BLOCK RESPONSE TO REINJECTION IN A FRACTURED GEOTHERMAL RESERVOIR

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INTRODUCTION

While fractured porous formations constitute an important class of geothermal reservoirs, a generally applicable methodology for accurately describing their performance has yet to appear in the literature. As a preliminary step in our attempt to unravel the fundamental physical principles involved in fractured media simulation, we herein describe an elementary yet interesting numerical experiment. A porous media block containing superheated steam is suddenly enclosed by reinjection water of lower enthalpy. The objectives of the experiment are 1) to examine the response of the porous block to reinjection and 2) to study the influence of block geometry on the simulated behavior. The numerical experiments are conducted using a one-dimensional multiphase (steam-water) model. With the model defined in Cartesian coordinates, we investigate a porous block delimited by two parallel fractures of infinite extent: the model defined in radial coordinates is employed to examine a cylindrical geometry. The reservoir parameters and reinjection water properties are chosen to be representative of those encountered in The Geysers geothermal field in California.

GOVERNING EQUATIONS

The governing equations and auxiliary conditions employed in the numerical experiments are the following:

For the Cartesian case:

(1a) $$\frac{\partial}{\partial x}\left(\tau \frac{\partial p}{\partial x}\right) = \frac{\partial F}{\partial t}$$

(1b) $$\frac{\partial}{\partial x}\left(\lambda \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(\beta \frac{\partial h}{\partial x}\right) = \frac{\partial G}{\partial t}$$

(1c) $$h(\ell,t) = \bar{h}\ , \quad \frac{\partial T}{\partial x}(0,t) = 0$$

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(1d) $p(\ell,t) = \bar{p}$, $\frac{\partial p}{\partial x}(0,t) = 0$

(1e) $p(x,0) = p_0(x)$, $h(x,0) = h_0(x)$ $0 \leq x \leq \ell$

and for the radial case, (assuming pressure and enthalpy are independent of θ, z),

(2a) $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau \frac{\partial p}{\partial r}\right) = \frac{\partial F}{\partial t}$$

(2b) $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial p}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\beta \frac{\partial h}{\partial r}\right) = \frac{\partial G}{\partial t}$$

(2c) $h(r_e,t) = \bar{h}$, $\frac{\partial T}{\partial r}(0,t) = 0$

(2d) $p(r_e,t) = \bar{p}$, $\frac{\partial p}{\partial r}(0,t) = 0$

(2e) $p(r,0) = p_0(r)$, $h(r,0) = h_0(r)$ $0 \leq r \leq r_e$

where p is the fluid pressure

h is the enthalpy of the fluid mixture

$\tau = \tau_w + \tau_s$

$\tau_\alpha = \frac{\rho_\alpha k k_{r\alpha}}{\mu_\alpha}$ $\alpha = s,w$

$F = \rho\phi$

$\rho = S_w\rho_w + S_s\rho_s$

$$\lambda = \begin{cases} K \frac{\partial T}{\partial p} + h_s^* \tau_s + h_w^* \tau_w & h_w \leq h \leq h_s^* \\ K \frac{\partial T}{\partial p} + h\tau & h < h_w^* \text{ or } h > h_s^* \end{cases}$$

$$\beta = K \frac{\partial T}{\partial h}$$

$$G = \phi \rho h + (1-\phi)\rho_r h_r$$

S_α is the saturation of the α phase

ρ_α is the mass density of the α phase

k is the saturated permeability

ϕ is the porosity of the porous media

$k_{r\alpha}$ is the relative permeability of the α phase

μ_α is the dynamic viscosity of the α phase

h_α^* is the saturated enthalpy of the α phase

T is the temperature, and

K is thermal conductivity

The constitutive relationship for relative permeability is assumed to depend on water saturation in the following manner,

(3a) $$k_{rw} = S_w^2$$

(3b) $$k_{rs} = (1-S_w)^2$$

SOLUTION PROCEDURE

Given the necessary thermodynamic information and reservoir parameters, Eqs. 1 and 3 or 2 and 3 constitute a complete mathematical description of the fractured porous media model. These equations are approximated using finite-difference methods. The thermodynamic regression relationships and further details regarding the computer code are found in Shapiro, 1979.

SIMULATION

The reservoir properties employed in the two models are given in Table 1. The initial and boundary conditions are found in Table 2.

