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INTERSTITIAL FLUID PRESSURE SIGNAL PROPAGATION ALONG FRACTURE LADDERS

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Abstract Arrays of interconnected permeable fracture spaces that form fracture ladders propagate small amplitude fluid pressure signals in much the same way as slabs of porous formations. Data from geothermal fields in Iceland indicate that the fracture width there is of the order of 1 to 2 mm and the signal diffusivity 20 to 100 m^2/s. Well interference tests are not likely to furnish data to distinguish between fracture ladders and equivalent porous slabs.

Introduction The majority of medium to high-temperature geothermal systems are embedded in formations of igneous origin that generally are characterized by a fracture dominated fluid conductivity. The fractures are of elastomechanical/tectonic and/or thermoelastic or possibly chemoelastic origin. The fracture conductivity is invariably highly heterogeneous, anisotropic and is quite often confined to flat sheet-like structures such as fault zones and volcanic dikes. Quantitative relations relevant to axisymmetric Darcy type flow in homogeneous/isotropic porous media generally do not apply to such situations and an uncritical standard type interpretation of well test data from fractured reservoirs is therefore likely to lead to faulty conclusions. Unfortunately, since little is known about the dimensions and distribution of fractures in the various types of natural settings, it is difficult or even impossible to derive relevant quantitative relations. It is, nevertheless, of considerable interest to obtain some measure of the discrepancy that would result from an application of the standard interpretational procedures. The purpose of this short note is to discuss a few very simple concepts and relations that are useful in the present context.

The Fracture Ladder For the present purpose, we will consider a specific case of a composite fracture conductor consisting of a linear array of interconnected individual fracture spaces of similar dimensions as displayed in Fig. 1. We will refer to this system as a fracture ladder. The individual fractures or ladder-elements are assumed to have a quasi-rectangular shape with a characteristic edge length L. The two surfaces are welded together at the edges. The width of the open fracture space may vary over the L x L element area, and there may even be some asperities where the opposite surfaces meet, but we assume that they touch without a solid weld. From the elastomechanical point of view, the fracture element acts as an open space of an edge length L. Moreover, we assume that with regard to fluid flow, the fracture element has a well defined average flow width h. No specific assumptions have to be made as to the type of interconnection except that the fluid can flow freely between adjacent ladder elements and that the specific flow conductance can be taken to be approximately uniform over 'the length of the ladder. Obviously, it is possible to generalize this model by envisioning a system of parallel ladders that are interconnected along their entire length and form a fracture sheet.

The principal physical parameters of the linear ladder consisting of one strand of elements are easily defined. At steady-state laminar flow conditions, the vertically integrated fluid conductivity of a fracture space of width h between two parallel planes that would be referred to as the transmissivity c_F is obtained on the basis of the well known cubic law (Lamb, 1932)

$$c_F = h^3/12\nu \qquad (1)$$

where ν is the kinematic viscosity of the fluid. At these conditions, the mass flow through a unit length of the fracture is $q = c_F \nabla p$ where ∇p is the pressure gradient, and hence the local flow over the ladder is $Q = Lc_F \nabla p$.

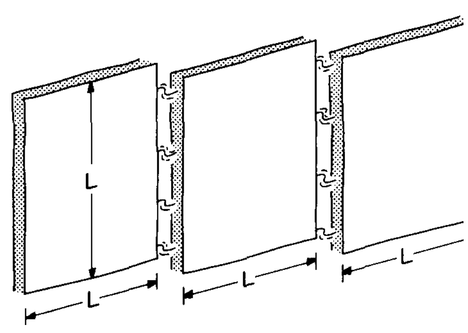

Figure 1 The Fracture Ladder

To obtain the hydraulic capacitivity or storage coefficient of the ladder, we assume that the element walls are elastic Hookean with a rigidity μ. Moreover, let the volume elastance of a fracture space of volume V be defined by $e = dV/dp$ where p is the internal pressure that is assumed to be uniform, Since no analytical expression is available for the elastance of a rectangular fracture element, we will resort to approximating the element by a circular or penny-shaped element of equal area such that the diameter is 1.12L. The elastance of the penny-shaped cavity of diameter d has been obtained by Sneddon (1346) as $e = d^3/4\mu$ where Poisson's relation of equal Lame parameters has been assumed. Based on this result the elastance of a fracture element would approximately be $e = (1.12L)^3/4\mu$ or about $L^3/3\mu$. Hence, the elastance per unit area, that is, the capacitivity is

$$s_F = (L/3\mu) + h\kappa \quad (2)$$

where κ is the compressibility of the fluid. We can usually take that μ is of the order of 10^{10} to 2×10^{10} Pa and assuming that the fluid is liquid water with $\kappa = 5 \times 10^{-10}$ Pa^{-1}, the product $\kappa\mu$ = 5 to 10. The second term on the right of *(2)* can then be neglected when L>>30h. In general, this condition holds and we will therefore simplify the expression for s_F by neglecting the fluid compressibility term. It is to be noted that the above expression for s_F neglects the possible presence of satellite fracture that may contribute to the capacitivity.

