energy, body force, and partial heating. The right-hand side terms denote the exchange of mass energy and momentum between the phases and topophases. The subscripted term indicates exchange from all other phases in the λ topophase, while the superscripted term means exchange from all other topophases for all phases. Note that, for example,

(5) $$\sum_\alpha (R_\alpha + R^\lambda) = R^\lambda,$$

$$\sum_\lambda (R_\alpha + R^\lambda) = R_\alpha,$$

$$\sum_\alpha \sum_\lambda (R_\alpha + R^\lambda) = 0$$

The symbols ∂_t and $\underline{\nabla}\cdot$ are the partial derivative with respect to time and the divergence operator.

SUMMING OVER PHASES, DISTINCT TOPOPHASES

We now consider an approach for fractured media which echoes current methodology in simple porous medium analyses of geothermal systems. The procedure is to sum the balances (1)-(3) over the subscript α, i.e., we will obtain one equation for the fractures and another for the matrix, each of which embodies the three phases rock, liquid, and vapor. In the ensuing sections, we will only write the energy balance to conserve space and because the mass and momentum equations are similarly obtained.

We define the inner part of the stress and the heating by

(6a) $$\underline{\underline{t}}_I^\lambda := \sum_\alpha \underline{\underline{t}}_\alpha^\lambda$$

(6b) $$\underline{q}_I^\lambda := \sum_\alpha \underline{q}_\alpha^\lambda$$

The phase-summed energy equation can now be written

(7) $$\partial_t[\rho_R^\lambda(e_R^\lambda + \tfrac{1}{2}|\underline{v}_R^\lambda|^2) + \rho_L^\lambda(e_L^\lambda + \tfrac{1}{2}|\underline{v}_L^\lambda|^2) + \rho_v^\lambda(e_v^\lambda + \tfrac{1}{2}|\underline{v}_v^\lambda|)] + \underline{\nabla}\cdot[\rho_R^\lambda \underline{v}_R^\lambda(e_R^\lambda + \tfrac{1}{2}|\underline{v}_R^\lambda|^2) + \rho_L^\lambda \underline{v}_L^\lambda(e_L^\lambda + \tfrac{1}{2}|\underline{v}_L^\lambda|^2) + \rho_v^\lambda \underline{v}_L^\lambda(e_v + \tfrac{1}{2}\underline{v}_v^\lambda|^2)] - \underline{\nabla}\cdot\underline{q}_I - \underline{\nabla}\cdot(\underline{\underline{t}}_R^\lambda\cdot\underline{v}_R^\lambda + \underline{\underline{t}}_L^\lambda\cdot\underline{v}_L^\lambda + \underline{\underline{t}}_v^\lambda\cdot\underline{v}_v^\lambda) - (\rho_R^\lambda \underline{v}_R^\lambda + \rho_L^\lambda \underline{v}_L^\lambda + \rho_v^\lambda \underline{v}_v^\lambda)\cdot\underline{g} = T^\lambda$$

subject to

(8) $$\sum_\lambda T^\lambda = 0.$$

While these equations retain the basic formal structure of equations (1) - (3), they appear to be easier to treat. The reason is that the inter-phase exchange terms are summed leaving as residues only inter-topophase terms R^λ, $\underline{P}^\lambda$, and T^λ. Unfortunately, the constitutive expressions for these terms are not well understood.

Equation (7) can be written in terms of phase-averaged variables. To expedite this procedure we introduce new notation:

(9) $$\rho^\lambda := \sum_\alpha \rho_\alpha^\lambda$$

(10) $$\rho^\lambda \underline{v}^\lambda := \sum_\alpha \rho_\alpha^\lambda \underline{v}_\alpha^\lambda$$

(11) $$\underline{u}_\alpha^\lambda := \underline{v}_\alpha^\lambda - \underline{v}^\lambda$$

(12) $$\underline{\underline{t}}^\lambda := \underline{\underline{t}}_I^\lambda - \sum_\alpha (\rho_\alpha^\lambda \underline{u}_\alpha^\lambda \underline{u}_\alpha^\lambda)$$

(13) $$\rho^\lambda e^\lambda := \sum_\alpha (\rho_\alpha^\lambda e_\alpha^\lambda + \tfrac{1}{2}\rho_\alpha^\lambda|\underline{u}_\alpha^\lambda|^2)$$

(14) $$\underline{q}^\lambda := \underline{q}_I^\lambda + \sum_{\alpha_2} [\underline{\underline{t}}_\alpha^\lambda\cdot\underline{u}_\alpha^\lambda - (\rho_\alpha^\lambda e_\alpha^\lambda + \tfrac{1}{2}\rho_\alpha^\lambda|\underline{u}_\alpha^\lambda|^2)\underline{u}_\alpha^\lambda].$$

