VALIDATION AND COMPARISON OF DIFFERENT MODELS OF THE WAIRAKEI GEOTHERMAL RESERVOIR

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INTRODUCTION

Records of pressure, temperature, and mass and energy discharge for the Wairakei geothermal field in New Zealand have been available since 1953. Power production began in 1958 and today Wairakei remains as one of the few liquid-dominated systems to be exploited for electrical power. Numerous types of conceptual and mathematical models have been developed to describe the significant features of the Wairakei field under natural and exploited conditions. On the whole, most existing models have yielded results which are reasonably compatible with the observed pressure versus discharge response, be they 0, 1, or 2 dimensional. In such a situation, the choice of model to be used for prediction of future response or for analysis of optimum field management procedures is a difficult task.

In this paper we concentrate on the zero-dimensional, or lumped parameter models of the Wairakei reservoir and use system identification techniques to find the best analytical expression for the average pressure response. We show that the corresponding conceptual model may be interpreted as a slow-drainage model in which portions of the reservoir where two-phase conditions develop serve mainly as a source of liquid for the underlying single-phase region from which most of the production is obtained. A more detailed discussion of these techniques and comparisions with the results obtained from other lumped and distributed parameter models will be given in a forthcoming paper by Fradkin and Sorey (to be submitted to Water Resources Research).

RESERVOIR DESCRIPTION

Hyrothermal phenomena at Wairakei are related to a column of hot water rising over a magmatic heat source at a depth of about 10 km in a cold water environment. The areal extent of the hot fluid column is about 15 km^2 near the surface, although an additional hot water area of comparable size underlies the Tauhara region to the southeast and is connected hydraulically to the Wairakei field. Most of the production wells drilled at Wairakei range in depth from 0.2 km to 1.5 km.

Prior to 1953, the vertical pressure gradient in the reservoir was slightly in excess of hydrostatic, producing an upflow of liquid to feed the natural surface discharge features. By 1958 a steam zone had formed at shallow levels over portions of the reservoir, and by 1962 two-phase conditions extended over most of the reservoir area (Grant, 1979). Above the two-phase zone cool ground water occurs in the upper 0.1-0.2 km except near natural dicharge vents where most of the liquid upflow has now been replaced by steam discharge. The depth to which two-phase conditions extend at present varies with location but is generally close to 0.60 km below land surface (Donaldson and Grant, 1979).

Production of hot water and steam from the Wairakei reservoir is obtained from a sequence of Pleistocene volcanic rocks consisting of pumice breccias, welded tuffs, and flows. Most of the permeable features tapped by wells are fault and fissure zones and contact zones between different volcanic units. A significant feature of the response to development at Wairakei is that differences in pressure between wells in various parts of the reservoir have remained significantly less than the average decline in pressure with time (Bolton 1970). Two factors appear to be responsible for the uniform nature of the pressure response. The first is a high degree of lateral permeability connecting wells in different parts of the field. The second is the dominance of single-phase (liquid) conditions in the diffusion of pressure changes across the reservoir.

In most of the previous modeling efforts, the average pressure data of Bolton (1970) have been used as the principal or the only datum for model verification. To provide a more detailed and consistent pressure history, we have calculated average reservoir pressures for each month between 1959-1977 based on measurements from a group of about 50 deep wells within the Western Production Area (as defined by Bolton, 1970). For each well, measured downhole pressures were extrapolated to a common reference level of 274 m below mean sea-level (RL-274 m). In this data set the variations in pressures between wells at RL-274 averaged about ± 1 bar. The resultant representative reservoir pressure history shown in figure 1 shows the smoothing introduced by Bolton (1970).

Variations in total mass produced are also plotted in figure 1. Maximum production from the field was obtained in 1963 and a partial shutdown occurred for three months in 1968. The average enthalpy of the discharged fluid, initially near the the enthalpy of liquid water at 253^{o}C, has increased less than 5 percent due to the contribution of wells feeding from two-phase portions of the reservoir.

LUMPED PARAMETER MODELS

Previous models of Wairakei differ slightly in definition of the reservoir boundries and each treats the reservoir rock as a porous medium (although the drainage models described below allow for two-phase flow in a fractured media). The main difference between models lies in the assumed state of the reservoir fluid. The reservoir is correspondingly termed liquid, two-phase, or vapor if it is saturated with liquid, water and steam, or steam only. We use the term mixed reservoir if the state of the fluid varies with location. Because it is not always easy to determine the extent to which two-phase conditions develop within a reservoir during exploitation, the Wairakei reservoir has been modeled as two-phase by some and as liquid or mixed by others. The available data, including the areal uniformity of pressure response at deeper levels, the average and well-by-well discharge enthalpy histories, and the repeat gravity measurements (described later), indicate that Wairakei is a mixed reservoir for which certain simplifying assumptions can be made to obtain a physically meaningful lumped parameter model of the average reservoir pressure response.

