A TECHNICAL ANALYSIS OF GEOTHERMAL PRODUCTION DATA BY DECLINE CURVE METHODS

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This project is an attempt to test traditional oil field decline curve methods on geothermal production data and develop new methods if necessary. We have large data sets from Wairakei, New Zealand and Cerro Prieto, Mexico along with a handful of sets from Larderello and Otake. Other data are being sought.

Arps's (1945, 1956) equations are the basis for most decline techniques. They are the solutions to the differential equation

$$a = Kq^{b} = -dq/dt/q \qquad (1)$$

where a is the fractional decline.
For b=0 we get the exponential form

$$q = q_i \exp(-at) \qquad (2)$$

For b > 0, we get the hyperbolic form

$$q = q_i(1 + ba_it)^{-1/b} \qquad (3)$$

where $b = da^{-1}/dt$ and a_i = initial fractional decline. The equations were derived empirically and no physical basis was ascribed to them until Fetkovich (1973) showed that the exponential form is a longtime solution of the constant pressure case. He developed log-log type curves for dimensionless time vs. dimensionless flow rate for $0 \leq b \leq 1$. See Fig. 1. Gentry and McCray (1978) discussed cases where b could be greater than 1 and presented an equation which might give better descriptions of decline behavior than Arps's.

$$\frac{N_p}{q_i t} = \left(\frac{q}{q_i}\right)^{\alpha} \qquad (4)$$

Our initial approach was to analyze all the data using equations 2 and 3 and then to try to develop correction terms to take temperature and geological effects into account. We started with the 34 highest producers at Wairakei and fit their production data using equation 2. Some results for Wairakei and Cerro Prieto are shown in Table 1 and Fig. 2 and 3. In addition to making least squares fit we graphed the data in the following ways, 1) q vs t, cartesian, 2) q vs t, semi-log, 3) q vs t, log-log, 4) N_p vs t, log-log, 5) N_p vs q, log-log, and 6) q_i/q vs N_p/q_it, semi-log. We tried to fit the log-log graphs on the several type curves with very little success because of data scatter. Other data sets

Editor's Note: the Fetkovich study was based on a study by Tsarevich and Kuranov (1958), who first explained the analytical basis for exponential depletion.

tested gave no better results than Wairakei. However, the computer fits indicate that many wells at Wairakei can be considered to be declining exponentially. This fits with Grant's comment (1979) that the New Zealand DSIR regards the field to be declining exponentially. See Fig. 4.

The hyperbolic equation was more difficult to deal with than the exponential, but we now have a working program in which an objective function is defined and then minimized to find the optimal values for q_i, a_i, and b. Using the data for Pozo M-15, Cerro Prieto (cf Rivera-R, 1977, well A) we got $q_i = 229$, $a_i = 0.05$, and $b = 7$ with $R^2 = 0.97$ indicating an excellent fit. The relation between b and a given in equation 3 does not hold in this kind of a fit because a_i, b, and q_i were only constrained to be non-negative. No relation between a and b was prescribed. We earlier attempted to find a_i, b, and q_i from the data following Guerrero (1961) but the scatter made the procedure chancy at best. Gentry's and McCray's (1979) graphical methods for a_i were also equivocal for our data.

In testing equation 3 we had hoped to use b as some sort of a correlating parameter perhaps discriminating between highly and slightly fractured reservoirs; we can make no such conclusions at this time. If the hyperbolic equation is used it should be regarded only as a curve fit and be used accordingly.

We do not yet have sufficient vapor-dominated field data to test Pruess's et al (1979 a,b) contention that P/z vs G curves should not be used for steam fields in the same way that they are used in gas field analysis. Their argument that temperature drop not loss of mass controls production in a steam field which has a boiling water interface seems reasonable and should be further examined.

We are now trying to develop a model following Bodvarsson's linearized free surface approximation (1977, 1978) using Green's functions to model the pressure behavior of a field as a function of flow rate. The equations we are looking at are

$$G = \frac{-1}{2\pi\Phi\rho}(wt + d)\left(x^2 + y^2 + (wt + d)^2\right)^{-3/2} U_+(t) \qquad (5)$$

$$p(t) = \int G(t - \tau)q(\tau)d\tau \qquad (6)$$

where
G = Green's function
p = pressure
w = sinking velocity = permeability x gravity/kinematic viscosity x porosity - $\frac{kg}{\nu\Phi}$
d = depth to producing zone
t = time
U = unit impulse function

See Fig. 5 for a graph of p vs t for Bore 18 at Wairakei.

