RESERVOIR EXPLORATION/TESTING BY ELASTOMECHANICAL METHODS

Gunnar Bodvarsson, School of Oceanography,
Oregon State University, Corvallis, OR

Introduction

The recovery of geothermal energy for power generation invariably involves the withdrawal of very substantial amounts of heat/water from the subsurface. For example, the commercial operation of a 100 MW power plant in a liquid dominated reservoir situation will require the recovery of about $5x10^{16}$J/yr of heat and a mass flow of water of about $5x10^{10}$kg/yr. A number of conspicuous phenomena are associated with this rate of production. Usually there are changes in the subsurface temperature, a significant rate of lowering of the local reservoir fluid pressure and an associated lowering of the ground water surface in the production region. Moreover, there are more subtle effects such as subsidence, strain and tilt of the ground surface, changes in the local gravity field and changes in the subsurface electrical conductivity. All these phenomena represent responses of the reservoir to production and convey information on the evolution of the system. The recording of these signals can therefore be helpful in reservoir monitoring during production. Obviously, the evolutionary pressure phenomena are known to everyone engaged in reservoir engineering and the recording of reservoir pressure in available boreholes and of the groundwater level are standard tools in production monitoring. Although ground subsidence and changes in the gravity field have been recorded in a few cases, the elastomechanical phenomena have not received much attention.

The purpose of this brief note is to present a preliminary evaluation of the potential of the elastomechanical methods in practical reservoir engineering and related areas. Assuming simple relevant situations, the strength of the field signals will be estimated and compared to other ground surface data such as gravity and D.C. electrical signals that are also of interest in reservoir monitoring. Because of greater difficulty in observing surface strain, we will limit our discussion to vertical ground displacement and tilt signals.

Basic Relations

For the sake of brevity we will consider only liquid dominated reservoirs embedded in porous/permeable half-spaces that are ultrasimple in the sense that the formation can be taken to respond in bulk as a homogeneous and isotropic Hookean solid. Because of the two-phase situation, all elastic parameters are composite and will consequently have to be defined properly. Moreover, Hooke's law must be generalized to include the effects of the pore or fracture fluid (see for example, Nur and Byerlee, 1971). In the present note, where only orders of magnitude are of interest, we will circumvent a more detailed discussion of these aspects by assuming that the saturated formation has empirically well defined effective elastic parameters. Armed with this set of quite strong but useful assumptions, we are now in the position of considering the effects listed in the introduction above.

Let the (x,y) plane of our coordinate system be placed in the surface Σ of the half-space with the z-axis vertically down. The general field point is P = (x,y,z). We assume that the reservoir is in equilibrium at the onset of production at time t = 0. Having at a later time produced a certain mass of fluid, we can assume that the subsurface temperature field has been perturbed by an amount T(P) and the subsurface fluid pressure field by p(P). Clearly, since the fluid pressure vanishes in the drained formation there is a discontinuity in the pressure field at the groundwater level.

Let the formation displacement vector resulting from both perturbations be $\vec{u}(P) = (u,v,w)(P)$. Moreover, let λ and μ be the effective Lamé parameters, k the effective bulk modulus, ν the Poisson ratio and α the effective thermal expansivity of the fluid/rock systems. Since the half-space is assumed to be Hookean, the elastomechanical equations for the displacement are

$$\mu\Pi\vec{u} - (\lambda+\mu)\nabla\nabla\cdot\vec{u} = \vec{b}, \qquad (1)$$

where $\Pi = -\nabla^2$ is the Laplacian operator and $\vec{b}$ is the body force density field resulting from the temperature and pressure perturbations. The boundary condition at the ground surface Σ is that of no stress. In the present case, $\vec{b}$ is a sum of two terms, one of thermoelastic origin associated with the perturbation temperature field T and a second one that results from the perturbation p of the pore pressure. Because of the discontinuity at the groundwater surface, it is convenient to split the second contribution into two parts, one associated with the perturbation pressure field in the wet formation and the other one resulting from the draining of the formations by the subsidence of the groundwater level. We have thus

$$\vec{b} = -\nabla f, \qquad (2)$$

where

$$f = f_T + f_p + f_g \qquad (3)$$

where the f's are scalars and the subscript T refers to temperature, p to fluid pressure, and g to groundwater level.

