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ANALYTICAL APPROACH TO THE SIMULATION OF LABORATORY STEAM-FLOW EXPERIMENTS

Allen F. Moench and William N. Herkelrath

U.S. Geological Survey
345 Middlefield Road
Menlo Park, CA 94025

Abstract A partial differential equation with pressure as the dependent variable is derived for the flow of steam in porous materials under conditions of low liquid-water saturation. The equation includes effects of vapor-pressure lowering and latent heat of adsorption. An apparent steam diffusivity is obtained that includes these effects.

The equation is tested using pressure-transient experiments conducted in the laboratory at 100°C. The experiments, described in earlier Workshop summaries, were run by bringing a cylinder of porous material to a uniform pressure and then making a step increase in pressure at one end of the sample while monitoring the pressure response at the other end.

Because the apparent steam diffusivity was found to be nearly constant over the pressure range of the experiments, it was possible to use a simple analytical solution to simulate the experimental results.

Introduction Laboratory steam-flow experiments conducted on a cylinder of porous material have been described in a series of reports by Herkelrath and Moench (1978, 1980, 1981). In these experiments the sample was brought to a uniform pressure and then subjected to a step increase in pressure at one end while monitoring the pressure response at the other end. The experiments were run at pressures less than saturated vapor pressure and at various temperatures. Results showed that the pressure pulse propagated through the material 10 to 25 times slower than predicted by standard noncondensable gas-flow theory. Numerical simulation supported the hypothesis that the delay was due to adsorption of the steam by the porous matrix.

The purpose of this paper is to develop a linear equation for the simulation of experiments run at 100°C with initial pressures of 0.483 and 0.684 bars and final pressures of 0.951 and 0.963 bars, respectively.

Theory The analytical approach used in this report requires a linearized differential equation for one-dimensional, planar steam flow in a porous medium. The derivation of this equation makes use of equations given by Herkelrath and Moench (1981), reproduced below for convenience.

The governing equation for steam flow was

$$\frac{\partial}{\partial z}\left(\rho_v \frac{KK_{rv}}{\mu_v} \frac{\partial P}{\partial z}\right) = \phi \frac{\partial\left[\rho_v(1-S)\right]}{\partial t} + q' \quad . \qquad (1)$$

Symbols are defined in the nomenclature. A similar equation could be written for the flow of liquid; however, because adsorbed water was assumed immobile and incompressible, the equation reduced to

$$q' = \phi\rho_\ell \frac{\partial S}{\partial t} \quad . \qquad (2)$$

Temperature changes in the porous medium were assumed to occur only as a result of phase changes, hence the following simplified form of the energy equation was used:

$$L_v q' = H_c \frac{\partial T}{\partial t} \quad . \qquad (3)$$

In addition to the above, it was necessary to consider steam pressure as a function not only of temperature but also of the amount of adsorbed water:

$$P = P(T,S) = R(S)\, P_o(T) \quad . \qquad (4)$$

The functional relationship, $R(S)$, was established experimentally for the sample material. It was described by the following empirical relationship:

$$R(S) = 10^{-\left(10^{(A-S)/B}\right)} \quad . \qquad (5)$$

Using the chain rule of partial differentiation (4) is expanded as

$$\frac{\partial P}{\partial t} = \left(\frac{\partial P}{\partial S}\right)_T \frac{\partial S}{\partial t} + \left(\frac{\partial P}{\partial T}\right)_S \frac{\partial T}{\partial t} \quad . \qquad (6)$$

Combining (2) and (3), and substituting the results into (6) yields.

$$\frac{\partial P}{\partial t} = \left[\left(\frac{\partial P}{\partial S}\right)_T + \left(\frac{\partial P}{\partial T}\right)_S \phi\rho_\ell \frac{L_v}{H_c}\right] \frac{\partial S}{\partial t} \quad . \qquad (7)$$

