TWO PHASE FLOW IN POROUS MEDIA AND THE CONCEPT OF RELATIVE PERMEABILITIES

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INTRODUCTION

New equations for the two phase flow of water and steam are presented. The new equations coincide with those already in use for the case of horizontal flow but are different from those for vertical flow. It is shown that the usual equations can only be valid when the two phases are flowing in separate channels, where the channel dimensions are large compared with the grain size of the porous media, and in such a case the relative permeabilities should vary only slightly with the saturation ratio. It is shown that the actual variation of relative permeabilities with saturation ratio suggests a flow model where the flow channel dimensions are of the same order of magnitude as the grain size. On this basis a new set of equations is proposed, which with the associated flow model explain relative permeabilities qualitatively. In addition they show that water can flow upwards in two phase flow where the pressure gradient is less than hydrostatic. In a simple two phase flow test it is demonstrated that this happens as predicted by the new equation set.

RELATIVE PERMEABILITIES

Relative permeabilities are permeability reduction factors, when two phases are flowing simultaneously. They are used in connection with Darcy's law, as it has been used for two phase flow, as follows:

$$\hat{V}_w = \frac{k\hat{k}_{rw}}{\mu_w}\left(-\frac{\partial p}{\partial z} - \rho_w g\right) \quad (1)$$

$$\hat{V}_s = \frac{k\hat{k}_{rs}}{\mu_s}\left(-\frac{\partial p}{\partial z} - \rho_s g\right) \quad (2)$$

The velocities, $\hat{V}_w$ and $\hat{V}_s$ are the average water and steam velocity with respect to the total seepage area. Another possibility of extending Darcy's law to include two phase flow is to use the average velocity with respect to the actual seepage area eqs. 1 and 2 would then be

$$V_w = \frac{kk_{rw}}{\mu_w}\left(-\frac{\partial p}{\partial z} - \rho_w g\right) \quad (3)$$

$$V_s = \frac{kk_{rs}}{\mu_s}\left(-\frac{\partial p}{\partial z} - \rho_s g\right) \quad (4)$$

where k_{rw} and k_{rs} are defined as follows:

$$\hat{k}_{rw} = S_w k_{rw} \quad (5)$$

$$\hat{k}_{rs} = (1-S_w)\ k_{rs} \qquad (6)$$

Both definitions of the relative permeabilities will be used in the following discussion to avoid confusion. As can be seen from eqs. 1 and 2 the only forces acting on the two phases are the resistance force and the force of gravity. There are no interfacial forces between the two phases. The only flow model, where no forces between the phases occur is when water and steam flow in separated large channels . In such a case the relative permeabilities would only be area reduction factors defined as:

$$\hat{k}_{rw} = S_w \qquad (7)$$

$$\hat{k}_{rs} = (1-S_w) \qquad (8)$$

Most laboratory measurements of relative permeabilities seem to indicate that they are dependant on saturation to the power two or higher. Wyckoff and Botset (1936) investigated the flow of mixtures of liquid (water) and gas (carbon dioxide) and their results for relative permeabilities are given in fig. 1, which will be used for reference permeability curves in the following. The separate channel model is therefore not viable, as it is not in agreement with measurements. For further references see for example Chen et al. (1978) and Counsil and Ramey (1979), in which measurements of relative permeabilities for steam and water are given.

MACROSCOPIC FLOW MODEL

Since the two phases are not flowing in large separated channels, there must exist some interfacial force between them and the phases will be flowing at different velocities with some slip between them. One possibility is to use a macroscopic flow-model, in which it is envisaged that there are large bubbles of steam in water or large drops of water in steam. Here large is used in a relative sense in comparison with the grain size. If we consider for example the case of large bubbles of steam in water, Yih (1965, p. 216) gives for the velocity slip between the phases for horizontal flow as:

$$S = \frac{\hat{V}_s}{\hat{V}_w} = \frac{3\mu_w}{2\mu_s+\mu_w} \qquad (9)$$

By assuming horizontal flow the slip factor is obtained from eqs. 1 and 2:

$$S = \frac{\hat{V}_s}{\hat{V}_w} = \frac{\mu_w}{\mu_s}\ \frac{\hat{k}_{rs}}{\hat{k}_{rw}} \qquad (10)$$