The Wairakei reservoir is defined as a vertical cylinder extending from the initial level of the boiling surface ($H=H_o$) down to the level 1.5 km below mean land surface ($H=0$). The reservoir area is assumed constant with depth and equal to the area of the hot fluid column within the depths of drilling. We first make the simplifying assumptions that changes in the mass of steam within the reservoir and cooling due to vaporization are negligible and that recharge flows only into the liquid portions of the reservoir. Then integration of the mass conservation equation over the reservoir volume leads to the following global conservation equation:

$$\frac{dM_w}{dt} = r_w - q \tag{1}$$

where $M_w = A \int^{H_o} S\phi\rho_w dh$ is the total liquid mass, r_w is the total liquid recharge rate, and q is the total mass discharge rate. We will assume further that porosity ϕ is constant and that the liquid saturation S and and density vary only with depth (and time).

Assuming isothermicity, the energy conservation equation can be neglected. Temperature changes in the deeper, liquid-fed bores have been small; but production from shallow two-phase bores has produced local declines in pressure and temperature in the two-phase zone which may be responsible for the present-day decline in field output with stabilized reservoir pressures (Grant, 1978).

The recharge rate from the surrounding cold ground-water system induced by changes in pressure in the hot liquid portion of the reservoir was given by Grant (1977) as

$$r_w = K(P_i - P) \tag{2}$$

where the recharge coefficient $K = 0.4\pi R\rho_w k/\mu_w$ and P_i is the initial reservoir pressure at RL-274 m. R represents the equivalent radius of the hot reservoir and k the average permeability of the field. Significant horizontal inflow of cold water should occur to depths of about twice the maximum producing level and the constant 0.4 is a scaling factor which takes into account the reduction in horizontal inflow with depth (Grant, 1977). The gradual reduction in the area of hot water due to the induced cold water recharge is shown by Robinson (1977) to be slow compared with the 20 year production history at Wairakei.

Three representative lumped parameter models of the pressure response at Wairakei can be compared with reference to the mathematical model described by equations 1 and 2. If storage changes due to water decompression are assumed dominant, equation 1 becomes

$$AH_o\phi\left(\frac{d\rho_w}{dP}\right)_T \frac{dP}{dt} = K(P_i - P) - q \tag{3}$$

representing the Wairakei model of Whiting and Ramey (1969). If storage changes due to a decline in the boiling level are assumed dominant, the following equation is obtained for the case of instantaneous drainage of liquid from the two-phase zone (Grant, 1977)

$$\frac{A\phi}{g} \frac{dP}{dt} = K(P_i - P) - q \tag{4}$$

Alternatively if drainage from the two-phase zone is not instantaneous, variations in the liquid saturation above the declining boiling level must be described. Following McNabb (1975), this can be done for a system of rapidly draining fractures surrounding less permeable porous blocks. An equation of the form

$$\frac{dP}{dt} = a(P_i - P) + bq + c\frac{dq}{dt} \tag{5}$$

is obtained for the case where the relative permeability to liquid varies linearly with S. The coefficients in (5) can be expressed in terms of the parameters $A\phi$,K and S_o(the residual liquid saturation for the blocks).

MODEL VALIDATION

Several techniques have been used to validate different models of the Wairakei response. With the distributed models of Mercer and Faust (1979), Pritchett, Garg, and Brownell (1976), and Garg, Rice, and Pritchett (1979), the direct approach involving trial and error adjustment of parameters to obtain a visually satisfying fit to the pressure history has been used. In the case of the previous lumped parameter models, least-squares regression was used to fit the pressure curve. Several factors complicate the analysis of the Wairakei pressure data, including the presence of linear feedback, in which the discharge rate becomes linearly dependent on the reservoir pressure when the number of wells discharging remains unchanged. In the presence of linear feedback, arbitrarily large biases in parameter estimates can be introduced though the fit remains good. Feedback and other identifiable problems account for the fact that analysis of different portions of the Wairakei pressure data often lead to significantly different results.

System identification techniques based on the work of Young (1972) can be used to find the best type of simple equation and unbiased parameter estimates for the pressure and discharge data shown in figure 1. The best lumped parameter model has proved to be the discretised form of equation 5, with constant coefficients. This slow drainage model is also capable of predicting the observed response during part of the production history based on parameter estimates determined from a different part of the history. Previous Wairakei models did not possess this predictive power.

As a part of the validation process, the interpretation of parameters according to a conceptual model should lead to meaningful values for the physical parameters. For the lumped parameter models discussed previously, fitting the decompression model based on equation 3 to the pressure history yields an unreasonably large value for the parameter group $A\phi$ of 1.2×10^5 km^2. Fitting the instantaneous draninage model based on equation 4 and assuming $A = 15$ km^2, yields $\phi = .09$ and k = 50md. In this case parameter values are not unreasonable, the relatively low value of porosity being consistent with the inference that only the permeable fractures can drain rapidly.

