# PRODUCTION DECLINE ANALYSIS USING INFLUENCE FUNCTIONS

Elliot J. Zais
Elliot Zais & Assoc., Inc.
7915 NW Siskin Drive
Corvallis OR 97330

We previously reported (Zais, 1979) that Arps's exponential equation works quite well on geothermal production data. The hyperbolic equation should probably not be used. In this paper we show the progress made in using influence functions to describe reservoir production behavior.

Influence functions arise in many areas of science and engineering under the names of Green's function, memory function, response function, and resistance function. They are used to relate the response of a system to an input. In a geothermal reservoir we can find an influence function relating reservoir pressure to production rate. Once this function is calculated, it can be used to predict the behavior of the reservoir. van Everdingen and Hurst (1949) developed an influence function method for aquifer drive petroleum reservoirs using Laplace transforms. Hutchinson and Sikora (1959) and Jargon and van Poolen (1965) developed algebraic water drive analysis methods for reservoirs and individual wells, respectively.

Coats et al (1964) made a major improvement over the algebraic methods mentioned above by requiring derivative constraints. His formulation may be written as

$$\Delta p = \int q(\tau)\, \frac{dF(t-\tau)}{d\tau} d\tau = \int \frac{dq(t-\tau)}{d\tau} F(\tau)\, d\tau \qquad (1)$$

$$\Delta p_i \approx \Sigma\, (q_j - q_{j-1}) F_{i-j+1} \qquad (2)$$

with the constraints

$$F > 0,\ t > 0 \qquad (3)$$

$$\frac{dF}{dt} \geq 0$$

$$\frac{d^2F}{dt^2} \leq 0$$

These constraints are necessary to ensure that the function derived is physically meaningful. We analyzed some data from Cerro Prieto both with and without the constraints. The derived influence functions are shown in Figure 1. The irregular curve is the "best fit" to data, but it is physically meaningless and cannot be extrapolated.

Some other results obtained by Coats's method are shown in Figures 2 and 3. The $\rho$ is a measure of goodness of fit and is defined as

$$\rho = \frac{1}{n} \Sigma \frac{1}{\Delta p_i}(u_i + v_i) \qquad (4)$$

where $u_i$ and $v_i$ are slack variables in the linear programming formulation.

We used three different kinds of extrapolations for the influence functions assuming 1) a bounded aquifer, 2) an aquifer which crops out, and 3) an infinite aquifer. Geologic data can help in choosing the correct extrapolation.

The methods so far discussed are all "black box" methods because no physical model of the reservoir mechanism is assumed. We also studied a linearized free surface (LFS) model proposed by Bodvarsson (1977). If a reservoir has a free liquid surface with relatively small vertical movement, the equations describing the movement can be linearized. The equation for surface drawdown at a general field point P becomes

$$h(t) = \int G(t - \tau)q(\tau)d\tau \qquad (5)$$

$$\approx \Sigma G(t - \tau)q(\tau)\Delta\tau \qquad (6)$$

q(t) is a production rate and G(t) is a Green's function which can be defined for an infinite half space, a reservoir with an impermeable bottom layer, and other configurations as necessary. The equations for the first two cases are

$$G = \frac{-1}{2\pi\phi\rho} (wt + d)(x^2 + y^2 + (wt + d)^2)^{-3/2}, \quad t > 0 \qquad (7)$$

$$G = \frac{-1}{2\pi\phi\rho}\left[(wt + d)(x^2 + y^2 + (wt + d)^2)^{-3/2} + \frac{2H - dwt}{(x^2 + y^2 + (2H - d + wt)^2)}^{-3/2}\right] \qquad (8)$$

w is the sinking velocity and equals $\frac{kg}{\gamma\phi}$ = permeability x gravity divided by kinematic viscosity x porosity. The other symbols are distances as shown in Figures 4a and 4b.

Only the Wairakei data set (Pritchett et al) was complete enough for us to use the LFS method. The results are shown in Figure 5. Curve 1 is the observed pressure drawdown, curve 2 the calculated drawdown using equation 7, and curve 3 the calculated drawdown using equation 8. The fit is quite good over 25 years.

