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SIMULATION OF FLOW IN FRACTURED POROUS MEDIA

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Introduction While flow in fractured porous media is a phenomenon often encountered in reservoir simulation, there exists no generally accepted simulation methodology. One can catalogue existing approaches as either discrete fracture or continuum. As the name implies the *discrete fracture model* considers each fracture as a geometrically well-defined entity wherein the fluid behavior is described using some variant of classical fluid mechanics. The geometry of the porous blocks is also assumed known and the pore fluid behavior is determined via the equations describing the physics of flow through porous media. The two systems are coupled through conservation constraints along the fracture-porous block interface. Discrete fracture models have been popular for some time. Early work was conducted by Berman (1953) and Crawford and Collins (1954); recently Grisak and Pickens (1980) used this approach to examine mass transport.

The *continuum model*, sometimes referred to as the double porosity model, does not attempt to describe the behavior in each porous block or fracture explicitly. Rather one abandons this detailed level of observation and alternatively examines the physical phenomenon from a more distant perspective. At this higher level of observation, one considers only the average properties of the pores and fractures. These properties are in turn represented by functions which are assumed to satisfy certain smoothness conditions consistent with the fundamental postulates of continuum mechanics. This approach relies more heavily on constitutive theory to establish meaningful experiments to determine these property functions. The concept of the continuum model, as applied to fractured reservoirs, is generally attributed to Barenblatt and Zheltov (1960). Only recently however have the mathematical-physical underpinnings of this approach been carefully examined. Duguid and Lee (1977) were the first to recognize the necessity of adhering to continuum principles in equation formulation A recent summary of work in this area can be found in Shapiro (1981).

The Model Problem Although both modelling approaches have received considerable attention, little effort has been expended in studying the relationship between them. The outstanding question is whether the continuum model can adequately represent the mathematical-physical behavior of the discrete system. To address this problem, we have constructed a discrete fracture model and a corresponding continuum model. The discrete fracture model is shown in figure 1. It consists of a set of infinitely long prisms with square cross-sections.

The equations describing fluid flow in the fractures are, for the x coordinate direction

$$(1)\quad D_t\rho + D_x(\rho\bar{v}_x) = \frac{2}{\ell}\,\rho v_y\Big|_{y=0}$$

(mass conservation)

and

$$(2)\quad (D_t\bar{v}_x + \bar{v}_x D_x\bar{v}_x) + D_x P - (\tfrac{4}{3}\mu + \lambda)D_x^2\bar{v}_x + \frac{12\bar{v}_x}{\ell^2}\mu + \bar{v}_x\left(\frac{2}{\ell}\rho v_y\Big|_{y=0}\right) = 0$$

(momentum conservation)

where ρ is fluid density,

P is fluid pressure,

$\bar{v}_x$ is the average fluid velocity,

μ is the shear fluid viscosity,

λ is the bulk fluid viscosity,

ℓ is the fracture thickness, and

$D_t(\cdot)$ and $D_x(\cdot)$ are partial derivatives in time and the x coordinate direction respectively.

A similar set of equations can be written for the y direction.

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The equation describing flow in the porous blocks is

$$(3)\quad (c_f\phi + \rho\beta)D_t p - \rho\frac{k}{\mu}\nabla^2 p = 0$$

where c_f is medium compressibility,

β is fluid compressibility,

k is matrix permeability, and

ϕ is porosity.

The continuum equations for the porous medium and fractures are given by (Shapiro, 1981).

$$(4)\quad D_t(\rho^k\phi^k) - \frac{\rho^p k^p}{\mu}\nabla^2 p^p = -\rho^f\phi^f \sum_{\beta=f,p}\left(\alpha^\beta c_f p^\beta - \Lambda^\beta \frac{k^\beta}{\mu^\beta\phi^\beta}\nabla^2 p^\beta\right)$$

(porous medium)

$$(5)\quad \phi^f\frac{D}{Dt}(\rho^f) - \frac{\rho^f k^f}{\mu^f}\nabla^2 p^f = -\rho^f\phi^f \sum_{\beta=f,p}\left(\alpha^\beta c_f p^\beta - \Lambda^\beta \frac{k^\beta}{\mu^\beta\phi^\beta}\nabla^2 p^\beta\right)$$

(fractured medium)

where α^β and Λ^β are coefficients in the mass exchange function.
The terms on the right hand side of (4) and (5) represent the interaction between the blocks and the fractures.

