BOILING HEAT TRANSFER FROM A DIKE

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Introduction

It is known that hot dike complexes are one type of heat sources in a volcanic geothermal reservoir. To extract energy from a geothermal reservoir, it is important to know the cooling rate of the intrusives by groundwater. With a maximum temperature of 1,200°C, the intrusives are most likely to be cooled by film boiling during the initial stages and by single-phase convection during the later stages when its temperature becomes lower that the saturated temperature corresponding to the pressure. At the Second Workshop on Geothermal Reservoir Engineering, Cheng [1] has reported that the cooling rate of a dike by free convection of single-phase groundwater can be computed according to

$$q_w = 0.444(T_w - T_\infty)^{3/2}\sqrt{\frac{k_m K\beta g C_p}{\nu x}} , \tag{1}$$

$$\bar{q}_w = 0.888(T_w - T_\infty)^{3/2}\sqrt{\frac{k_m K\beta g C_p}{\nu L}} , \tag{2}$$

and the thickness of the hot water zone is

$$\delta = 6.31\sqrt{\frac{\mu\alpha x}{K\rho_\infty \beta g(T_w - T_\infty)}} , \tag{3}$$

where g is the gravitational acceleration; q_w and $\bar{q}_w$ are the local and the average surface heat flux; T_w and T_∞ are the temperatures of the groundwater at the surface and at a great distance from the surface; K, k_m and α are the permeability, the equivalent thermal conductivity, and thermal diffusivity of the formation; c_p, β, and ν are the specific heat, the thermal expansion coefficient, and the kinematic viscosity of the groundwater; L and x are the height of the dike and the locality on the dike measured from the bottom; ρ_∞ is the density of the groundwater at infinity.

Recently, we have obtained an analytical solution for boiling heat transfer from a vertical isothermal surface in a porous medium filled with a subcooled liquid [2,3]. In this paper we shall briefly summarize the results obtained, and carry out numerical computations of boiling heat transfer from a dike.

Film Boiling from a Vertical Plate in a Porous Medium

Consider the problem of steady cooling of an isothermal vertical surface embedded in a porous medium filled with a subcooled liquid as shown in Fig. 1. When the wall temperature T_w is sufficiently higher than the saturation temperature T_s (corresponding to its pressure), a vapor film with thickness δ_V will form adjacent to the vertical surface. To investigate the heat transfer characteristics of this problem, Cheng and his co-worker [2,3] have made the following assumptions: (1) A distinct boundary exists between the vapor and the subcooled liquid with no mixed zone in between. (2) The interface at $y = \delta_V$ is smooth and stable, and is at a constant temperature T_S. (3) Boundary layer approximations are applicable. (4) Bousinesq approximations are invoked in the liquid phase so that density is assumed to be constant except in the buoyancy force term where density is assumed to be linearly proportional to temperature. (5) All other properties of the liquid and vapor phases and the porous medium are constant. (6) Darcy's law is applicable to both phases. It is worth noting that assumptions (1)-(5) are the usual approximations used in the classical film boiling literature. Note that as a result of the first approximation, the mathematical formulation of the problem is considerably simplified. This is because the complexity of the relative permeability no longer comes into the picture and that single-phase equations can be applied separately to the vapor and liquid phases. As a result, both the vapor and the liquid phases admit similarity solutions in terms of the dimensionless boundary layer thickness parameter $\eta_{V\delta}$ which is defined as $\eta_{V\delta} = \sqrt{Ra_{x,v}}\,\delta_V/x$. This parameter is an implicit function of three other dimensionless parameters: Sh, Sc, and R, where $Sh = c_{pv}(T_w - T_S)/h_{fg}$, $Sc = c_{pL}(T_S - T_\infty)/h_{fg}$,

$$R = \frac{\rho_v}{\rho_\infty}\left[\frac{\mu_L \alpha_v (\rho_\infty - \rho_v) c_{pL}}{\mu_v \alpha_L \rho_\infty \beta_L h_{fg}}\right]^{1/2}, \quad c_{pL} = k_{mL}/\rho_\infty \alpha_L \text{ and } c_{pv} = k_{mv}/\rho_\infty \alpha_v$$

with the subscript "v" denoting quantities associated with the vapor phase while the subscript "L" denoting quantities associated with the liquid phase. The functional relationship between $\eta_{V\delta}$, Sh (a measure of degree of wall superheating), Sc (a measure of the extent of the subcooling of the surrounding fluid), and R (a property ratio) is illustrated in Fig. 2.

