IS GEOTHERMAL SIMULATION A "CATASTROPHE"?

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INTRODUCTION

All numerical simulators of geothermal reservoirs depend upon an accurate representation of the thermodynamics of steam–water systems. These relationships are required to render tractable the system of balance equations derived from the physics of flow through porous media. While it is generally recognized that the steam–water system (i.e. two phase) is not in thermodynamic equilibrium, equilibrium thermodynamics are employed in its description. In this paper, we present an alternative view based on non–equilibrium thermodynamics. The underpinnings of this approach are found in a branch of topology generally referred to as "catastrophe theory". [Thom, 1975]

THERMODYNAMICS

Consider the thermodynamic relationships presented in figure 1. Generally available theory dictates the use of the curve (A–B–D–F–G) in desribing the change in density encountered in moving from the single–phase water region (A–B) through the two phase region (B–D–F) to the single phase steam region (F–G). Thus, there is one pressure and temperature for the steam–water mixture irrespective of its liquid--gas composition. This is the curve (A–B–D–F–G) that is now employed in all geothermal simulators known to the authors. Note that the "kinks" encountered at points B and F lead to discontinuous derivatives with respect to the thermodynamic variables; this, in turn, generates serious numerical problems.

With the recognition that "metastable" water and steam states could, and indeed do, exist in the two–phase region, attention focused on the use of Van der Waals' equation to describe the thermodynamics of the two phase region. This curve (A–B–C–D–E–F–G) indicates that metastable water can exist along (B–C) and metastable steam along (E–F). The curve (C–D–E) does not correspond to a stable or even metastable state and is never observed. While these curves are of theoretical interest they do not, in and of themselves, enhance our understanding of the non–equilibrium two–phase region because the dynamics of the system are lacking.

CATASTROPHE SURFACES

The dynamics of the two–phase region can be uncovered using catastrophe theory. While the mathematical foundations of catastrophe theory are rather abstract, the practical ramifications are easily understood. The catastrophe surface representing the Van der Waals' equation for the steam–water system is presented in figure 2. It is simply the three–dimensional representation of the information presented in figure 1; temperature has been added as a third coordinate. In the nomenclature of catastrophe theory this surface is referred to as the "slow manifold".

The information presented in figures 1 and 2 is combined in figure 3. The letter nomenclature is the same as that appearing in figure 1. The reader is encouraged at this point to examine the three figures until the surface appearing in figure 2 is evident in figure 3. It is this figure that will guide us through the two phase region.

THERMODYNAMIC EVOLUTION

We begin by identifying a location in thermodynamic space of figure 3 designated by the numeral 1. This represents the initial condition or state at a physical point in our geothermal simulator. In this instance the point is in the hot water region of the well documented Arihara experiment [Arihara et al, 1976]. As the numerical simulation of this experiment evolves through time the pressure decreases dramatically and the thermodynamic state of the point of observation evolves along the trajectory defined by the curve (1–2). At the point denoted by the numeral 2, a second phase begins to appear. According to the catastrophe theory concept at this point, the specific volume of the steam phase is denoted by the thermodynamic state at point 4 and that of the water phase by the location 2. The trajectory continues along the line (2–3). During this interval of time the evolution of the specific volume of the water phase is described by the curve (2–3) and that of the steam phase by the curve (4–5). At the point 3, the thermodynamic trajectory describing the behavior at our observation point suddenly changes. Suddenly the state moves along the line (3–5) and only steam exists at this point in the system. Thereafter, the evolution of the state of the system at the observation point is described by the trajectory (5–6).

While this description may appear quite straight–forward, it is certainly unorthodox. It proposes an evolution in water and steam densities within the two–phase region whereas current thinking assumes the steam and water densities do not change within the two phase region and are described by the two points 2 and 5 respectively as when the two fluids are maintained at equilibrium. The difference in our approach arises from our intention to model behavior in the two–phase region when the fluids are not in equilibrium. Our approach also requires the existence of different pressures for water and steam within the two–phase region. In other words, the pressure of the steam associated with the point 4 is quite different than that of the water described by point 2 although these thermodynamic states would coexist in the proposed thermodynamic model. The existence of a higher pressure in the steam phase than the water phase is apparent to anyone who has watched a pan of boiling water. Because the steam forms a bubble, it must have a higher pressure than the surrounding water.

PRACTICAL SIGNIFICANCE

Let us now investigate the practical ramifications of the above theoretical argument. Figure 4 presents saturation profiles computed for the Arihara experiment using the standard approach and the methodology founded on Van der Waals' equation and catastrophe theory. While the solutions are similar in shape, they are quite different in magnitude. Unfortunately, because the exact solution is unknown we cannot determine unequivocally which solution is more accurate.

A second more subtle difference in the two approaches involves the numerical treatment of the phase change. Because there is no sudden discontinuity in the derivatives of the thermodynamic variables when one crosses the two–phase boundary, the non–linear aspects of the problem are less troublesome. Early test runs suggest an order of magnitude larger time steps can be accommodated with the new approach as compared to the same simulator formulated using the standard equilibrium thermodynamic methodology.