By comparing eqs. 9 and 10 we obtain:

$$\frac{\hat{k}_{rs}}{\hat{k}_{rw}} = \frac{3}{\frac{\mu_s}{\mu_w}\left(2\,\frac{\mu_s}{\mu_w}+1\right)} \qquad (11)$$

Eq. 11 then gives the ratio of the relative permeabilities independent of saturation, which is clearly not in agreement with measurements as indicated by fig. 1 The macroscopic flow model is thus not viable and the flow channels must be of the same size as the grain dimensions and a microscopic flow model is more relevant. The relative permeability factors would in this case be porosity reduction factors and as indicated by most research results the permeability is a function of porosity to the third power. The relative permeabilities would then fit the experimental data in fig. 1 as shown by Irmay (1954).

MICROSCOPIC FLOW MODEL

In this model it is assumed that the water is in contact with the solid skeleton and that the steam forms channels in the water without contacting the solid skeleton. The forces per volume acting in this case would be as follows.

1) Force between solid and water:

$$\frac{V_w \mu_w}{k\, k_w} \qquad \text{for water}$$

where k_w is a permeability reduction factor for water.

2) Force between water and steam:

$$-\frac{1}{S_w}(C_1 V_w - C_2 V_s) \qquad \text{for water}$$

$$\frac{1}{1-S_w}(C_1 V_w - C_2 V_s) \qquad \text{for steam}$$

where C_1 and C_2 are certain unknown constants.

3) Forces of gravity and pressure:

$$I_w = -\frac{\partial p}{\partial z} - \rho_w g \qquad \text{for water}$$

$$I_s = -\frac{\partial p}{\partial z} - \rho_s g \qquad \text{for steam}$$

Taking the force balance for steam and water we obtain:

$$\frac{V_w \mu_w}{k\, k_w} = I_w - (C_1 V_w - C_2 V_s)\frac{1}{S_w} \qquad (12)$$

$$0 = I_s + (C_1 V_w - C_2 V_s)\frac{1}{1-S_w} \qquad (13)$$

Inserting eq. 13 into eq. 12 and rearranging the terms gives:

$$V_w = -\frac{kk_w}{\mu_w S_w}\left(\frac{\partial p}{\partial z} + (S_w\rho_w + (1-S_w)\rho_s)g\right) \tag{15}$$

Eq. 13 then gives for the steam phase

$$V_s = \frac{1-S_w}{C_2} I_s + \alpha V_w \tag{16}$$

where α is defined as the ratio between the two constants C_1 and C_2. Here α is the ratio between the maximum velocity of water and the mean water velocity. In a laminar flow of water and steam in a circular pipe α would be equal to 2 Eq. 16 can be rewritten by redefining the constant C_2, as

$$V_s = \frac{kk_s}{\mu_s(1-S_w)} I_s + \alpha V_w \tag{17}$$

where k_s is some permeability reduction factor. Using the definition of I_s eq. 17 becomes

$$V_s = -\frac{kk_s}{\mu_s(1-S_w)}\left(\frac{\partial p}{\partial z} + \rho_s g\right) + \alpha V_w \tag{18}$$

by comparing eqs. 15 and 18 with eqs. 3 and 4 the relative permeabilities in the case of horizontal flow are:

$$k_w = \hat{k}_{rw} \tag{19}$$

$$k_s = \hat{k}_{rs} - \alpha \frac{\mu s}{\mu_w} \frac{1-S_w}{S_w} \hat{k}_{rw} \tag{20}$$

If we take for example, $\alpha = 2$, $\frac{\mu_s}{\mu_w} = 1/6$ and $\hat{k}_{rw} = S_w^3$, the maximum value of the last term in eq. 20 becomes equal to 0.05, and can thus be neglected. Eq. 20 then becomes

$$k_s \simeq \hat{k}_{rs} \tag{21}$$

In the case of horizontal flow eqs. 15 and 18 reduce to eqs. 3 and 4. The necessary condition to maintain upward flow of water then becomes

$$-\frac{\partial p}{\partial z} > (S_w\rho_w + (1-S_w)\rho_s)g \tag{22}$$

which can be compared with the necessary condition derived from eq. 1:

$$-\frac{\partial p}{\partial z} > \rho_w g \tag{23}$$

According to eq. 23 the pressure gradient must be greater than hydrostatic pressure in order to obtain upward flow of water. Eq. 22, which is derived from eq. 15, allows water to flow vertically upwards, although the pressure gradient is less than hydrostatic.