In addition we are looking at influence functions as discussed by Coats et al (1964), Jargon and Van Poolen (1965) and Hutchinson and Sikora (1959). The relevant equations are

$$\Delta p = \int_0^{\tau} q(T) \quad F(\tau - T) \; dT$$

$$\Delta p_i = \sum_{j-1}^{i} (q_i - g_{j-1}) \; F_{i-j+1}$$

with the constraints

$$F(t) \geq 0$$

$$\frac{dF(t)}{d_t} \geq 0$$

$$\frac{d^2F(t)}{dt^2} \geq 0$$

The influence functions should be found using linear programming with the above constraints to prevent physically meaningless results from being generated. See Fig. 6 for the results obtained from fitting 15 years of total field data from Wairakei.

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TABLE 1 Exponential Fit

Well Name	Percent Decline/year	R^2
Bore 18	5.8	0.75
Bore 26B	6.3	0.68
Bore 42	21.0	0.80
Bore 56	11.8	0.86
Bore 72	7.2	0.90
Pozo M-15	17.5	0.59
Pozo M-34	27.2	0.84

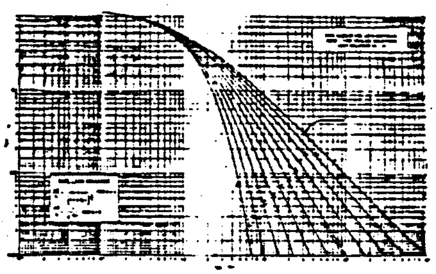

Figure 1

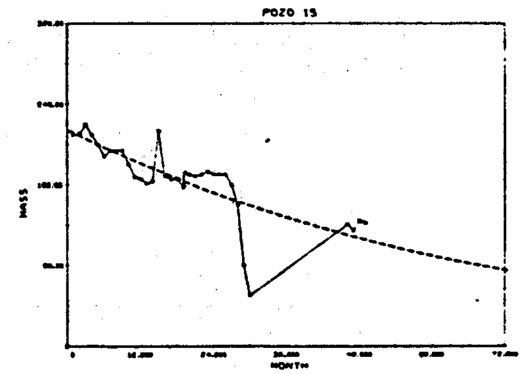

Figure 2

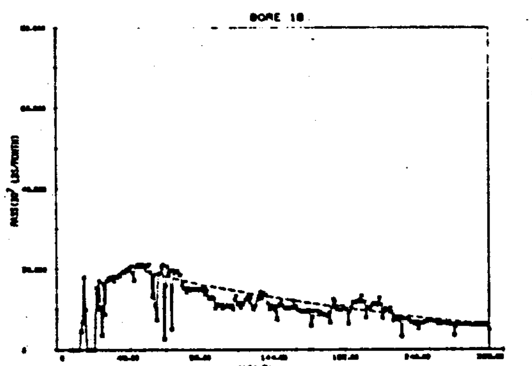

Figure 3

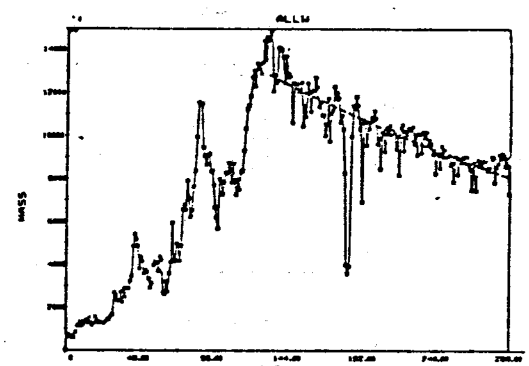

Figure 4

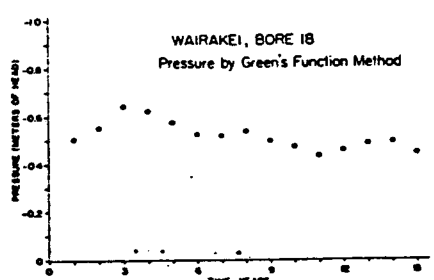

Figure 5

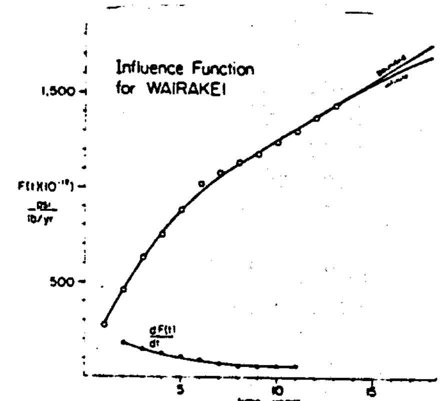

Figure 6

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