The local and average boiling heat transfer rates from a heated surface in a geothermal reservoir can be shown to be

$$q_w = \frac{(T_w - T_s)}{\operatorname{erf}(\frac{\eta_{v\delta}}{2})}\sqrt{\frac{k_{mv} K (\rho_L - \rho_v) g\, c_{pv}}{\pi \nu_v x}}\ , \tag{4}$$

$$\bar{q}_w = \frac{2(T_w - T_s)}{\operatorname{erf}(\frac{\eta_{v\delta}}{2})}\sqrt{\frac{k_{mv} K (\rho_L - \rho_v) g\, c_{pv}}{\pi \nu_v L}}\ . \tag{5}$$

Equations (4) and (5) can be rewritten in dimensionless form as

$$\frac{Nu_x}{\sqrt{Ra_{x,v}}} = \frac{1}{\sqrt{\pi}\operatorname{erf}(\frac{\eta_{v\delta}}{2})}\ , \tag{6}$$

$$\frac{\overline{Nu}}{\sqrt{Ra}} = \frac{2}{\sqrt{\pi}\operatorname{erf}(\frac{\eta_{v\delta}}{2})}\ , \tag{7}$$

where Nu_x and $\overline{Nu}$ are the local and average Nusselt numbers while $Ra_{x,v}$ and Ra are the Rayleigh numbers based on x and L respectively, i.e.,

$$Nu_x \equiv \frac{q_w x}{k_{m,v}(T_w - T_s)} , \quad \overline{Nu} \equiv \frac{\bar{q}_w L}{k_{m,v}(T_w - T_s)} \tag{8a,b}$$

and

$$Ra_{x,v} \equiv \frac{K(\rho_L - \rho_v) g x}{\mu_v \alpha_v} , \quad Ra \equiv \frac{K(\rho_L - \rho_v) g L}{\mu_v \alpha_v} \tag{9a,b}$$

It is convenient to multiply Eq. (6) by $\eta_{v\delta}$ and plot the quantity $Nu_x \eta_{v\delta}/\sqrt{Ra_{x,v}}$ versus $\eta_{v\delta}$ which is presented in Fig. 3. On the other hand, the quantity $Nu_x/(Ra_{x,v})$ versus Sh at R = 0.5 and with different values of Sc is presented in Fig. 4.