Parameter Estimation and Analysis The immediate objective is to determine theability of equations (4) and (5) to describe the physical response of the fractured porous medium system represented by equations (1), (2) and (3). To examine this hypothesis, we determine the unknown parameters α^β and Λ^β using the solution to (1), (2) and (3). In other words we use the discrete fracture model and equations (1), (2) and (3) as our experimental observations and solve for the unknown parameters. TO establish the robustness of the continuum model we subsequently compare the solutions obtained using the two approaches. The parameters used in this mathematical experiment are listed in table 1. The two solutions are presented in figure 2.

It is apparent from figure 2 that the continuum model generated a solution qualitatively similar to that generated by the discrete fracture model. Experiments conducted using REVs of different sizes indicate that the continuum model solution is relatively unaffected by this parameter and that the continuum parameters are temporally stable.

Comparison with the earlier continuum representation of Barenblatt indicates that his formulation generates a solution somewhat different than either the discrete fracture or continuum formulations presented herein.

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Table 1

Experimental Properties

Property	Symbol	Value
1) Porous medium properties		
Porosity	ϕ	0.20
Permeability	k	1.0E-08cm^2
Matrix and Fluid Compressibility	$(\phi c_f + \rho\beta)$	1.0E-11gm/dyne-cm
2) Fracture properties		
Thickness	ℓ	0.01 cm
Fracture spacing	L	8.0 E01cm
Fluid velocity at inlet	v_0	1.0 cm/sec
3) Fluid properties		
Compressibility	c_f	0.48E-10cm^2/dyne
Viscosity	μ	1.3E-02 dyne/cm^2-sec
Reference density	ρ_0	1.0 gm/cm^3
Reference pressure	P_0	0.0 dyne/cm^2

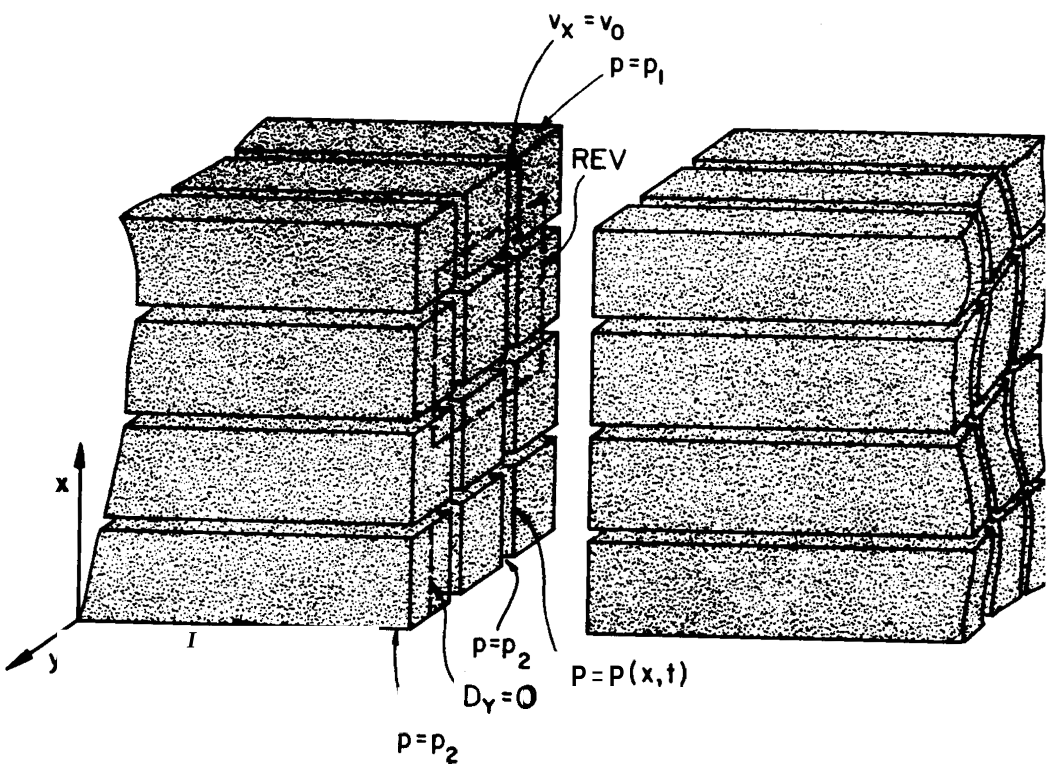

Figure 1: Diagrammatic representation of a discrete fracture system.