On the basis of (1) and the simplified version of *(2)* follows the laminar flow diffusivity of the ladder

$$a_F = c_F/\rho s_F = h^3\mu/4L\eta \quad (3)$$

where η is the absolute viscosity of the fluid. Moreover, there may be leakage from the fracture ladder into the adjacent formation. On a linear laminar flow model, the fluid loss per unit area of the ladder would be characterized by a coefficient b such that the leakage is bp where p is the fluid pressure in the ladder. We have no way of arriving at any expressions for this coefficient that has to be treated as a purely experimental parameter.

The Ladder and the Porous Slab It is interesting to compare the parameters of the fracture ladder to those of a homogeneous/isotropic porous slab with Darcy type flow of the thickness H, permeability k and hydraulic capacitivity (storage coefficient) s. The thickness of the slab of equal transmissivity is obtained by

$$kH/\nu = h^3/12\nu \quad (4)$$

such that

$$H = h^3/12k \quad (5)$$

Moreover, assuming equal transmissivity, the ratio of the diffusivities is

$$a_S/a_F = L/3\mu sH \quad (6)$$

Finally assuming equal diffusivities, the ratio of the transmissivities is

$$c_S/c_F = 3\mu sH/L \quad (7)$$

Borehole/Fracture Contacts It is quite evident that because of the small cross sections available, the local flow-velocities from fracture spaces into boreholes may be quite high and the flow regime therefore highly turbulent. The above relation for the laminar type transmissivity is then invalid and has to be revised. The resulting relatively large fracture/borehole contact resistance can be derived as follows.

Consider a borehole of diameter D which cuts a horizontal fracture of width h as shown in Fig. 2. Let the fluid be incompressible, of density ρ and the mass flow out of the fracture be M. Moreover, let the fluid pressure at a distance r from the center of the hole be p(r) and the fluid velocity there be v(r).

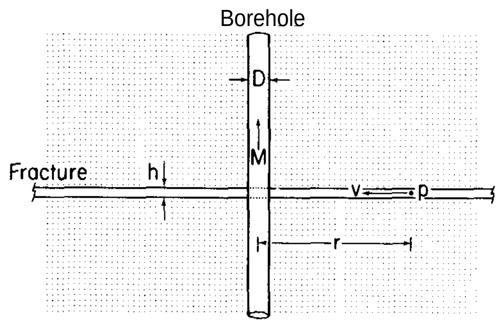

Figure 2 Borehole/Fracture Contact

We have then

$$M = 2\pi rh\rho v \quad \text{or} \quad v = M/2\pi rh\rho \quad (8)$$

The pressure loss over the distance dr is due to the conversion of potential energy into kinetic energy and friction heat, viz.

$$dp = -\rho v dv + (f\rho v^2 dr/2h) \quad (9)$$

the wall friction is represented by the second term on the right of this equation that is derived in the same manner as for the case of pipes and where f is the friction coefficient of the fracture. Assuming that the formation pressure at a large distance from the borehole is p_o this equation is easily integrated for p and we obtain

$$p = p_o - (M^2/8\pi^2h^2\rho)[(1/r^2) + (f/hr)] \quad (10)$$

If the pressure in the borehole is p_b, the following expression is obtained for the mass flow into the hole

$$M = \pi\sqrt{2}\, hD\, [\rho(p_o-p_b)/(1+(fD/2h))]^{1/2} \quad (11)$$

Abbreviating $(p_o-p_b) = \Delta p$ and $(1/\pi\sqrt{2}) = 0.23$, we define the contact resistance of the borehole

$$R = \Delta p/M = (0.23/hD)[(\Delta p/\rho)(1+(fD/2h))]^{1/2} \quad (12)$$

Clearly, these results hold only for the turbulent region around the borehole. Little data is available on the values of the friction coefficient **f** for natural fractures, but based on experimental data for pipes with rough walls, we can expect that $f \sim 0.05$ to 0.10 (see, for example Moody, 1947). It should be pointed out that because of the quadratic terms in (9), the mass flow M is not a linear function of Δp and R therefore depends on Δp. The principal application of equation (11) is for the estimating of the fracture width h in field cases where M and Ap are known.

