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DISPERSION IN TRACER FLOW IN FRACTURED GEOTHERMAL SYSTEMS

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Abstract Recent field experiments in Japan have emphasized the importance of performing tracer tests in any geothermal utilization in which reinjection is in use or is planned. This is because rapid short-circuiting between reinjection and production wells may occur due to the fractured nature of the system. In cases where fracturing is such that preferred pathways exist in the reservoir, the result may be a rapid thermal drawdown of the field production. Tracer testing provides a method of evaluating the magnitude of such problems. Previous methods used to analyze the Onuma, Hatchobaru, and Otake tracer tests have used early and long time data, this paper discusses the use of the field concentration/time profile in fractured systems, and the likely forms of dispersion likely to dominate in the process.

Introduction Reinjection of waste hot water is practiced in many liquid-dominated geothermal fields (namely Ahuachapan, Otake, Onuma, Kakkonda, Onikobe, Hatchobaru, Mak-Ban, East Mesa, Brawley, and Raft River). The fundamental purpose of reinjection is to dispose of the unused hot water, although it has often been suggested that the reservoir productivity may be increased concurrently. In fact there has been only scant evidence to show support of reservoir performance by reinjection (see Horne 1981), and in fact in some cases it has been seen to be detrimental to production due to early invasion of the cooler injected water through high permeability paths in the reservoir. Furthermore, observations on the effects of reinjection have emphasized the need to pay close attention to the fractured nature of geothermal reservoirs.

The benevolence or malevolence of reinjection in geothermal reservoirs has been seen to be closely related to the degree of fracturing. The degree of fracturing has been most successfully determined by using tracer tests. For example, tracer tests summarized in Horne (1981) indicated a high degree of fracturing in Wairakei, Kakkonda, and Hatchobaru, a moderate or mixed degree in Onuma and Ahuachapan, and a low degree in Otake. In view of the subsequent experience in reinjection it was concluded that understanding the fracture system through the use of tracers should be the first step in designing a reinjection program. Unfortunately however, the methods of analysis appropriate to tracer flow in fractured systems are not yet fully developed, most surveys to date having used only the early time (or in one instance the late time) data. A method of analyzing the full tracer return profile is demonstrated in this work, and a discussion offered on the form of the appropriate transfer function.

Existing Tracer Analysis Methods The classic petroleum reservoir methods for analysis of tracer tests have commonly been based on uniform "sweep" flow through a porous medium in a given configuration (usually a 5-spot) - see for example Brigham and Smith (1965), Baldwin (1966) and Wagner (1977). In these analyses the system is modelled as a "stack" of non-connecting layers of porous media which are uniform but which nevertheless have differing properties. The tracer "breaks through" different layers at different times, giving rise to the characteristic multiple hump return illustrated in Figure 1. Geothermal systems however show very different returns because of the limitation of flow to fractures and commonly show a single hump return as in Figure 2. The absence of more than one strong tracer return itself emphasizes the highly fractured nature of geothermal reservoirs. It is clearly inappropriate to use the uniform sweep model of the petroleum industry in such instances.

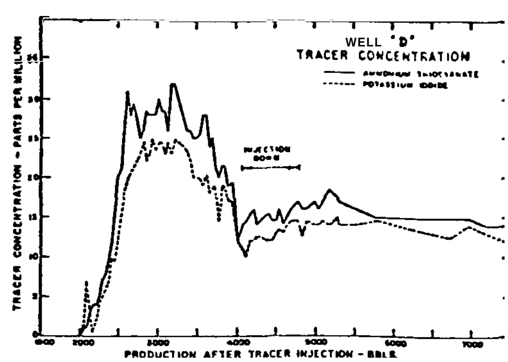

Figure 1: Multiple breakthrough tracer return - from Brigham & Smith (1965)

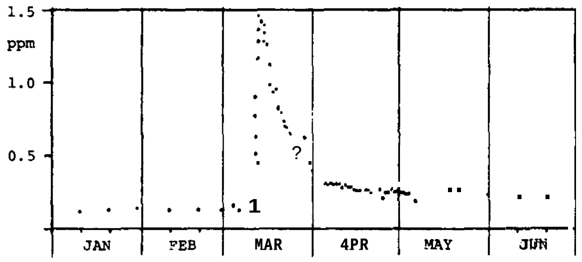

Figure 2: Onuma geothermal tracer test - from Ito, Kubota & Kurosawa (1978)

Without resorting to a flow model there are two important items of informatin which can be derived from a tracer test. The first is simply the speed of first return between an injector and permeability connection between the two. A "connectivity" map of the reservoir can be drawn from such results. The second item is the long term dilution of tracer in a test in which the produced tracer is recycled (as is often the case in an operational system). This long term dilution has been used by Ito, Kubota, and Kurosawa (1978) to estimate the volume of fluid circulating through the system at Onuma field.