On the basis of the theory of thermoelasticity (Boley and Weiner, 1960) we obtain the first factor

$$f_T = \alpha k T \qquad (4)$$

There is some uncertainty as to the proper form of f_p. This factor depends quite heavily on pore/fracture geometry and connectivity. For the present purpose we will adapt the classical procedure of Biot (1941) by assuming

$$f_p = \theta p \tag{5}$$

where θ is a positive dimenionless factor quantizing the effects of the pore pressure p on the rock matrix. Clearly, this factor is less than unity, but actual values may vary within rather wide limits. Few experimental results are available. We can only quote elastomechanical data collected by Rice and Cleary (1976) from which values of $\theta = 0.2$ to 0.8 can be inferred. The lower values apply to samples of marble and granites whereas the higher values are obtained for sandstones. Along the same lines the third factor on the right of (3) can also be estimated on the basis of (5) where p is then the negative hydrostatic pore pressure in the drained rock above the groundwater level. Given p(P) and T(P), the problem of solving (1) for the displacement vector $\vec{u}(P)$ is now well defined.

A simple procedure of solving (1) at the above defined conditions has been presented by Bodvarsson (1976). We will refrain from discussing details of the method and quote only the result of main interest, that is, the expression for the vertical displacement component w at the ground surface Σ. Let $S = (x,y,o)$ be a field point on Σ and $Q = (x',y',z')$ be the source point; then

$$w(S) = \int g_w(S,Q)f(Q)dv_Q \tag{6}$$

where $dv_Q = dx'dy'dz'$ and $g_w(S,P)$ is the appropriate Green's function

$$g_w(S,Q) = [(1-\nu)/\pi(\lambda+2\mu)]z'/r_{SQ}^3 , \tag{7}$$

where

$$r_{SQ}^2 = (x-x')^2 + (y-y')^2 + (z')^2 \tag{8}$$

is the distance from Q to S. The tilt vector $\vec{t}(S)$ is obtained from (6) by $\vec{t} = -\nabla_h w$ where $\nabla_h = (\partial_x,\partial_x,o)$, the horizontal gradient in Σ. This vector can thus be expressed in the same form as (6) by

$$\vec{t}(S) = \int \vec{h}(S,Q)f(Q)dv_Q \tag{9}$$

where $\vec{h} = -\nabla_h g$.

Simple Situations

To present an overview of relevant field amplitudes, we will consider ground surface displacement and tilt fields generated by temperature or pressure perturbations within bounded compact source regions. Moreover, let the average depth of the source region be rather larger than its greatest linear dimension. Obviously, this situation is best portrayed by a spherically symmetric source region of radius R placed such that the depth d of the center is a few times larger than R. In the case of the first two factors in (3) the integrals in (6) and (9) can then be quite well approximated by simple expressions. With regard to the ground surface displacement we are most interested in the maximum value w_m that is obtained at the point 0 vertically above the center of the region. Assuming the Poisson relation $\lambda = \mu$ that applies quite well to common rock, and taking that $z' \simeq r_{OQ} \simeq d$, we obtain on the basis of (6) and (7) that

$$w_m = (15/36\pi)\Delta V/d^2 = 0.13\Delta V/d^2 \qquad (10)$$

where

$$\Delta V = (1/k)\int f dV_Q \qquad (11)$$

is the total volume increment. In the particular case that f is homogeneous over a spherical region of radius R equation (10) leads to

$$w_m = (5f/9k)R^3/d^2 \qquad (12)$$

A little algebra reveals that the maximum ground surface tilt is obtained at a distance (d/2) from the center 0 and amounts to

$$t_m = 0.11\Delta V/d^3 \qquad (13)$$

or in the spherical case

$$t_m = (0.48f/k)R^3/d^3 \qquad (14)$$

As already emphasized these expressions hold for the temperature and pressure perturbations involving f_T and f_P in equation (3). Since the third factor f_g results from the lowering of the groundwater level the source region for this effect is quite different from the compact region assumed above in the case of f_T and f_P. In fact, f_g is generated by an unloading of the half-space because of the drainage of the formations above the groundwater level. This results in two effects. First, a contraction of the drained formation as indicated directly by equations (3) and (5). Second, the unloading results in an upward displacement of the ground surface due to elastic rebound of the entire half-space. The second effect can be estimated on the basis of the Bussinesq formula for the surface loaded half-space (Love, 1927). Although analytical expressions are available for the form of the drained volume (Bodvarsson, 1977), we will here assume that to the first approximation, the form is that of a disk of thickness h and a diameter D. Using (3), (5) and the Bussingesq formula in combination with Poisson's relations we obtain then upon some algebra the maximum vertical ground surface displacement that occurs at the center of the disk

$$w_m = \phi g\rho h[(\theta h/5\mu) - (3D/8\mu)] \qquad (15)$$

where ϕ is the porosity, g the accelation of gravity and ρ the density of the fluid. It is important to emphasize again that this result is based on the assumption of a compact homogeneous/isotropic half-space. The second term in (15) will dominate under such circumstances. However, the presence of unconsolidated non-Hookean surface layers would invalidate the first term in (15) and generally lead to very much greater displacements as observed in some areas (see for example Hatton, 1970).