CONCLUSIONS

Geothermal simulation which accounts for the non–equilibrium thermodynamic nature of the steam–water system generates saturation profiles similar to but distinctly different from those obtained using standard methodology. The approach is intuitively simple, mathematically rigorous, and gives rise to a system of algebraic equations more amenable to solution than those generally encountered in alternative formulations.

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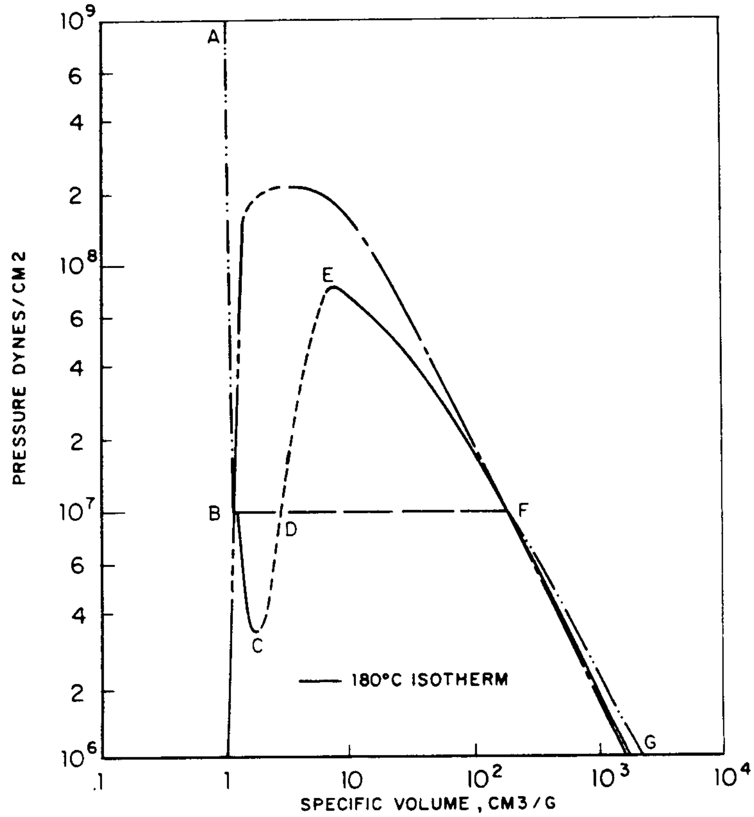

Figure 1: Thermodynamic relationships for a steam-water system employing equilibrium theory and Van der Waals' state equation.

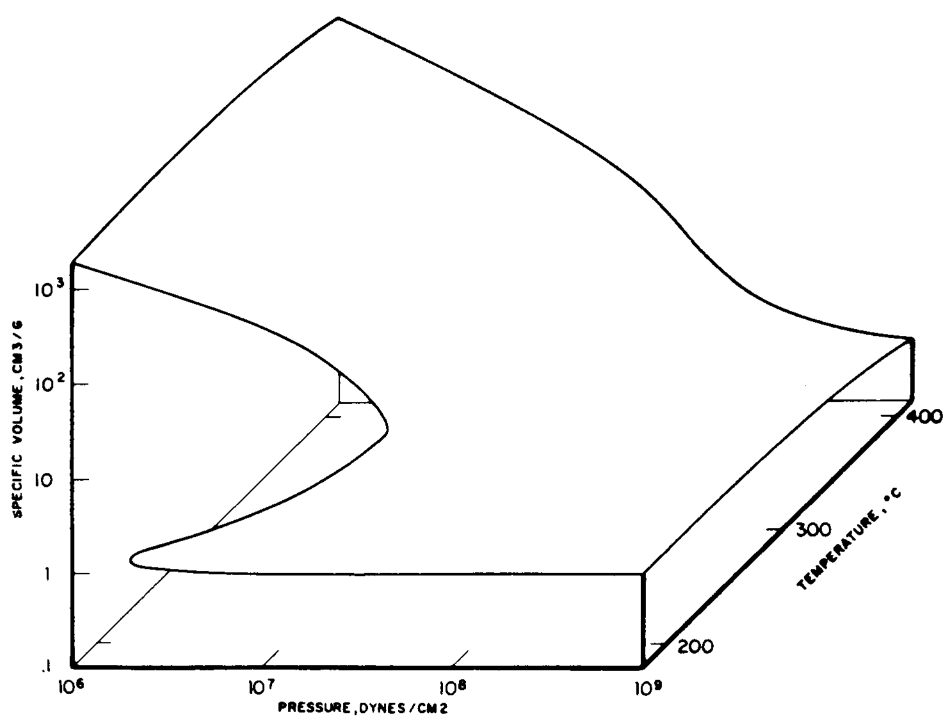

Figure 2: Geometry of the Van der Waals' state equation.