These two calculations involve the use of only the early and late measurements in the test. In the petroleum literature, Brigham and Smith (1965) demonstrated a method of "matching" the intermediate time tracer return concentration, essentially by a trial and error apaproach. Their analysis was based on a model of the reservoir as a series of non-connecting layers with different permeabilities which gave rise to separate "peaks' in tracer return. Yuen, Brigham, and Cinco (1979) later extended the method of analysis to calculate the permeabilities of various layers by matching the concentrations at the various peaks with the analytical solution. Both of these studies considered a 5-spot configuration, however the methodology would be applicable to other configurations. Despite the considerable extra information that this "intermediate time" analysis can provide, it still essentially uses only data at the peaks of the response curve. Also, the analysis is based on a layer model that may hold for hydrocarbon reservoirs, but which would be inappropriate for most geothermal systems.

<u>Features of Geothermal Tracer Tests</u> Geothermal reservoirs are usually very highly fractured. As a result, and as an indication of this fact, the tracer response almost always shows just a single peak. Thus, although the early and late time analyses are still possible, the analysis of the single peak concentration would provide little extra information, and does in any case require the formulation of a flow model. Thus, there exists a need to formulate a means of analyzing the shape of the single humped tracer response with specific reference to flow in fractures. An attempt to isolate the features of tracer transport in fractures is reported here.

<u>Tracer Transport in Fractures</u> Methods of signal analysis are readily applicable to the interpretation of tracer return concentration histories, reducing the observed profile to the sum of its component signals. For example, the tracer concentration in a producing well that receives flow from an injection well through two intervening fractures will demonstrate the superposed transfer function corresponding to tracer flow through those two fractures. The difficulty in decoupling the response into its component parts depends on defining the features of those component parts. For example, tester, Bivins, and Potter (1979) describe a method to represent the tracer concentration C at a production point in terms of M independent components, thus:

$$C = \sum_{j=1}^{M} \xi_j C_j (X_m, \theta_j, Pe_j) \qquad (1)$$

where ξ_j is the fraction of flow in "path"; and non-dimensional distance and time are defined by:

$$X_j = \frac{x_j}{L_j} \qquad (2)$$

$$\theta_j = \frac{q_j t}{V_j} \qquad (3)$$

where x_j, L_j, q_j and V_j are the position within, length of, flow rate through and volume of the j-th "path" through the system. Pe_j represents the Peclet number of flow through the j-th path, defined as:

$$Pe_j = \frac{u_j L_j}{\eta_j} \qquad (4)$$

where η_j is the diffusivity (or dispersion coefficient) of tracer during transport.

Tester, Bivins, and Potter (1979) proposed the analysis of N measured values of exit tracer concentration C_i by minimizing the objective function F, where:

$$F = \sum_{i=1}^{N} (C - C_i^*)^2 \qquad (5)$$

and C is given by equation (1). Decision variables will be Pe_j, q_j, and V_j.

This method can straightforwardly provide estimates of the Peclet numbers associated with the various flow paths, and their relative (but not absolute) rates of flaw and relative (but not absolute) values. It does however depend strongly upon the transfer function $C_j(X_j, \ _j, Pe_j)$ assumed in equation (1) for the transport of the tracer. Brigham and Smith (1965) based their determination of the transfer function on flow through a porous medium between wells in a five spot formation. Tester, Bivins, and Potter (1979) determined transfer function for one- and two-dimensional flow through porous media.

Convective Dispersion in Fracture Flow These two studies do not, however, correctly represent the flow through a fracture in that a tracer front is modelled as propagating perpendicularly to the direction of flow. In a fracture, however, in either laminar or turbulent flow, the fluid will be transported faster in the center of the fracture than on the walls (in fact, due to boundary layer effects, it will not be transported along the walls at all). This is illustrated in Figure 3.

