Three-Dimensional Geothermal Reservoir Simulation

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The authors present a three-dimensional model for reservoir simulation to show some behaviors of fluid flow in a geothermal reservoir, assuming water influx and heat conduction from heat sources under the reservoir.

Basic Equations The following three equations describe a system, where mass transfer and heat conduction occur.

Mass conservation equation

$$-\frac{\partial}{\partial t}(\phi S_w \rho_w + \phi S_g \rho_g) = div(\rho_w \vec{v}_w + \rho_g \vec{v}_g) + q_w + q_g + q_{we} \quad \text{----- (1)}$$

Energy balance equation

$$-\frac{\partial}{\partial t}[\phi S_w \rho_w U_w + \phi S_g \rho_g U_g + (1-\phi)(\rho C_p)_f T] = div(H_w \rho_w \vec{v}_w + H_g \rho_g \vec{v}_g) + div\,\vec{t} + q_H + q_L \quad \text{----- (2)}$$

Equation of state (in case of water-steam equilibrium)

$$P = P_s(T) \quad \text{----- (3)}$$

where, $\vec{v} = -\frac{K k_r}{\mu}\nabla\Phi$, $\vec{t} = -T_c \nabla T$ and $S_w + S_g = 1$

The boundary conditions for solving the above equations are $\partial\Phi/\partial n = 0$ and $\partial T/\partial n = 0$ for mass and heat flow, respectively. Both mass and heat production terms are also considered to account for water encroachment and heat flow from the boundaries.

Potential equilibrium and heat equilibrium are adopted for the initial conditions which in turn imply no mass flow and steady state heat flow.

Difference Equations Equations 1 through 3 are approximated by the finite difference equations as follows.[3)]

$$\frac{V}{\Delta t}\delta(\phi\rho_w S_w + \phi\rho_g S_g) = \Delta T_w \Delta\Phi_w + \Delta T_g \Delta\Phi_g - Q_w - Q_g - Q_{we} \quad \text{----- (4)}$$

$$\frac{V}{\Delta t}\delta[\phi\rho_w S_w U_w + \phi\rho_g S_g U_g + (1-\phi)(\rho C_p)_f T] = \Delta H_w T_w \Delta\Phi_w + \Delta H_g T_g \Delta\Phi_g + \Delta T_c \Delta T - Q_H - Q_L \quad \text{----- (5)}$$

$$-P'_{ST}\,\delta T + \delta P = -P^n \qquad \text{----- (6)}$$

The right hand sides of the eqs. 4 and 5 are expressed in terms of δP using $\Delta\Phi = \Delta\delta P + \Delta P^n - \rho g\,\Delta Z$. The eqs. 4 and 5 can be written in the following matrix form[2] with δS_w, δT and δP as independent variables.

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}\begin{pmatrix} \delta S_w \\ \delta T \\ \delta P \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \\ 0 \end{pmatrix} + \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} \qquad \text{----- (7)}$$

where, $Y_1 = \Delta(T_w + T_g)\Delta\delta P$ and $Y_2 = \Delta(H_w T_w + H_g T_g)\Delta\delta P$

By elimination, eq. 7 is transformed into

$$\begin{pmatrix} 1 & C'_{12} & C'_{13} \\ 0 & 1 & C'_{23} \\ 0 & 0 & C'_{33} \end{pmatrix}\begin{pmatrix} \delta S_w \\ \delta T \\ \delta P \end{pmatrix}$$

$$= \begin{pmatrix} B_{11} & 0 & 0 \\ B_{21} & B_{22} & 0 \\ B_{31} & B_{32} & B_{33} \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \\ 0 \end{pmatrix} + \begin{pmatrix} R'_1 \\ R'_2 \\ R'_3 \end{pmatrix}$$

The third row of the above equation contains only one independent variable, δP, which satisfies the following set of finite difference equations.

$$f\,\delta P_{k-1} + d\,\delta P_{j-1} + b\,\delta P_{i-1} + a\,\delta P + c\,\delta P_{i+1} + e\,\delta P_{j+1} + g\,\delta P_{k+1} = R'_3$$

These equations are solved by the direct method.[6] Fig. 1 is a simplified flow chart that shows the program's basic logic.