Application to Heat Transfer from a Dike

Consider a dike 100 m in height with an average surface temperature of 400°C is intruded into an aquifer (with $K = 10^{-12} m^2$, $k_{m,L} = 2.65$ J/sec-K, and $k_{m,v} = 1.6$ J/sec-K) at a temperature of 20°C. Suppose that the mean static pressure along the dike is at 10 atmospheric pressure and the saturated temperature corresponding to this mean pressure is 180°C. To apply the theory to this problem, we shall evaluate the properties of the vapor and liquid layers at their mean temperatures. Thus, the density, viscosity, and specific heat of vapor will be evaluated at the mean temperature of $(T_w+T_s)/2 = 290$°C while that of the liquid phase at $(T_s+T_\infty)/2 = 100$°C. At these temperatures, we found that $\rho_v = 0.004$ g/cc, $c_{pv} = 2.16$ J/g-k, $\mu_v = 1.96 \times 10^{-4}$ g/cm-sec; $c_{pL} = 4.22$ J/g-K, $\mu_L = 2.74 \times 10^{-3}$ g/cm-sec, $\rho_\infty = 0.9574$ g/cc and $\beta_{L\infty} = 4.67 \times 10^{-4}$/C, $h_{fg} = 2019$ J/g. With these values, we obtain Sh = 0.2364, Sc = 0.3323, and R = 0.565, and consequently we can determine $\eta_{v\delta}$ from Fig. 2 to get $\eta_{v\delta} = 0.49$. With this value of $\eta_{v\delta}$ we obtain $Nu_x = 33.3$ at x = 100 m where $Ra_{x,v} = 258$ and $Ra_{x,L} = 425$. The vapor film boundary layer thickness can be determined from the definition of $\eta_{v\delta}$ which gives $\delta_v = 0.49x/\sqrt{Ra_{x,v}}$ and the vapor boundary layer thickness is given by $\delta_L = \frac{5x}{\sqrt{Ra_{x,L}}}$. This is plotted in Fig. 5 where it is shown that $\delta_v = 3$ m and $\delta_L = 24.3$ m at x = 100 m. The vertical velocity profiles for the vapor and the liquid phases at x = 100 m are plotted in Fig. 6. It is shown that there is a velocity discontinuity at the vapor-liquid interface which is a consequence of the Darcy's law. The vertical velocity in the vapor phase is shown to be much higher than that in the liquid phase because buoyancy force is larger and viscosity is lower for vapor.

Concluding Remarks

The validity of the present theory depends critically on the assumptions of (1) the non-existence of a two-phase zone in the boundary layer, and (2) the vapor-liquid interface being stable and smooth. For the classical film boiling problems, the first assumption has been widely accepted while the second assumption is more difficult to be met in reality since bubbles near the interface may be formed (resulting in a wavy shape) or detached (resulting in an unsteady behavior). These assumptions are certainly plausible for the corresponding problems in a porous medium. Its accuracy, however, can only be determined by future experiments.

References

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2. Cheng, P., "Film Condensation Along an Inclined Surface in a Porous Medium," accepted for publication in Int. J. Heat Mass Transfer (1980).

3. Cheng, P., and Verma, A.K., "The Effect of Subcooled Liquid on Film Boiling about a Vertical Heated Surface in a Porous Medium," accepted for publication in Int. J. Heat Mass Transfer (1980).