FLOW TESTS

To investigate the behaviour of two phase vertical flow, a test was performed in a vertical 4" circular tube in the laboratory. The set-up is schematically shown in fig. 2.

Pressure and temperature are measured in four different levels. The steam phase and water phase are separated and measured at the top end. The flow is always upwards.

A porous gravel test medium of crushed basalt was used. The permeability was measured as being 545 Darcy. The test was not intended to show the variation of relative permeabilities with saturation so saturation was not measured, but test with porous media of other permeabilities needs to be made. The reason for completely omitting the saturation measurements in this work is the uncertainty of the known methods in use at present to measure this parameter.

The results of the tests are shown in table 1. They fall into three categories, according to the no-slip saturation, which is the saturation one would have if both water and steam were flowing with the same velocity. In the 8 first runs, S_w^o is 2% or lower. The flow is mostly steam with very little water flow and in one case the relative permeability for water proves to be negative, which means that the water should flow downwards according to eq. 3., but in fact actually flows upwards. Otherwise the k_{rw} values behave normally, but are much higher than would be expected for these very low water saturations.

In the next five runs water and steam are flowing in more equal proportions. The k_{rw} values are all much too high, i.e. a lot more water is flowing than can be explained by eq. 3.

In the next six runs there is very little or no steam. The k_{rw} values are practically equal to one as would be expected.

The test shows clearly that eqs. 3 and 4 cannot explain the results except in the six cases where the flow is almost purely water. The relative permeabilities calculated from eqs. 15 and 18 behave in accordance with theory. To demonstrate this k_{rs} is calculated, by assuming that the value of k_{rw} is equal to S_w^3 as seems to be the conclusion in the majority of cases of investigation. In these cases both S_w and k_{rw} can be calculated and the results are shown in fig. 3, and the result is quite in agreement with theory and experiments. This means that the microscopic flow model eqs. 15 and 18 fully explain the test results, whereas the macroscopic flow model (eqs. 1-4) does not.

DISCUSSION

In the literature there are two different definitions in use for relative permeability. The relative permeabilities as defined in these different ways, are related, but have very different numerical values (see eqs. 5 and 6). Careful distinction between these two is necessary especially when reporting research data. In many of the existing papers on the subject it is impossible to see wether k_{rw} and k_{rs} are being used or $\hat{k}_{rw}$ and $\hat{k}_{rs}$.

Reported laboratory tests on relative permeabilities show that they depend on saturation to the power two or higher. The eqs. 1-4 can only be valid when the fluid shear stress is transmitted directly to the solid skeleton. The relative permeability is actually a permeability reduction factor, and as it varies with saturation to the power two or higher it does not matter if the definition k or $\hat{k}$ is used, since the resulting shear stress in the water phase is always higher than if water was flowing alone with the same average velocity. From this one can conclude that a macroscopic flow model, i.e. a flow model where the water is assumed to flow more or less alone either in large channels or big droplets, does not explain the reported behaviour of the relative permeabilities.

On the other hand, if it assumed that the gas phase is deplacing the liquid phase from the pores then a microscopic flow model is envisaged. In this the steam may be flowing in more or less continuous channels or paths, but these have a cross-sectional dimension of the same order of magnitude as the pores, or less. The steam is flowing within the water, with a velocity higher than the water velocity. The resulting average velocity slip creates a shear stress between the two phases, this shear stress is transmitted through the water to the rock. It increases the velocity in the water and lessens the possibility of the heavier phase flowing downwards while the lighter phase is flowing upwards.

The microscopic model, eqs. 15 and 18, is distinctly different from the macroscopic model, eqs. 1-4, in the case of vertical flow. In the case of horizontal flow the two models yield the same equations, provided a correct definition of the relative permeabilities is used (eqs. 19 and 21). The conditions for maintaining upwards flow of water is markedly different for the two flow models (eqs.22 and 23). This is of great importance for the calculation of geothermal convection, and it also explains the "chimney effect" of geothermal areas, i.e. a pressure level within a geothermal field that is lower than the pressure level of all surrounding aquifers. When calculating the flow towards a well, or other flows under the influence of great pressure gradients, only small differences are to be expected between the results of the two models.