<u>Heat Loss</u> Heat flow perpendicular to the top and bottom boundaries is assumed. In calculating heat flow at the boundaries, some more blocks are added above and below the reservoir.

The heat conduction equation,

$$\frac{\partial}{\partial z}\left(T_c \frac{\partial T}{\partial z}\right) = \rho C_p \frac{\partial T}{\partial t}$$

is solved in such blocks with appropriate boundary and initial conditions at the newly formed boundaries. In this procedure, the heat flow at the new time step calculated with the following equation.

$$Q_L^{n+1} = Q_L^n + \alpha\,\delta T, \quad \alpha = A\cdot T_B/L\cdot \mathrm{erf}\,(L/2\sqrt{\kappa\Delta t})$$

Water Influx Based on the solution of the diffusivity equation for the linear flow case by van Everdingen and Hurst,[5] water influx is

$$Q_{we}^{n+1} = \frac{A\phi C_w(\Delta P + \Delta P^n)Q(t) - A\phi C_w \Delta P^n Q(t^n)}{\Delta t}$$

The reservoir is divided into 10 x 10 x 5 blocks in X - Y - Z directions, respectively (Fig. 2). The size of the blocks and physical properties of the rock and fluid are shown in Table 1. For simplicity, the model treats an initially hot-water system and the bottom hole pressure of the well is assumed to be constant. Plots of potential distributions at various times are shown in Fig. 3 without water influx, while Fig. 4 with water influx.

NOMENCLATURE

A = cross sectional area, cm^2
C = compressibility, vol/vol-atm
C_p = specific heat, cal/g-°C
H = enthalpy, cal/g
g = gravitatinal acceleration
K = absolute permeability, darcy
k_r = relative permeability
T_c = thermal conductivity, cal/cm-°C-sec
P = pressure, atm
Q = production rate, g/sec
Q_H = enthalpy production rate, $H_wQ_w + H_gQ_g$, cal/sec
Q_L = rate of heat loss to surroundings, cal/sec
$Q(t)$ = fluid influx, dimensionless
S_w = saturation, fraction
t = time, sec
T = temperature, °C
T_w = water transmissibility, $\rho \cdot AKk_r/\mu L$, g/atm-sec
T_g = steam transmissibility, g/atm-sec
U = internal energy, cal/g
V = bulk volume, cm^3
Z = depth, cm
Φ = potential, $P - \int \rho g \, dZ$, atm
ϕ = porosity, fraction
δ = time difference, $\delta P = P^{n+1} - P^n$
Δt = time increment, $t^{n+1} - t^n$
μ = viscosity, c.p
ρ = density, g/cm^3
k = diffusivity, $T_c/\rho C_p$, cm^2/sec
P'_{ST} = derivative of the saturated curve with respect to temperature

Subscripts
we = water encroachment, f = formation
w = water, g = steam, i,j,k = grid, n = time level

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Table 1. Example data of Physical Properties and Block Dimensions

Initial temperature (1,1,1)	= 260°C
Initial temperature gradient	= 5°C/100m
Initial pressure (1,1,1)	= 70 atm
Horizontal permeability	= 100 md
Vertical permeability	= 10 md
Porosity	= 0.38
Initial water saturation	= 1.0
Water compressibility	= 1.65×10^{-4} vol/vol-atm
Formation compressibility	= 4.4×10^{-5} vol/vol-atm
Thermal conductivity	= 1.53×10^{-3} cal/cm-°C-sec
NX = 10, NY = 10, NZ = 5	
ΔX = 150m, ΔY = 150m, ΔZ = 15m	
Initial production rate	= 1.2×10^{4} g/sec
Bottom-hole pressure	= 60 atm