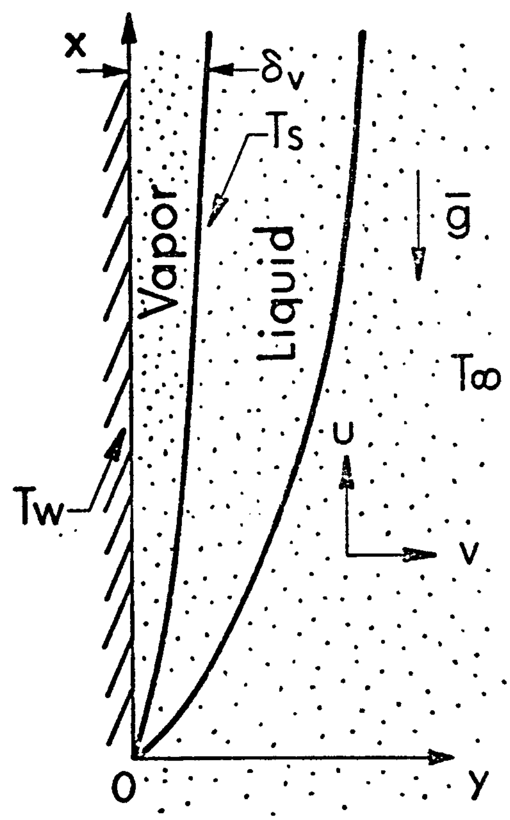

Fig. 1 Coordinate System

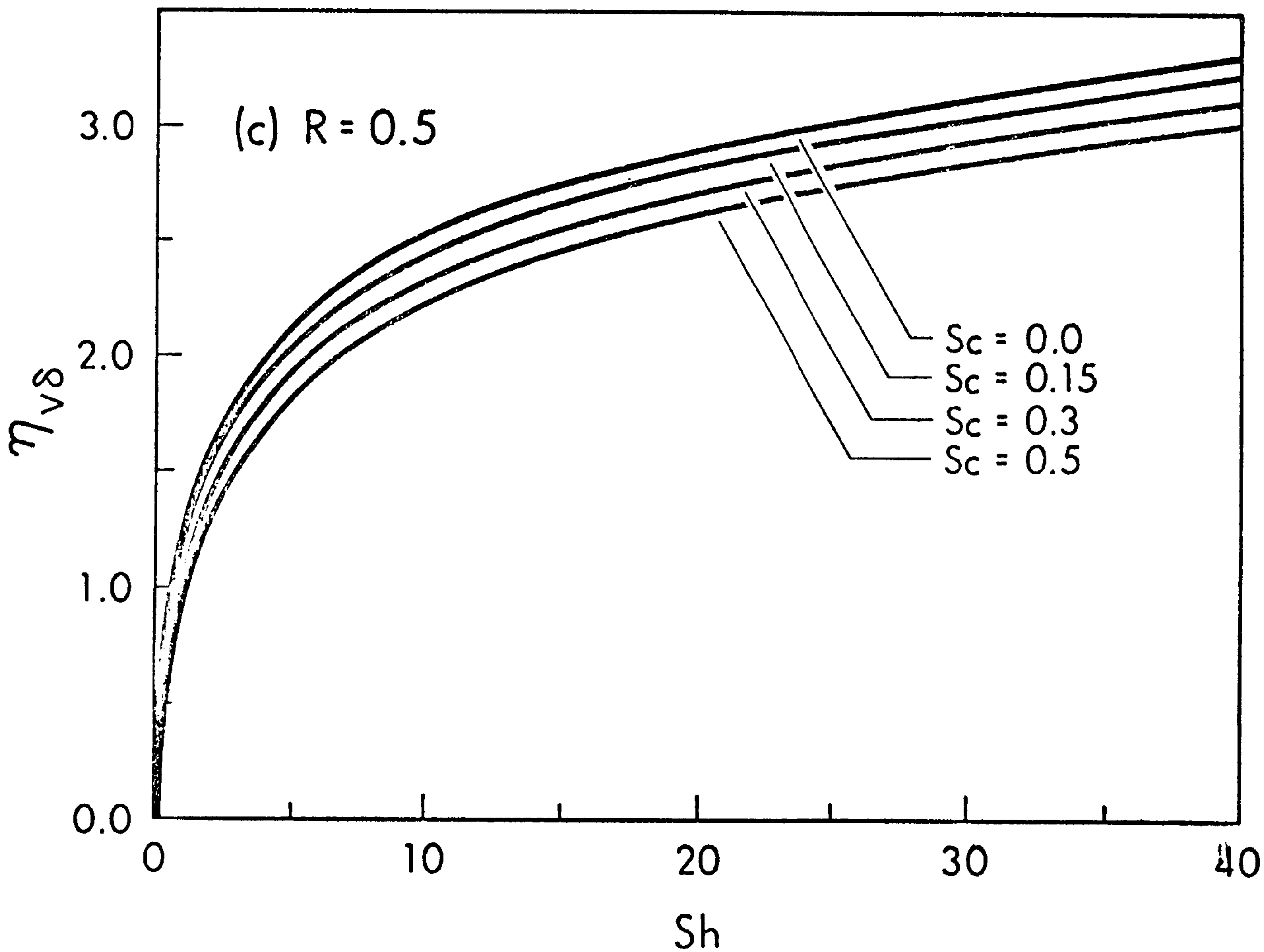