Solving (7) for $\partial S/\partial t$ and substituting in (1) yields

$$\frac{\partial}{\partial z}\left(\rho_v \frac{KK_{rv}}{\mu_v} \frac{\partial P}{\partial z}\right) = \phi \frac{\partial\left[\rho_v (1-S)\right]}{\partial t}$$

$$+ \phi\rho_\ell \left[\left(\frac{\partial P}{\partial S}\right)_T + \left(\frac{\partial P}{\partial T}\right)_S \phi\rho_\ell \frac{L_v}{H_c}\right]^{-1} \frac{\partial P}{\partial t} \quad . \qquad (8)$$

In the absence of vapor–pressure lowering (8) reduces to the equation derived by Moench and Atkinson (1978) for steam flow through porous materials. For the purposes of this paper the first term on the right-hand side of (8) can be neglected. Its omission in this study can be shown to decrease the numerical value of the right-hand side of (8) by at most 3%.

The left-hand side of (8) can be linearized by using pressure squared as the dependent variable. This yields an equation similar to that which describes the flow of noncondensable gas through a porous medium

$$\frac{\partial P^2}{\partial z^2} = \frac{1}{\alpha} \frac{\partial P^2}{\partial t} \qquad (9)$$

where in this case the pressure-dependent diffusivity is defined as:

$$\alpha = \rho_v \frac{KK_{rv}}{\mu_v \phi\rho_\ell} \left[\left(\frac{\partial P}{\partial S}\right)_T + \phi\rho_\ell \frac{L_v}{H_c} \left(\frac{\partial P}{\partial T}\right)_S\right] \quad . \qquad (10)$$

Permeability was found by Herkelrath and Moench (1981) to be a function of pressure as defined by Klinkenberg (1941):

$$K = K_o(1 + b/P) \quad . \qquad (11)$$

The slope of the isothermal vapor–pressure lowering curve (illustrated by Herkelrath and Moench, 1980, fig. 2) obtained from (4) and (5) is

$$\left(\frac{\partial P}{\partial S}\right)_T = - \frac{P \ln P/P_o(T)}{0.4343\ B} \quad . \qquad (12)$$

The change of pressure with temperature at constant saturation obtained from (4) is

$$\left(\frac{\partial P}{\partial T}\right)_S = R(S) \frac{\partial P_o(T)}{\partial T} + P_o(T) \left(\frac{\partial R(S)}{\partial T}\right)_S \quad . \qquad (13)$$

Preliminary data by Herkelrath and Moench (1980, fig. 2) show that the second term on the right-hand side of (13) is small compared with the first term. For purposes of this report it will be neglected. All parameters in (10) are obtained either by experimentation or from the steam tables (Keenan and others, 1969).

Evaluation of (10) shows that a is constant over a limited range of pressures. Thus a linear system is applicable and the governing differential equation can be written as:

$$\frac{\partial^2 P_D^2}{\partial z^2} = \frac{1}{a} \frac{\partial P_D^2}{\partial t} \quad , \quad 0 \le z \le L \qquad (14)$$

where the initial condition is

$$P_D^2 = 0 \quad , \ t=0, \ 0 \le z \le L \qquad (15)$$

and the boundary conditions are

$$P_D^2 = 1 \quad , \quad z=L \qquad (16)$$

$$\frac{\partial P_D^2}{\partial z} = 0 \quad . \quad z=0 \qquad (17)$$

The dimensionless pressure is defined as

$$P_D^2 = \frac{P^2 - P_i^2}{P_f^2 - P_i^2} \qquad (18)$$

The solution to (14) subject to (15)-(17) evaluated at $z=0$ can be written down directly from the solution given by Carslaw and Jaeger (1959, p. 309):

$$P_D^2 = 2\sum_{n=0}^{\infty} (-1)^n \operatorname{erfc}\left[\frac{(2n+1)L}{2\sqrt{\alpha t}}\right] \quad . \qquad (19)$$

Results The apparent pressure-dependent diffusivity defined by (10) was evaluated at 100°C for the sample material described by Herkelrath and Moench (1981). Table 1 shows the values of the measured parameters needed in the calculations. Values of the apparent diffusivity evaluated over the pressure range of 0.05 to 0.95 bars are shown in fig. 1. The apparent steam diffusivity is nearly constant over the pressure range of 0.5 to 0.95 bars.