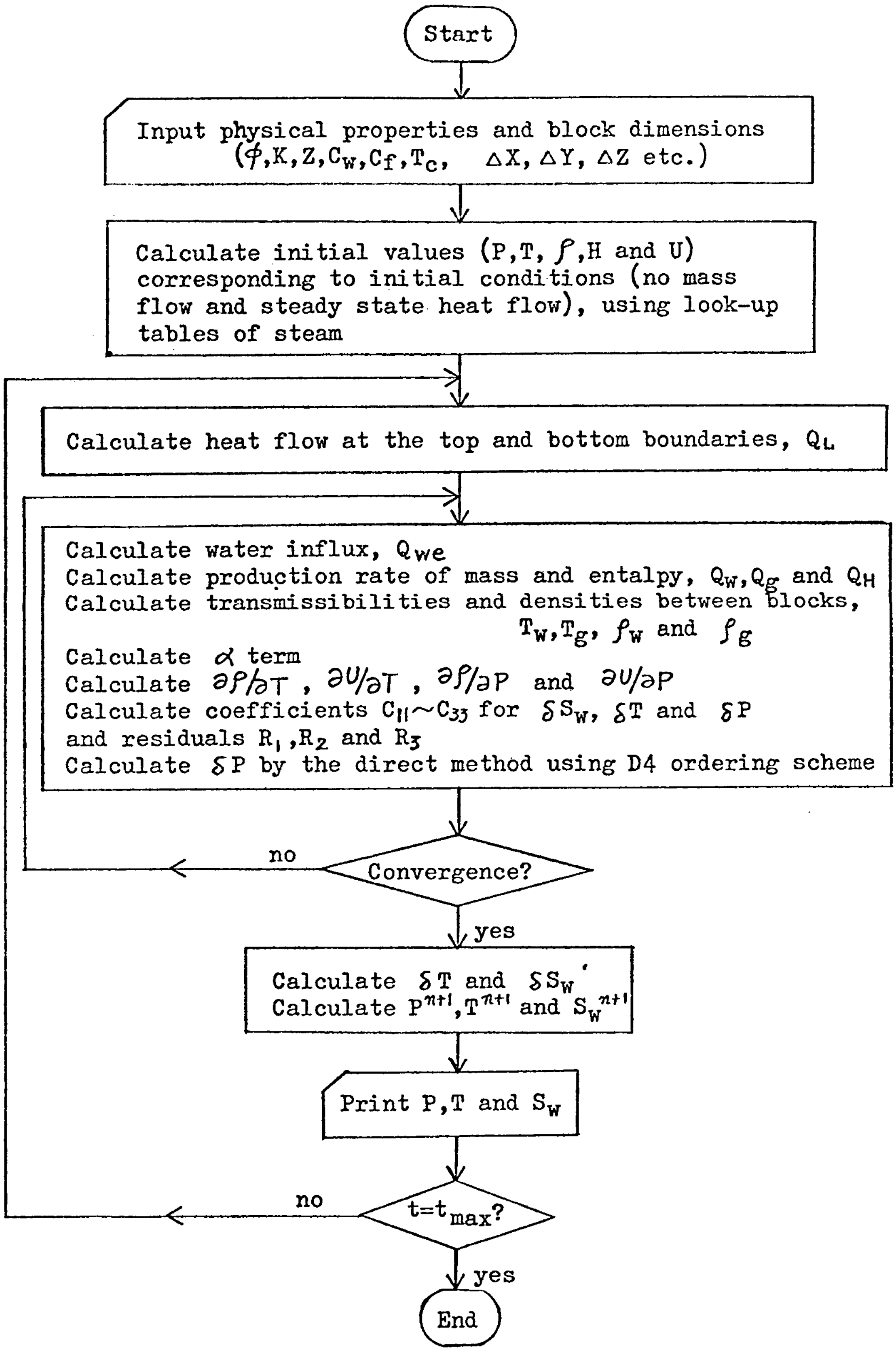

Fig. 1 Simplified Flow Chart

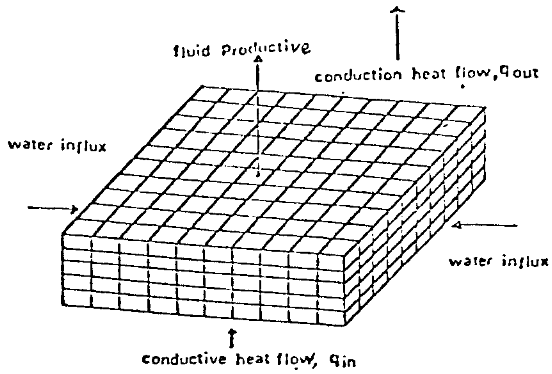

Fig.2 Geometry of 3-D Geothermal Reservoir Model

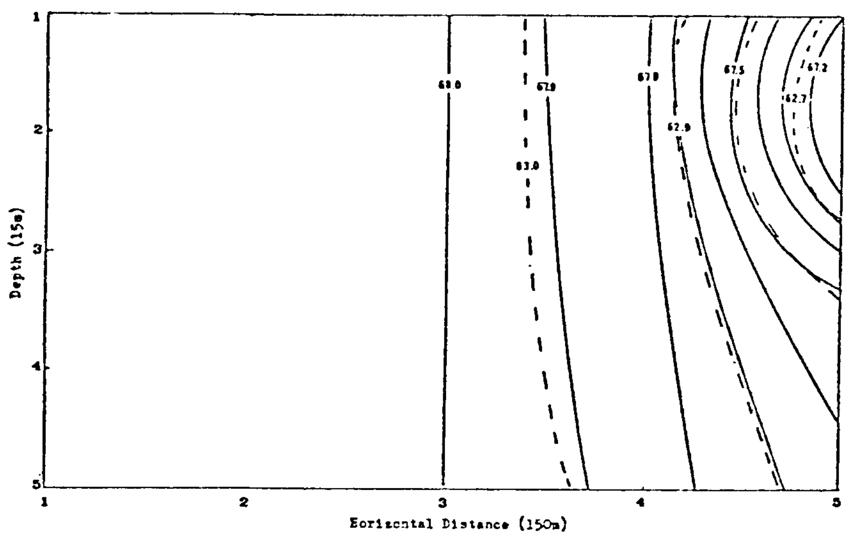

Fig. 3 Potential Distribution Behaviors (without water influx)