Fig. 2 $\eta_{v\delta}$ versus Sh and Sc at R = 0.5

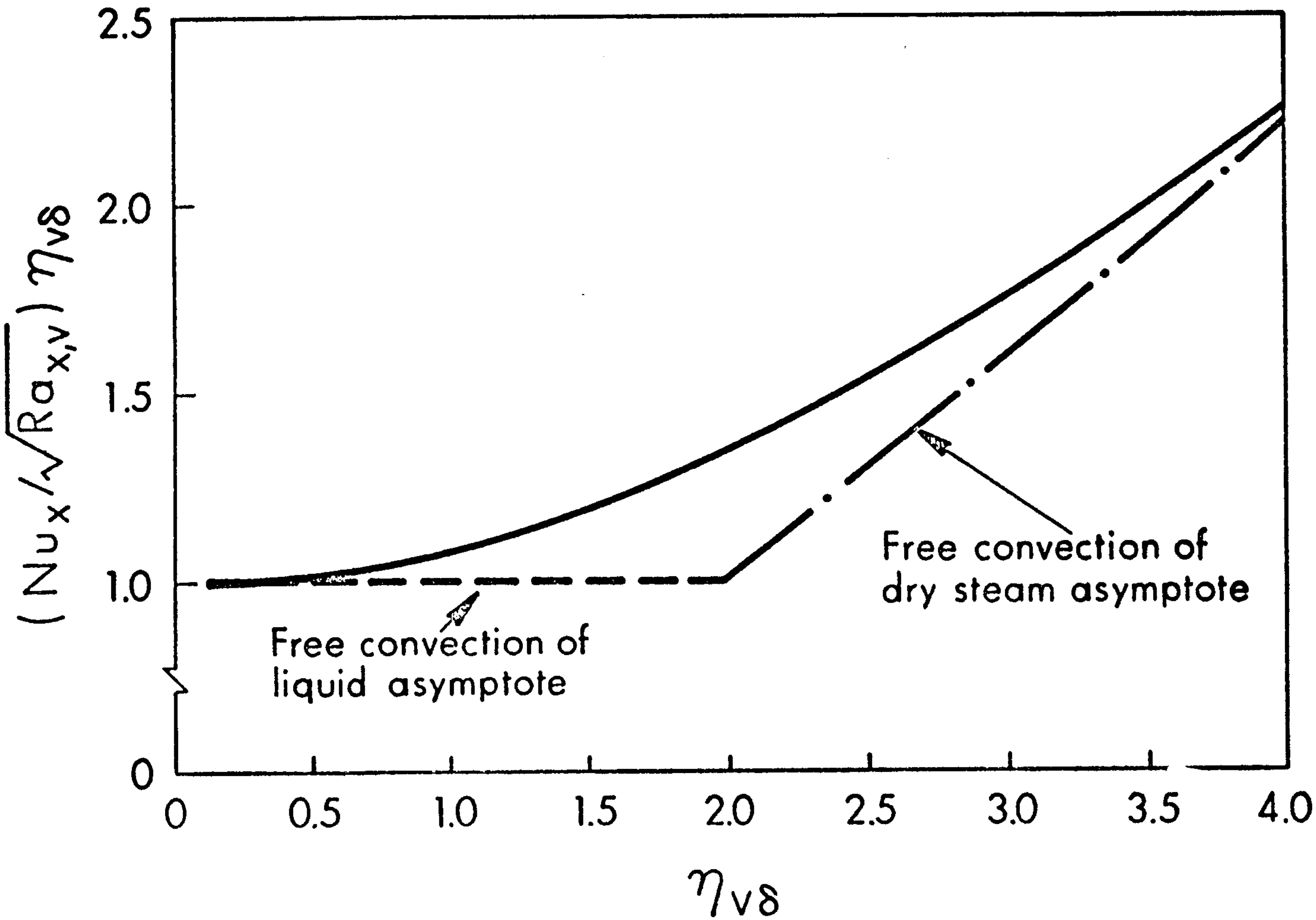

Fig. 3 Heat Transfer Rate versus $\eta_{v\delta}$

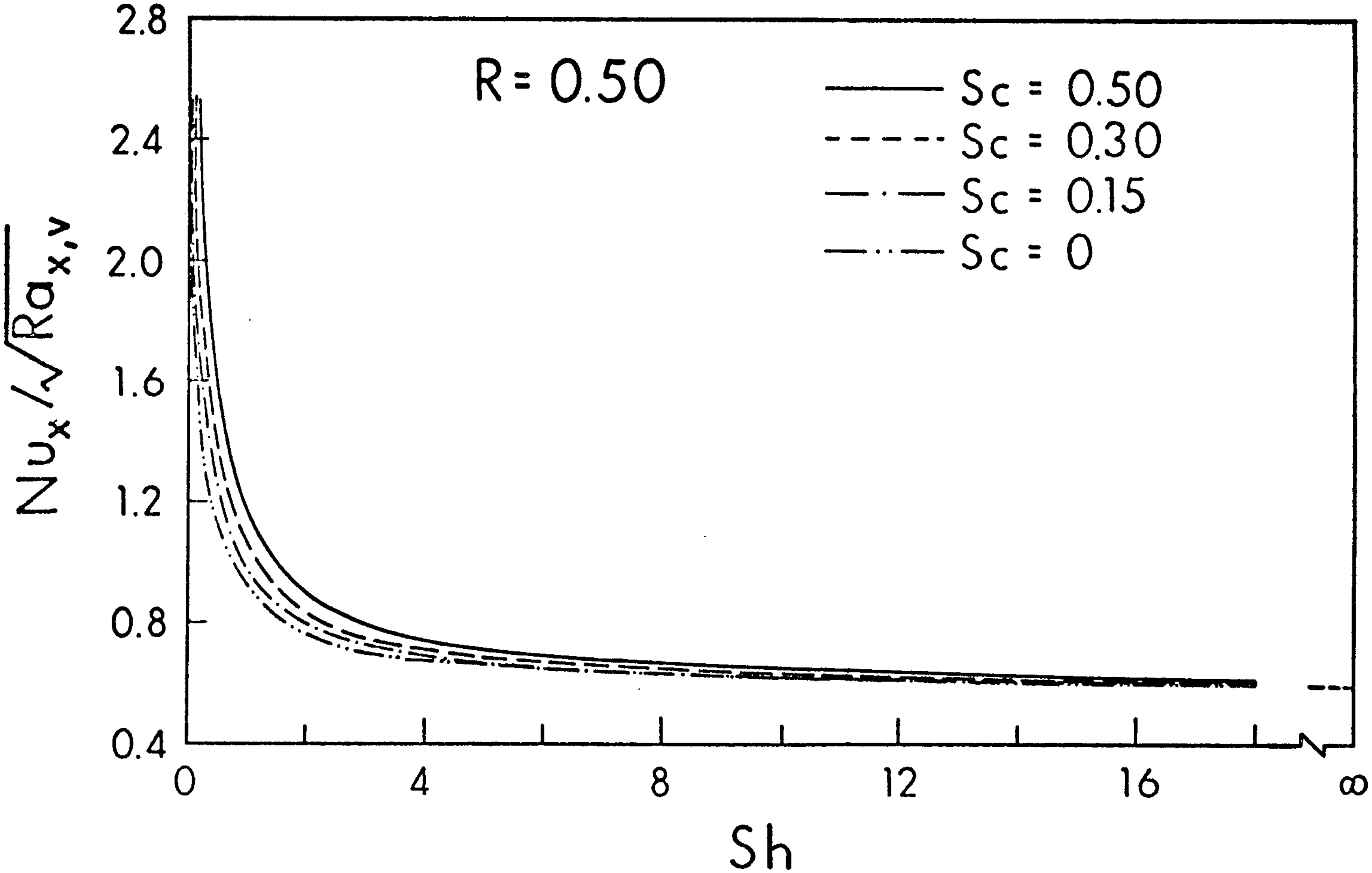

Fig. 4 Heat Transfer Rate versus Sh at Selected Values