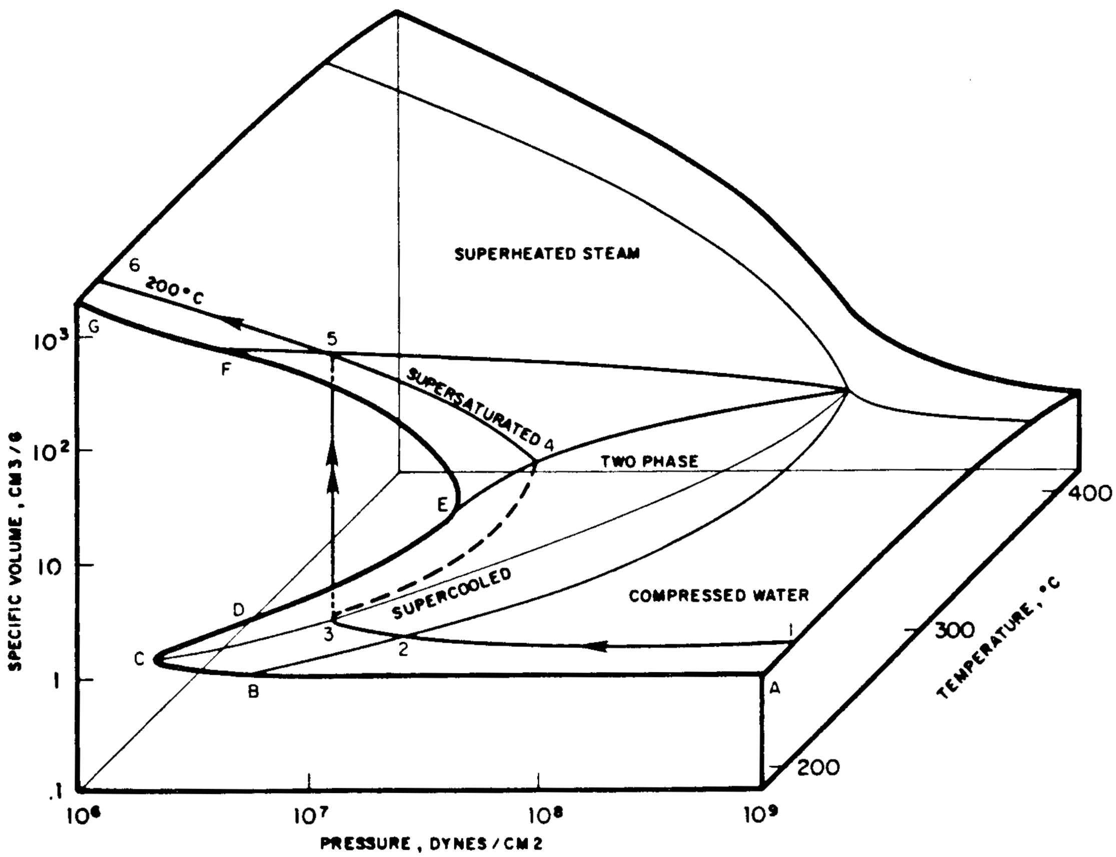

Figure 3: Morphological representation of steam-water dynamics as the Riemann-Hugoniot catastrophe.

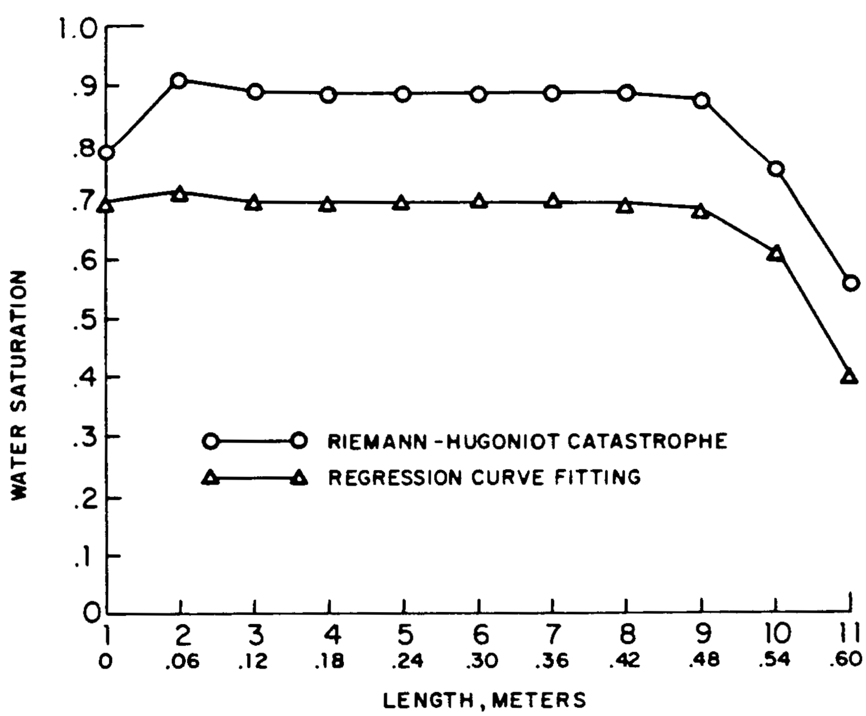

Figure 4: Contrast in saturation profiles from topological and standard statistical estimates of thermodynamic variables for the two-phase domain.