An estimate of the maximum ground surface tilt associated with the second term in (15) is given by

$$t_m = -3\phi g\rho h/4\mu \quad (16)$$

Gravity and D.C. Conduction Changes

Assuming the same situation as above, the lowering of the groundwater level leads to a reduced g-field at the center of the disk of

$$\Delta g = -2\pi\gamma\phi\rho h \quad (17)$$

where $\gamma = 6.67 \times 10^{-11}$ SI is the gravitation constant.

Referring again to a temperature perturbation T(P) within a compact source region, we can assume that the electrical conductivity within the region is increased by $\sigma_1(P)$ over the homogeneous half-space that has an uniform conductivity σ_o. Moreover, let there be an impressed horizontal unidirectional, uniform electrical field E_o that is scattered by the conductivity inhomogeneity. In the particular case that the source region is homogeneous spherical of radius R with the center at a depth d, the horizontal electrical field at the ground surface right above the center is given approximately by (Grant and West, 1965)

$$E = E_o(1-a) \quad (18)$$

where

$$a = [2\sigma_1/(\sigma_1+3\sigma_o)]R^3/d^3 \quad (19)$$

Discussion

Continued production of the magnitude referred to in the introduction may lead to average reservoir source region pressure declines of the order a few 10^5Pa. The groundwater level may be lowered by a few tens of meters and there may be a temperature decline of the order of a few tens of centigrades. Focusing our attention on the particular case of T = 50°C, p = 5×10^5Pa and assuming $k = 2 \times 10^{10}$Pa and $\theta = 0.4$, we find that $f_T = 100 f_P$. Clearly, the thermoelastic effects tend to dominate the pressure effects and thereby also the effects of the lowered groundwater level. Moreover, in conjunction with T = 50°C let $\alpha = 2 \times 10^{-5}/°C$, R = 300 m and d = 10^3m. Equation (10) and (13) give then the values w_m = 15 mm and t_m = 12 μrad. Referring to the same situation, let $\sigma_1 = -0.2\sigma_o$ and equation (19) gives a = -0.004. Assuming $\phi = 0.05$, h = 50 m, $\rho = 10^3 kg/m^3$, we obtain from (17) $\Delta g = 10^{-6}$ m/s^2 = 0.1 mgal.

Vertical ground displacement can be monitored with a precision of about one mm, surface tilt with a few 10^{-1} μrad, gravity with about 10 μgal and D.C. conduction in producing geothermal areas with no better precision than perhaps one part in 100. Although the above numerical data are grossly inadequate, the results, nevertheless, tend to indicate that in the absence of adequate borehole data, tilt observations at properly designed solid rock surface stations evidently offer the most appropriate method of monitoring large scale reservoir temperature evolutionary processes taking place at moderate depths.

References

Biot, M.A., 1941, General theory of three-dimensional consolidation, J. Appl. Phys., 12, 155-164.

Bodvarsson, G., 1976, Thermoelastic phenomena in geothermal systems, Second U.N. Symposium on the Development & Use of Geothermal Resources, San Francisco, 1975.

Bodvarsson, G., 1977, Unconfined aquifer flow with a linearized free surface condition, Jokull, 27:84-87.

Boley, B.A. and J.H. Weiner, 1960, Theory of Thermal Stresses, John Wiley, New York.

Grant, F.S. and G.F. West, 1965, Interpretation Theory in Applied Geophysics, McGraw-Hill Book Co., Toronto.

Hatton, J. W., 1970, Ground subsidence of a geothermal field during exploitation: UN Symposium on the Development and Utilization of Geothermal Resources, Pisa, Proceedings (Geothermics, Spec. Iss. 2), v. 2, pt. 2, p. 1294.

Love, A.E.H., 1927, A Treatise on the Mathematical Theory of Elasticity, 4th ed. Cambridge University Press, New York.

Nur, A. and J. D. Byerlee, 1971, An exact effective stress law for elastic deformation of rock with fluids, J. Geophys. Res., 76, 6414-6419.

Rice, J.R. and M.P. Cleary, 1976, Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents, Reviews of Geophysics and Space Physics, 14(2), 227-241.

Acknowledgement

This work was supported by the National Science Foundation under Grant No. EAR 77-23938.