Solid curve, after 10 days of production. Dashed curve, after 50 days of production.

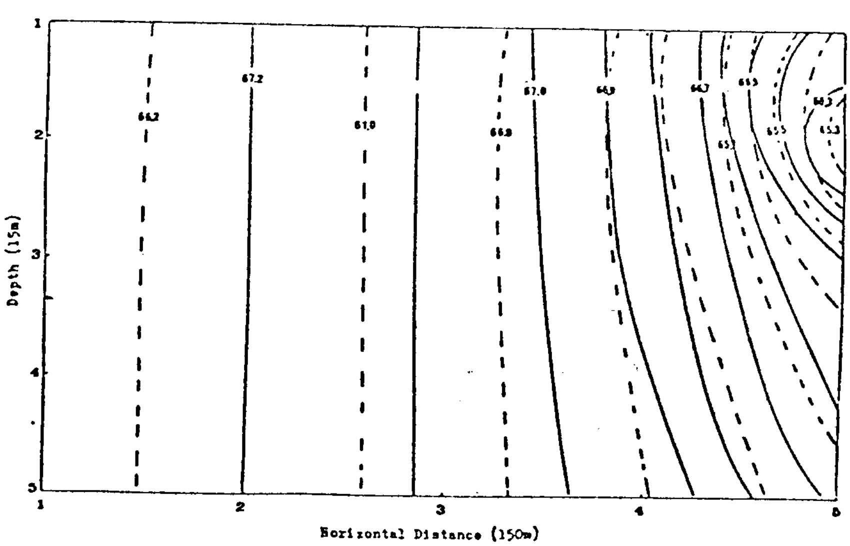

Fig. 4 Potential Distribution Behaviors (with water influx)
Solid curve, after 50 days of production. Dashed curve, after 100 days of production.