Steam-flow experiments similar to those described in detail by Herkelrath and Moench (1980, 1981) were run on the same sample of unconsolidated porous material at 100°C using different starting pressures. The experimental results of two runs with starting pressures of 0.483 and 0.684 bars and final pressures of 0.951 and 0.963 bars, respectively, are presented in fig. 2.

Also shown in fig. 2 are the results of computations using (19) and an average diffusivity of 15 cm²/sec. The reasonably close agreement between the experimental and analytical results tends to support the assumptions used in the development of the linearized governing differential equation.

Nomenclature

A & B	fitting factor in relative vapor-pressure function
b	Klinkenberg slip factor
H_c	heat capacity of porous medium

K	permeability
K_o	intrinsic permeability
K_{rv}	relative permeability to steam
L	sample length
L_v	latent heat of vaporization
P	steam pressure
	saturated vapor-pressure function
P_i	initial steam pressure
P_f	final steam pressure
	dimensionless steam pressure
q'	rate of steam adsorption
$R(S)$	relative vapor-pressure function
S	liquid saturation
T	Temperature
t	time
z	distance along sample
α	apparent steam diffusivity
μ_v	viscosity of steam
ϕ	porosity
ρ_v	density of steam
ρ_ℓ	density of liquid

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Table 1. Values of parameters*

A	8.65×10^{-3}	
B	2.30×10^{-2}	
b	0.14	bars
H_r	0.32	$cal/cm^3\,^oC$
K_{rv}	1	
K_o	3.60×10^{-8}	cm^2
L	61	cm
T	100	^oC
ϕ	0.42	

*remaining parameters are known properties of water at the prevailing temperature and pressure

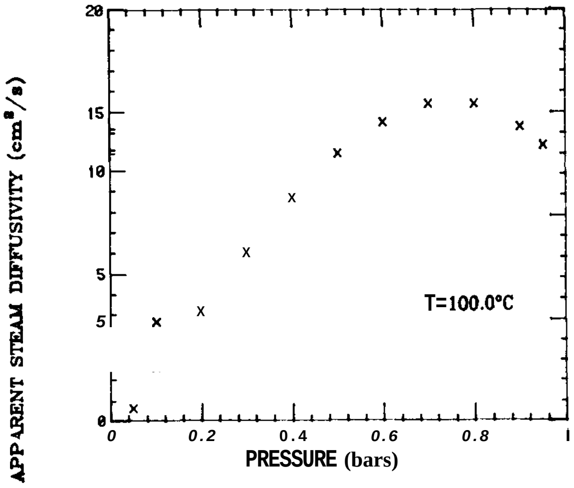

Figure 1. Calculated values of apparent steam diffusivity in the porous sample material versus pressure.

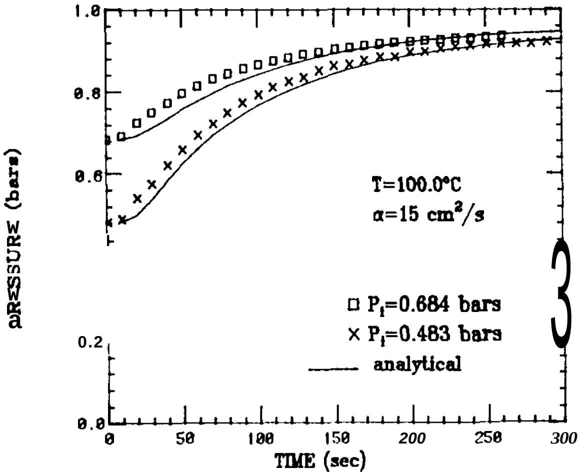

Figure 2. Analytical and experimental steam-pressure buildup at the closed end of the porous cylinder as a function of time since a step increase in pressure was impdsed at the other end.