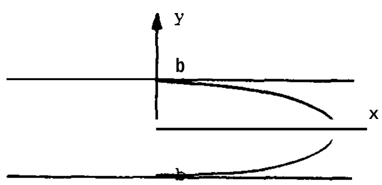

Figure 3: Fracture flow configuration

The profile across the span of the fracture for laminar flow is given by:

$$u(y) = -6\frac{U}{b^2}\left(y^2 - b^2\right) \tag{6}$$

where U is the average velocity and b is the half-width of the crack. Equation (6) is the well known parabolic velocity distribution and gives rise to a maximum velocity at the center of the crack (at $y=0$) of $3/2\ U$.

Now, if a continuous slug of tracer were to be injected at time $t=0$, the distance x moved by the tracer front, assuming no dispersion, will be given by:

$$x(y) = u(y)t \tag{7}$$

and thus the tracer will have "arrived" at x over a range of y given by the equation:

$$-6\frac{U}{b^2}\left(y^2 - b^2\right) = \frac{x}{t} \tag{8}$$

the solution of which is:

$$y = \pm\sqrt{b^2 - \frac{3b^2}{2U}\frac{x}{t}} \tag{9}$$

The mean concentration at point x is then given by:

$$C = \frac{y}{b} \tag{10}$$

$$\text{or} \quad C = \sqrt{1 - \frac{t^*}{t}} \quad \text{for } t>t^* \tag{11}$$

where t^* is the first arrival time of tracer and is given by:

$$t^* = \frac{2}{3}\frac{x}{u} \tag{12}$$

In a practical case, of course, the tracer would not be injected continuously nor would it be injected at a concentration of 100%. Equation (11) may however be used to superpose the behavior of the leading edge and trailing edge of a tracer slug of initial concentration C_o, after which:

$$C = C_o\left[H(t - t^*)\sqrt{1 - \frac{t^*}{t}} - H(t - \Delta t - t^*)\sqrt{1 - \frac{t^*}{t - \Delta t}}\right] \tag{13}$$

where $H(x)$ is the Heaviside step function:

$$H(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases} \tag{14}$$

and t is the length of time the tracer is injected.

Figure 4 shows the normalized tracer return concentration C/C_o as a function of normalized time t/t^* for various values of injection time t/t^*. The similarity between Figures 4 and 2 should be noted.

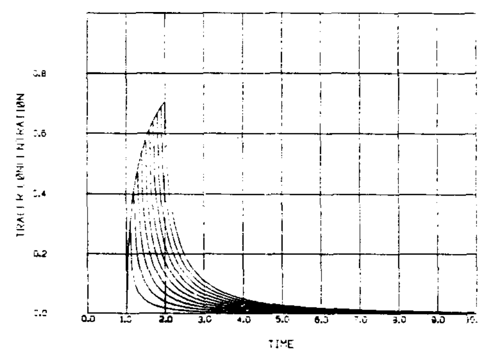

Figure 4: t/t^* 0.1 to 1.0, increments of 0.1

The end result of this non-uniform "convective" displacement of the tracer slug's leading and trailing edges is a dispersion of the tracer slug over the entire distance between the injection point and the observation point. This smearing of the slug may be termed the "convective dispersion" of the tracer, although it must be remembered that the effect is literally to disperse the tracer and not to diffuse it. The "dispersivity" of this process may not be determined in the common sense of the term, however a qualitative impression of its order of magnitude may be estimated. Comparing Equation (13) with the solution for a purely dispersive transport:

$$C = C_o \left\{ \text{erfc} \frac{L}{2\sqrt{\eta t}} - \text{erfc} \frac{L}{2\sqrt{\eta (t+\Delta t)}} \right\} \quad (15)$$

it is seen that, since the error function is roughly linear over its early range, there is a rough correspondence between the reciprocal square root of time terms in the two equations. Thus it may be observed that $4/L^2$ behaves like $1/t^*$ which is itself $1.5U/L$. The effective dispersivity of the convection process is therefore like:

$$\eta_{eff} = \frac{3}{8} UL \quad (16)$$

and the effective Peclet number is always of order $8/3$.