of Sc and at R = 0.5

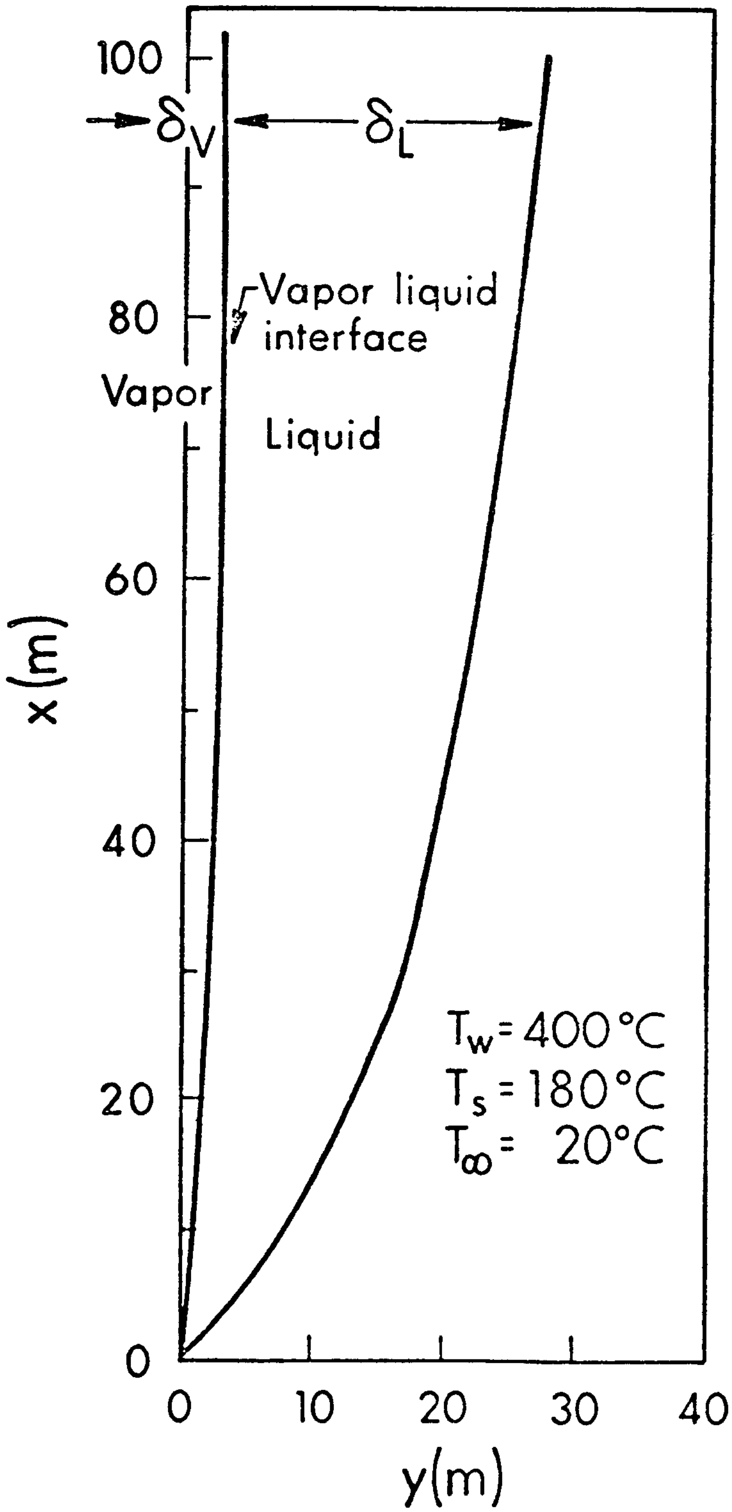

Fig. 5 Size of the Vapor Film and the Hot Water Zone