Note that

$$\sum_{\alpha} (\rho^{\lambda}_{\alpha} \underset{\sim}{u}^{\lambda}_{\alpha}) = \underset{\sim}{0}.$$

Summing equation (3) over α and using the above definitions we obtain

$$(15)\quad \partial_t(\rho^{\lambda}e^{\lambda} + \tfrac{1}{2}\rho^{\lambda}|\underset{\sim}{v}^{\lambda}|^2) + \underset{\sim}{\nabla}\cdot[(\rho^{\lambda}e^{\lambda} + \tfrac{1}{2}\rho^{\lambda}|\underset{\sim}{v}^{\lambda}|^2)\underset{\sim}{v}^{\lambda}] - \underset{\sim}{\nabla}\cdot\underset{\sim}{q}^{\lambda} - \underset{\sim}{\nabla}\cdot\underset{\approx}{t}^{\lambda}\cdot\underset{\sim}{v}^{\lambda} - \rho^{\lambda}\underset{\sim}{v}^{\lambda}\cdot\underset{\sim}{g} = T^{\lambda}$$

subject to equation (8).

SUMMING OVER TOPOPHASES, DISTINCT PHASES

We now present the second summing alternative designed to reduce the number of governing equations. In this case we will obtain balance statements for rock, liquid, and vapor. As before, we regretably begin the section with additional nomenclature:

$$(16)\quad \rho_{\alpha} := \sum_{\lambda} \rho^{\lambda}_{\alpha}$$

$$(17)\quad \rho_{\alpha}\underset{\sim}{v}_{\alpha} := \sum_{\lambda} \rho^{\lambda}_{\alpha}\underset{\sim}{v}^{\lambda}_{\alpha}$$

$$(18)\quad \underset{\sim}{\bar{u}}^{\lambda}_{\alpha} := \underset{\sim}{v}^{\lambda}_{\alpha} - \underset{\sim}{v}_{\alpha}$$

$$(19)\quad \underset{\approx}{t}_{\alpha} := \sum_{\lambda} (\underset{\approx}{t}^{\lambda}_{\alpha} - \rho^{\lambda}_{\alpha}\underset{\sim}{u}^{\lambda}_{\alpha}\underset{\sim}{u}^{\lambda}_{\alpha})$$

$$(20)\quad \rho_{\alpha}e_{\alpha} := \sum_{\lambda} (\rho^{\lambda}_{\alpha}e^{\lambda}_{\alpha} + \tfrac{1}{2}\rho^{\lambda}_{\alpha}|\underset{\sim}{\bar{u}}^{\lambda}_{\alpha}|^2)$$

$$(21)\quad \underset{\sim}{q}_{\alpha} := \sum_{\lambda}[\underset{\sim}{q}^{\lambda}_{\alpha} + \underset{\approx}{t}^{\lambda}_{\alpha}\cdot\underset{\sim}{\bar{u}}^{\lambda}_{\alpha} - \rho^{\lambda}_{\alpha}(e^{\lambda}_{\alpha} + \tfrac{1}{2}|\underset{\sim}{\bar{u}}^{\lambda}_{\alpha}|^2)\underset{\sim}{\bar{u}}^{\lambda}_{\alpha}].$$

In addition we note that

$$\sum_{\lambda} (\rho^{\lambda}_{\alpha}\underset{\sim}{u}^{\lambda}_{\alpha}) = \underset{\sim}{0}.$$

The governing energy equation becomes:

$$(22)\quad \partial_t(\rho_{\alpha}e_{\alpha} + \tfrac{1}{2}\rho_{\alpha}|\underset{\sim}{v}_{\alpha}|^2) + \underset{\sim}{\nabla}\cdot[\rho_{\alpha}\underset{\sim}{v}_{\alpha}(e_{\alpha} + \tfrac{1}{2}|\underset{\sim}{v}_{\alpha}|^2)] - \underset{\sim}{\nabla}\cdot\underset{\sim}{q}_{\alpha} - \underset{\sim}{\nabla}\cdot\underset{\approx}{t}_{\alpha}\cdot\underset{\sim}{v}_{\alpha} - \rho_{\alpha}\underset{\sim}{v}_{\alpha}\cdot\underset{\sim}{g} = T_{\alpha}$$

subject to

$$(23)\quad \sum_{\alpha} T_{\alpha} = 0.$$

SUMMING OVER PHASES AND TOPOPHASES

For those stubborn readers who have doggedly pursued the development to this point, we now combine balance equations (1) - (3) over all pseudophases; that is, over all phases (subscript α) and all topophases (superscript λ). The resulting lumped set of equations will have the form of a single-phase flow and energy transport equations of a porous medium continuum.