Interpretations of the coefficients in equation 5 in terms of the slow drainage model, assuming $A = 15$ km^2 and $S_o = .30$, yields $\phi = .24$ and k = 25 md. Porosity estimates based on this model apply to the combination of fractures and blocks and are in good agreement with average values from laboratory tests on cores. The permeability estimate of 25 md is consistent with values determined from other considerations (McNabb, Grant, and Robinson, 1975). The corresponding value for the recharge coefficient K in equation 2 is 0.06 cm sec; and from this, estimates of the ratio of recharge to production can be calculated as listed below.

1958-61	1961-67	1967-74
.28	.48	.80

Corresponding estimates based on the repeat gravity data of Hunt (1977) are

1958-61	1961-67	1967-74
.30 ± .15	.35 ± .15	.90 ± .15

Agreement with the gravity data is considerably better with this model than with the other lumped parameter models and also better than with the distributed parameter models noted previously. The gravity data provide an important constraint on these models. If the amounts of recharge are not simulated correctly, storage changes associated with the two-phase zone may be incorrectly described.

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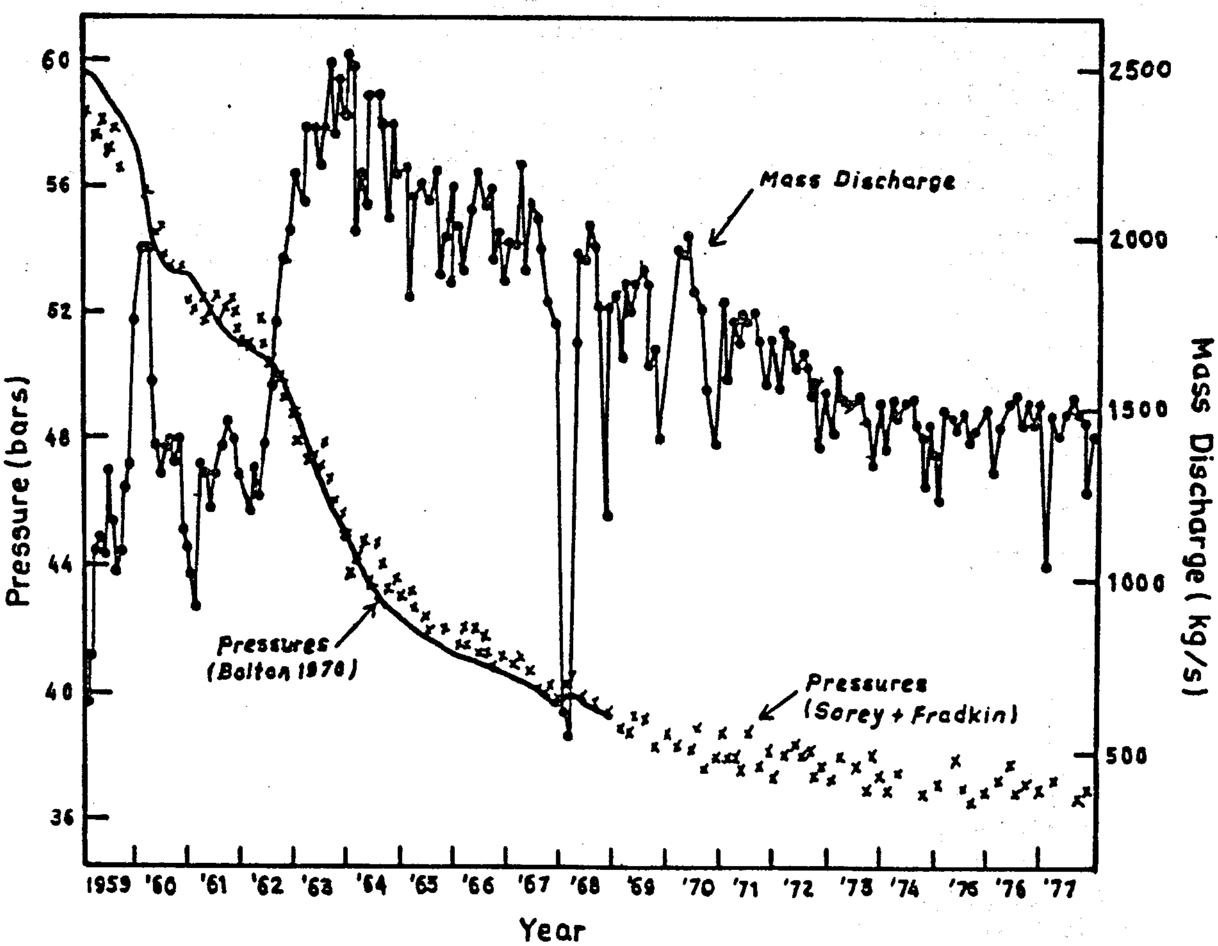

Figure 1.-- Monthly mean discharge and pressure for Wairakei reservoir