Flow tests, performed in a vertical tube support the theories put forward. In a third part of the tests measured relative permeabilities show no permeability reduction but rather an icrease in permeability, and this is accepted as impossible according to all researchers, the relative permeability being always less than one for all saturations. The 1/3 of the tests mentioned is performed under such conditions that neither the steam phase nor the water phase dominates the other, i.e. neither water saturation nor steam saturation are extremely low. The resulting 2/3 of the tests behave normally with respect to the relative permeabilities.

REFERENCES

Chen, H.K., Counsil, J.R., Ramey, H.J., Jr.: Experimental Steam-Water Relative Permeability Curves. Geothermal Resources Council, Transactions, Vol. 2, July 1978.

Counsil, J.R., Ramey, H.J., Jr.: Drainage Relative Permeabilities Obtained from Steam-Water Boiling Flow and External Gas Drive Experiments. Geothermal Resources Council, Transactions, Vol. 3, Sept. 1979.

Irmay, S.: On The Hydraulic Conductivity of Unsaturated Soils. Transactions, American Geophysical Union, Vol 35, Number 3, June 1954.

Wyckoff, R.D., Botset, H.G.: The Flow of Gas-Liquie Mixtures Through Unconsolidated Sands. Physics, Vol. 7, Sept. 1936.

Yih, Chia-Shun: Dynamics of Nonhomogeneous Fluids. Macmillan Co., N.Y., 1965.

NOMENCLATURE

g = Projection of the acceleration of gravity on the flow direction (m/sec^2).

k = Permeability (Darcy)

k_{rw}, k_{rs}, $\hat{k}_{rw}$, $\hat{k}_{rs}$ = Relative permeabilities for water and steam respectively.

$\dot{m}_w$, $\dot{m}_s$ = Mass flow of water and steam respectively (kg/sec).

p = Pressure (N/m^2).

S_w = Saturation of water.

S_w^o = No-slip saturation of water.

S = Slip factor.

$\hat{V}_w$, $\hat{V}_s$, V_w, V_s = Mean velocity of water and steam respectively (m/sec).

X = Mass fraction of steam.

z = Coordinate in the flow direction (m).

ρ_w, ρ_s = Density of water and steam respectively (kg/m^3).

μ_w, μ_s = Dynamic viscosity of water and steam respectively (kg/m sec).

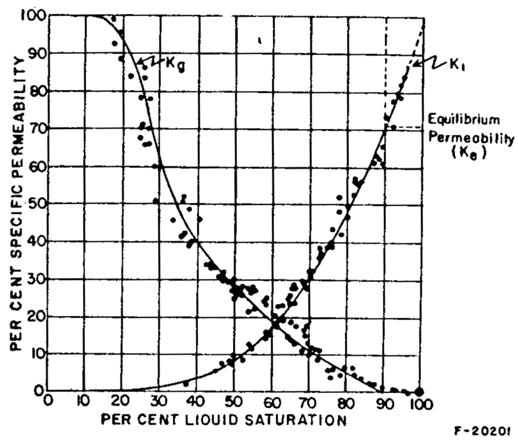

Fig. 1 Permeability - saturation relation for unconsolidated sand.
(Wyckoff and Botset (1936)).