Taylor Dispersion *in* Fracture Flow Even though molecular diffusion in the axial direction is several orders of magnitude smaller than all other effects (typically the Peclet number for molecular diffusion may be 10^{-9} compared to the value $8/3$ determined for convective dispersion), the convective smearing of the tracer gives rise to large concentration gradients across the narrow width of the fracture. With this large concentration gradient molecular diffusion tends to rapidly equalise the tracer concentration across the fracture, thus counteracting the effect of convective dispersion. Figure 5 shows the molecular diffusion of a tracer slug across the width of a cavity.

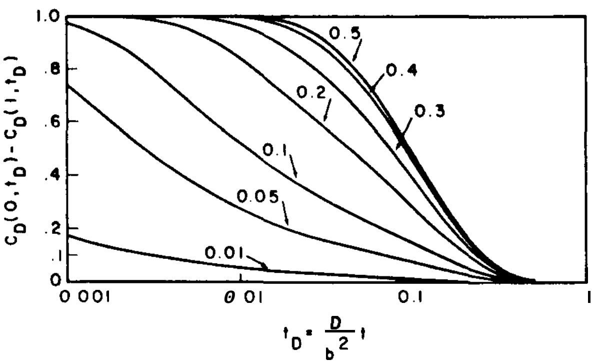

Figure 5: Concentration difference between wall and centerline as a function of time and initial tracer penetration across the fracture

For initial slug widths ranging from 0.01 to 0.5 of the total cavity width, the difference between the centerline and wall concentrations of tracer reduces effectively to zero within a dimensionless time t_D of 0.5 in every case. The molecular diffusivity D may be of order 10^{-5} cm^2/sec and the fracture half-width b may be of order 0.5 mm, suggesting that the transverse diffusion will equalise any concentration differences within 125 seconds (during which time the tracer slug could be considered to move no further than 40 cm). Clearly this transverse diffusion will rapidly overcome the convective dispersion in a field case.

This combination of transverse diffusion and convective dispersion is known as "Taylor Dispersion" and was described for pipe flow by Taylor (1953). The net result of Taylor dispersion is that the tracer front propogates with the mean speed of the flow in spite of the fact that the fastest moving fluid in the center of the channel moves at twice the speed in the case of pipe flow, or 3/2 times the speed in the case of fracture flow. The net longitudinal dispersion was determined by Taylor for pipe flow, and was derived during this investiqation for fracture flow to be:

$$\eta = \frac{2}{105} \frac{b^2 U^2}{D} \quad (17)$$

It should be noted that the Taylor dispersivity is inversely proportional to the molecular diffusivity, hence net dispersion is greater when molecular diffusion is smaller. This analysis cannot be extrapolated to zero molecular diffusivity because of assumptions made in the derivation, however the maximum dispersivity in that case would be that determined in Equation (16).

With a mean flow speed of 3 cm/sec and a fracture width of 1 mm, a typical value of the dispersivity would be 40 cm^2/sec. For a 100 m long fracture this would give rise to a Peclet number of order 1000.

Turbulence Turbulence would tend to increase the rate of tracer diffusion across the fracture and would thus decrease the total effective dispersivity and increase the Peclet number.

Discussion Various dispersion mechanisms have been discussed. Comparing their individual relevance to the field problem of flow through fractures it is seen that: (1) Longitudinal molecular diffusion is insignificant, (2) Taylor dispersion will dominate over convective dispersion, (3) Turbulence will reinforce the effects of Taylor dispersion, (4) The Peclet number for one-dimensional flaw will therefore typically be greater than 1000.

In actual fact, field studies on the fracture system at Los Alamos (Tester, Bivins and Potter, 1979) indicated values of the calculated Peclet numbers of order 2. The further reduction in total transfer Peclet number is due to the fact that the tracer spreads in two or three dimensions away from the injection point and converges again towards the production point, thus being further dispersed. The single field result from Los Alamos suggests that this multi-dimensional dispersion effect is in fact at least 3 orders of magnitude greater than the simple one-dimensional dispersion mechanisms. This conclusion is being tested in this continuing study by calculation of typical flow configurations. However the principal and somewhat unsatisfying conclusion of this work so far is that the tracer return is dominantly determined by the large scale flow configuration.

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