The full results of this project are available in Zais and Bodvarsson (1980).

Acknowledgements

This work was performed under a DOE contract administered by the Lawrence Berkeley Laboratory. We thank them for permission to present this report. Staff and facilities support were provided by the Geophysics Group, School of Oceanography, Oregon State University, Corvallis Oregon. Thanks especially to Gunnar Bodvarsson and his graduate student, William Avera.

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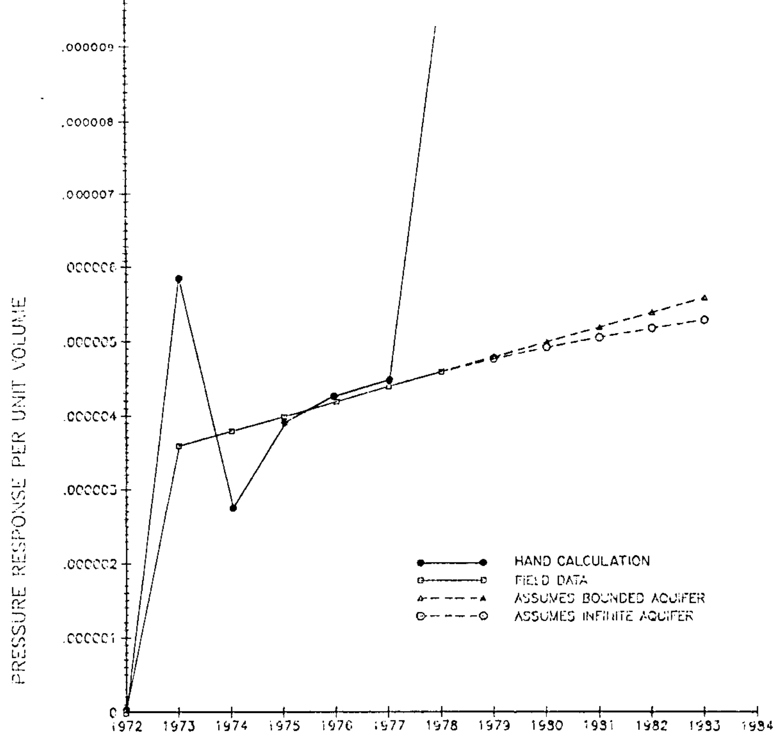
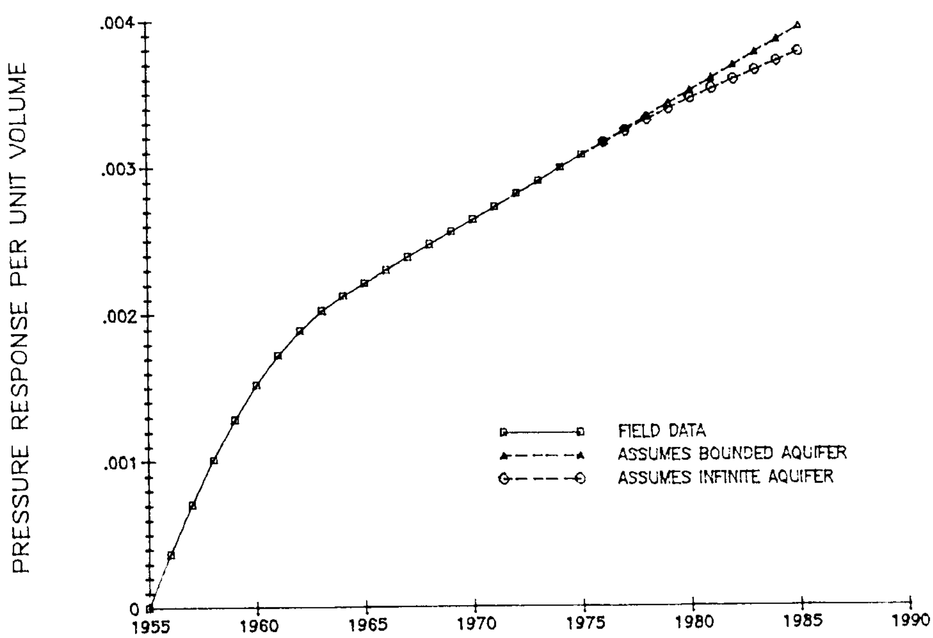