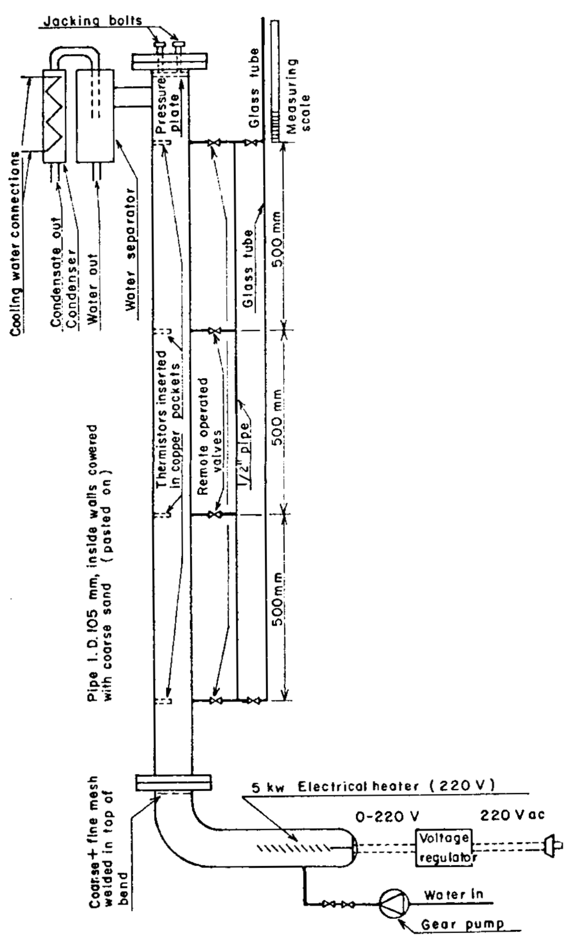

Fig. 2 Experimental setup for measuring two phase flow in porous media.

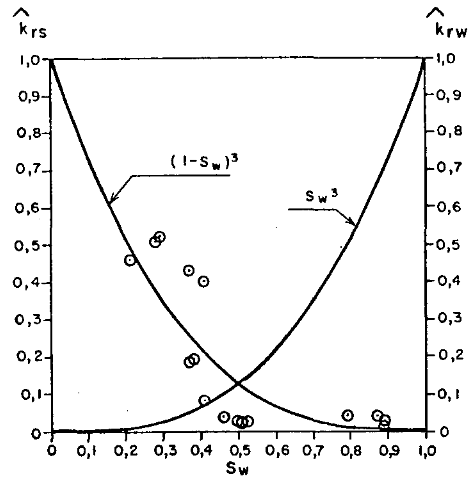

Fig. 3 Relative permeability for steam as calculated from experiments.

TABLE 1

Test no.	$\dot{m}_w$ kg/sec.	$\dot{m}_s$ kg/sec.	X %	I_w N/m^3	I_s N/m^3	S_w^o %	$\dot{m}/I_w$ m^2. sec	Relative permeab. for water	Dominating phase in volume
1	$1.11 \cdot 10^{-3}$	$1.36 \cdot 10^{-3}$	55.1	411.2	9810.08	0.05	$0.27 \cdot 10^{-5}$	0.17	Steam
2	2.44 -	1.23 -	33.5	235.0	9633.85	0.13	1.04 -	0.66	
3	2.77 -	1.23 -	30.8	274.1	9673.01	0.15	1.01 -	0.64	
4	5.00 -	1.04 -	17.2	391.6	9868,82	0.32	1.28 -	0.81	
5	7.22 -	1.01 -	12.3	920.3	10319.18	0.47	0.78 -	0.49	
6	5.00 -	0.44 -	8.1	254.5	9653.43	0.75	1.96 -	1.24	
7	5.55 -	0.46 -	7.7	332.9	9731.75	0.80	1.67 -	1.06	
8	6.11 -	0.19 -	3.0	-137.1	9261.81	2.10	-4.45 -	-2.82	
9	8.47 -	0.088 -	1.0	313.3	9712.17	6.04	2.70 -	1.71	Water and steam
10	10.42 -	0.082 -	0.8	391.6	9790.50	7.82	2.67 -	1.69	
11	10.69 -	0.064 -	0.6	391.6	9790.50	10.03	2.73 -	1.73	
12	11.25 -	0.061 -	0.5	391.6	9790.50	11.00	2.87 -	1.82	
13	10.55 -	0.052 -	0.5	391.6	9790.50	11.93	2.69 -	1.70	
14	39.09 -	0.16 -	0.4	2291.0	11689.85	13.95	1.71 -	1.08	Water
15	34.84 -	0.122 -	0.4	2506.0	11905.24	16.01	1.39 -	0.88	
16	38.18 -	0.115 -	0.3	2408.0	11807.34	18.14	1.59 -	1.00	
17	40.00 -	0.075 -	0.2	2428.0	11826.92	26.25	1.65 -	1.04	
18	40.00 -	0.03 -	0.1	2526.0	11924.82	47.09	1.58 -	1.00	
19	38.78 -	0.00 -	0.0	2448.0	11846.50	100.00	1.58 -	1.00	