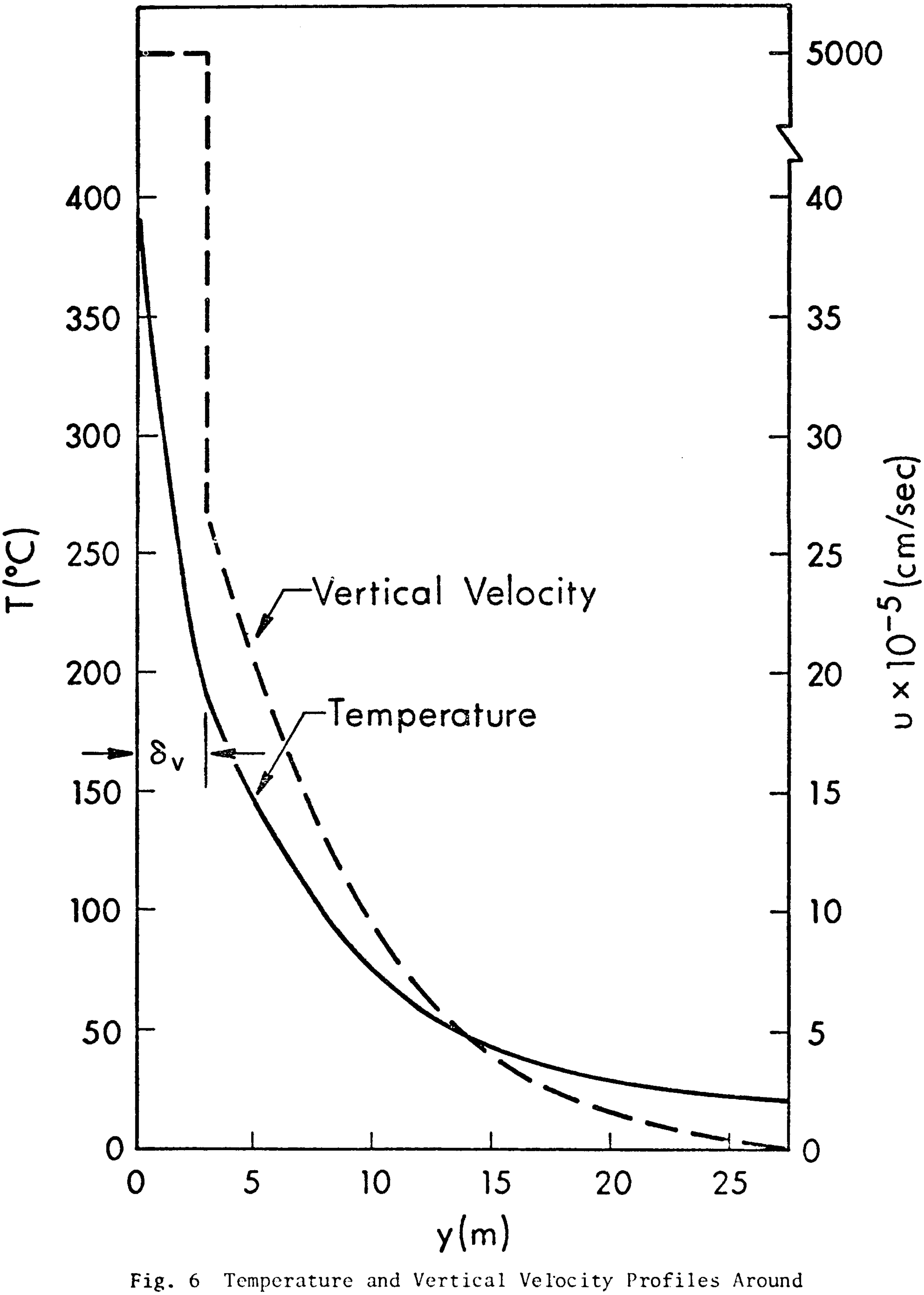

Fig. 6 Temperature and Vertical Velocity Profiles Around a Dike at x = 100 m