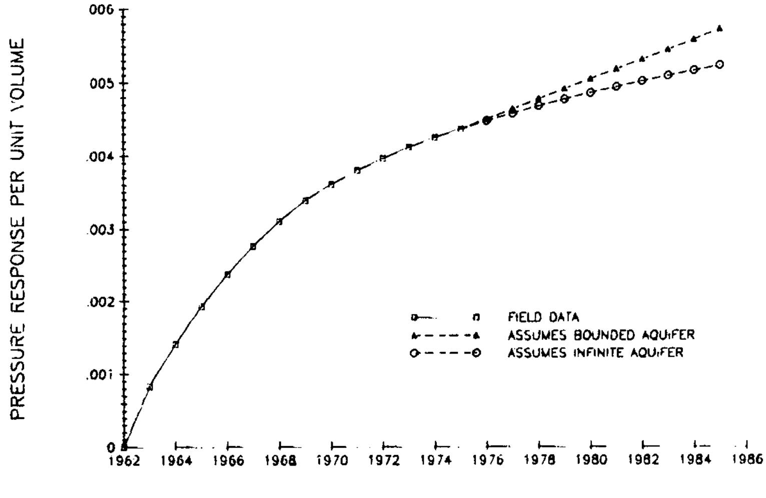

LIQUID INFLUENCE FUNCTION -- CERRO PRIETO TOTAL FIELD -- 1972-1978

**Figure 1** (Fitness measure $\rho$ =.100058)



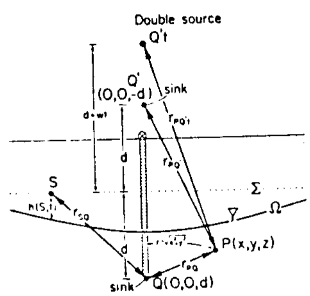

INFLUENCE FUNCTION -- WAIRAKEI TOTAL FIELD -- 1955 TO 1975

**Figure 2** (Fitness measure $\rho$ =.195863)

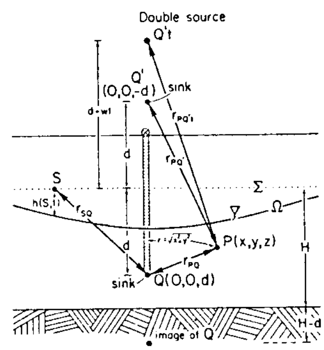



INFLUENCE FUNCTION -- BORE83 -- 1962 TO 1975

**Figure 3** (Fitness measure $\rho$ =.023756)




4a. Infinite half space for linearized free surface method.




4b. Reservoir half space for linearized free surface method with bottom layer. Q", the image of the image of Q with respect to the equilibrium Σ plane is not included. Q" t, the moving image of the image of Q, is also not included.

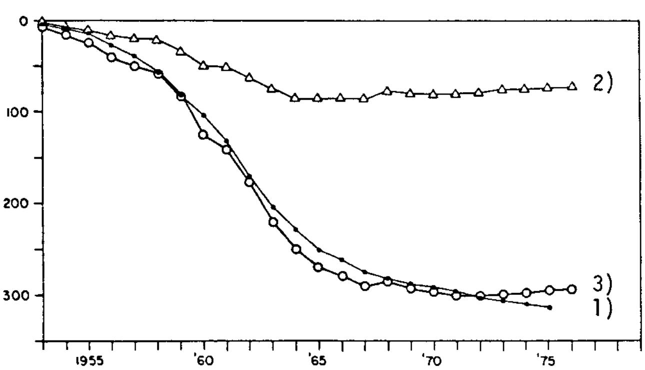



Fig. 5 Linearized free surface fits to Wairakei data