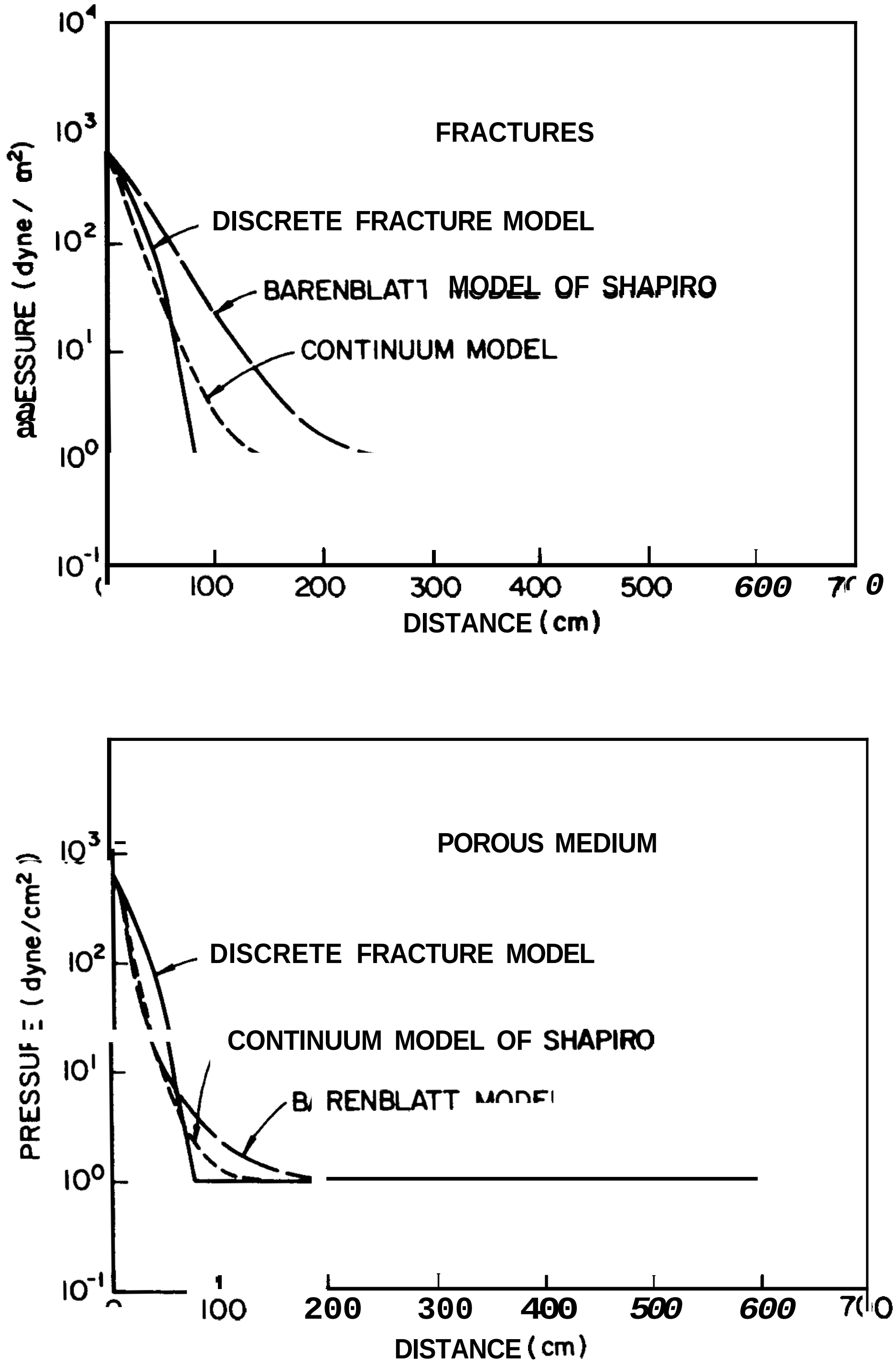

Figure 2: Pressure response calculated using discrete fracture and continuum models by Shapiro and Barenblatt.