Field Data Little information is available on the dimensions of fractures in nature. Perhaps the most accessible extensive data is on some of the geothermal reservoirs in Iceland (Thorsteinsson, 1976, Bjornsson, 1979). It is well known that the hydrological systems of Iceland are embedded in fracture dominated flood-basalts of late Tertiary to Pleistocene age. This material enables us to make attempts at estimating fracture widths in some of the Iceland reservoirs. Very briefly, we can proceed as follows.

(1) Borehole production data in various geothermal fields in Iceland indicate that major fracture conductors can produce mass flows from a few up to a few tens of kg/s at pressure differentials of a few 10^5 Pa. A figure of M = 10 kg/s at $\Delta p = 4 \times 10^5$ Pa is quite representative of the performance of a productive individual fracture in a borehole of D = 0.22 m. Assuming that the conditions set forth in the previous section hold, equation (11) with **f** = 0.06 gives then an estimate of h = 1.3 mm.

(2) Well interference testing in 4 geothermal fields in Southwestern Iceland have yielded transmissivities of $c_F = 2.6 \times 10^{-4}$ to 25×10^{-4} ms. Reinterpreting these results in terms of single fracture systems flowing water at 100°C with $\nu = 3 \times 10^{-7}$ m²/s, we obtain with the help of equation (1) above the estimate of h = 1 to *2* mm. Moreover, during the same tests, unit area capacitivities (storage coefficients) of $s_F = 1 \times 10^{-8}$ to 4×10^{-8} m/Pa were obtained. Equation (2) then yields estimates of L = 400 to 1600 m and the resulting diffusivities are a_F = 20 to 100 m²/s.

(3) It is of interest to note that fracture widths can also be estimated on the basis of the overall flow resistance in individual geothermal systems. Knowing the distance of recharge, the available pressure differential and other parameters, it is possible to arrive at estimates of an average h. The present writer has obtained along these lines results that compare well with the above estimates. Unfortunately, space does not permit a discussion of this method.

Signal Propagation On the above premises, we now arrive at the basic equation for the propagation of pressure signals along a fracture ladder. Assuming a homogeneous/isotropic flat ladder and neglecting inertia forces, the basic equation is the diffusion equation in two spatial dimensions for the fluid pressure **p**, viz. ,

$$\rho s_F \partial_t p + bp + c_F \Pi_2 p = m \qquad (13)$$

where ρ is the density of the fluid, $\Pi_2 = -\nabla_2$ is the Laplacian in two dimensions and m is a source density. The leakage term on the left can be eliminated by a transformation $p = u \cdot \exp(-bt)$ where u is a new dependent variable. The principal small amplitude propagation parameters, the penetration depth and the skin depth (assuming b = 0)

$$d_p = (a_F t)^{1/2} \quad \text{and} \quad d_s = (2a_F/\omega)^{1/2} \qquad (14)$$

where **t** is time and ω the angular frequency, follow then in the usual way.

The pressure signal diffusivities indicated by the Iceland data are quite high and signal propagation therefore rapid. For example, at a_F = 50 m²/s the penetration depth for a period of 10^4s is about 700 m. It is interesting to note that the fracture structures simulate porous slabs of H = 20 to 60 m and permeabilities of the order of $k = 10^{-11}$ = 10 darcy. The global permeabilities of the host systems appear, nevertheless, to be orders of magnitude smaller (Bodvarsson and Zais, 1978).

Under the circumstances assumed here, an interference test would not provide data to distinguish between the two models, the ladder and the slab, and an interpretation on the basis of slabs only can therefore lead *to* erroneous conclusions.

References

Bjornsson, A. , 1979, (editor), "The Akureyri District Heating System," Report OS JHD 7827 by the National Energy Authority, Reykjavik.

Bodvarsson, G. and E. Zais, 1978, "A Field Example of Free Surface Testing," Workshop on Geothermal Reservoir Engineering, December, 1978, Stanford University, Stanford, CA.

Lamb, H., 1932, "Hydrodynamics," 6th ed. , Dover Publications, New York, N.Y., 738 pp.

Moody, L. F., "An approximate formula for Pipe Friction Factors," Mech. Eng. 69 (1947) 1005.

Sneddon, I. N., 1946, "The Distribution of Stress in the Neighbourhood of a Crack in an Elastic Solid," Proc. Roy. Soc., 187, 229.

Thorsteinsson, T. , 1976, 'Redevelopment of the Reykir Hydrothermal System in Southwestern Iceland," Second U.N. Symposium on the Development and Use of Geothermal Resources, San Francisco, 1975.