We begin again with new nomenclature:

$$(24)\quad \rho := \sum_{\alpha}\sum_{\lambda} (\rho^{\lambda}_{\alpha})$$

$$(25)\quad \underset{\sim}{v} := \frac{1}{\rho}\sum_{\alpha}\sum_{\lambda} \rho^{\lambda}_{\alpha}\underset{\sim}{v}^{\lambda}_{\alpha}$$

$$(26)\quad \underset{\sim}{\tilde{u}}^{\lambda}_{\alpha} := \underset{\sim}{v}^{\lambda}_{\alpha} - \underset{\sim}{v}$$

$$(27)\quad \underset{\approx}{t} := \sum_{\alpha}\sum_{\lambda} (\underset{\approx}{t}^{\lambda}_{\alpha} - \rho^{\lambda}_{\alpha}\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha}\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha})$$

$$(28)\quad \rho e := \sum_{\alpha}\sum_{\lambda} (\rho^{\lambda}_{\alpha}e^{\lambda}_{\alpha} + \tfrac{1}{2}\rho^{\lambda}_{\alpha}|\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha}|^2)$$

$$(29)\quad \underset{\sim}{q} := \sum_{\alpha}\sum_{\lambda} [\underset{\sim}{q}^{\lambda}_{\alpha} + \underset{\approx}{t}^{\lambda}_{\alpha}\cdot\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha} - (\rho^{\lambda}_{\alpha}e^{\lambda}_{\alpha} + \tfrac{1}{2}\rho^{\lambda}_{\alpha}|\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha}|^2)\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha}].$$

Moreover it follows that

$$\sum_{\alpha}\sum_{\lambda} (\rho^{\lambda}_{\alpha}\underset{\sim}{\tilde{u}}^{\lambda}_{\alpha}) = \underset{\sim}{0}.$$

The governing energy equation is then

$$(30)\quad \partial_t(\rho e + \tfrac{1}{2}\rho|\underset{\sim}{v}|^2) + \underset{\sim}{\nabla}\cdot[(\rho e + \tfrac{1}{2}\rho|\underset{\sim}{v}|^2)\underset{\sim}{v}] - \underset{\sim}{\nabla}\cdot\underset{\sim}{q} - \underset{\sim}{\nabla}\cdot\underset{\approx}{t}\cdot\underset{\sim}{v} - \rho\underset{\sim}{v}\cdot\underset{\sim}{g} = 0.$$

DISCUSSION

Each set of equations has its advantages and disadvantages. The first set (1-3) is unattractive due to the number of equations which must be solved and the large number of exchange terms which must be specified.

The second set of equations represented by (7) or (15) offers a reasonable model. The topophase characteristics are kept clearly separate, so that in the absence of fractures the system of equations easily reduces to the standard geothermal equations. However, one is left with the problem of defining the exchange terms between the topophases.

The third set of equations has great initial appeal because the dependent variables are written for each phase, which coincides with field data (e.g., quality of produced fluid is

obtained, not quality of fluid produced from fractures). On the otherhand, a number of conceptual difficulties arise because the distinctive production mechanisms of fractures and matrix may be obscured. Moreover, there remains the difficulty of determining the exchange term between the phases.

The final set of equations corresponds to the most widespread technology for studying geothermal reservoirs -- fractured or not. The fractured reservoir is considered, in this scheme, to be a porous medium, and fractures or fracture zones are incorporated by adjustments to material properties.

Let us consider now the question of information transfer within the equations. Each equation summation has generated a new set of dependent variables which are related to the primitive variables through their defining equations. Each summed balance equation contains terms which describes the contributions of the primitive variables. Of particular interest are the heat flux terms $\underset{\sim}{q}^{\wedge}(14)$, $\underset{\sim}{q}_{\alpha}(21)$, and $q(29)$ which must be expressed in terms of the averaged variables to solve the resulting system of equations. The functional form of these constitutive relationships is generally diffusive (behaves like Fick's first law), i.e., the information lost in defining the new summed variables is imbedded in a diffusion-generating term. Consequently, the summed equations are inherently more diffusive than the primitive equations and their solutions will therefore be more smeared. We note that in current practice equation (7) rather than (15) is used (in the absence of fractures), avoiding the transfer of information to the heat flux terms. The outstanding issue now is the significance of this phenomenon for practical applications.

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	Liquid	Vapor	Solid	Combined Equations
Porous Blocks	Water in Pores 1a	Steam in Pores 2a	Solids in Blocks 3a	Steam, Water, and Rock Porous Medium Σa
Fractures	Water in Fractures 1b	Steam in Fractures 2b	Solids in Fractures 3b	Steam, Water, and Rock Fractured Medium Σb
"Effective" Single Porosity Model	Water in Pores and Fractures Σ1	Steam in Pores and Fractures Σ2	Solids in Blocks and Fractures Σ3	Steam, Water and Rock in Fractures Plus Porous Blocks Σ

Figure 1.