TABLE 1: RESERVOIR PROPERTIES

Porosity	0.0500
Rock density, ρ_r	2.19 gm/cm^3
Thermal conductivity	0.0377 joules/ (cm-sec-°C)
Saturated permeability	1.97×10^{-11} cm^2

TABLE 2: AUXILIARY CONDITIONS

		Cartesian	Radial
Initial Conditions	pressure p_0 (dynes/cm^2)	2.07×10^7	2.07×10^7
	enthalpy h_0 (Joules/gm)	2.85×10^3	2.85×10^3
Boundary Conditions	pressure $\bar{p}$ (dynes/cm^2)$\big\|_{\substack{r=10m \\ x=10m}}$	5.975×10^7	5.975×10^7
	enthalpy $\bar{h}$ (Joules/gm)$\big\|_{\substack{r=10m \\ x=10m}}$	8.84x10	8.84x10
	pressure gradient $\left.\frac{\partial p}{\partial x}\right\|_{\substack{x=0 \\ r=0}}$	0	0
	temperature gradient $\left.\frac{\partial T}{\partial x}\right\|_{\substack{x=0 \\ r=0}}$	0	0

It is apparent from Table 2 that the Cartesian and radial models employ the same auxiliary conditions. Thus a comparison of the two simulations demonstrates the significance of block geometry. We will return to this comparison shortly.

Let us first consider the behavior of the block delimited by parallel planar fractures, as shown in Figs. 1 and 2. The block is assumed to have a minimum dimension of 20 m from one fracture plane to the other. Initially, the system is in a superheated steam state corresponding to a pressure of 2.068×10^7 dynes/cm^2 (300 psia), a temperature of 230°C with a resultant enthalpy of 2.846×10^3 Joules/gm. At an arbitrary point in time, the block comes in contact with reinjected water with the following characteristics: the pressure is 5.975×10^7 dynes/cm^2 (866.7 psia, corresponding to a depth of 2000 ft), the temperature is 21.1°C, and the corresponding enthalpy is 8.843×10^1 Joules/gm. It is assumed that the fracture flow is adequate to approximately maintain the temperature and pressure of the fluid in the fractures. Because of symmetry, only one half of the region need be considered (i.e., a minimum dimension of 10 m).

Under the above conditions, there is an interesting exchange of energy. The pressure front propagates from the fracture toward the center of the block while the thermal energy flows in the opposite direction. This behavior is illustrated in the enthalpy and pressure curves of Figs. 1a and 1b. Figure 1b also indicates that the temperature profile propagates very slowly. Evidently, the block energy is utilized primarily to heat the infiltrating reinjected water. This water is subsequently cooled via conduction to the fracture.

The movement of the saturation profile is particularly interesting. Figure 1a illustrates a nearly step-like propagation of the liquid water from the fracture towards the center of the block. Note that the relative permeability functions require $k_{rw} \rightarrow 0$ as $S_w \rightarrow 0$. In as much as we assume S_w to be zero in the superheated steam region, k_{rw} is also initially zero. An alternative assumption regarding initial residual water in the block should result in a considerably different saturation profile.

The circular cylinder model is illustrated in Fig. 2. The fracture is assumed to define a block of circular cross-section having infinite length and a radius of 10 m. The boundary conditions imposed in this model are essentially identical to those employed in the parallel fracture case. In this somewhat busy figure, we attempt to illustrate how the four variables (pressure, enthalpy, temperature, and saturation) are interrelated as would be expected. We observe that the transition from superheated steam to water results in a marked change in the enthalpy. There is also a change in the slope of the pressure profile at the saturation front. The temperature, however, appears unaffected by the phase transition.

The significance of block geometry is illustrated through a comparison of Figs. 1 and 2. In general, there is no marked change in behavior between the two models: in the radial case the pressure in the superheated steam region appears somewhat higher. While two solutions for a common point in time are not available, it is nevertheless apparent that, to an elapsed time of 160 minutes, the propagation of energy and pressure are essentially the same in both models. One would anticipate a deviation in the response of the two models at a later time, but financial constraints precluded the further investigation of this phenomenon.

SUMMARY

Multiphase finite difference geothermal models are used to examine the thermal and mechanical energy transfer between a fracture and two porous blocks of differing geometry. One block is delimited by parallel fractures at a distance of 20 m; the second block is a cylinder with circular cross section of radius 10 m. The auxiliary conditions and reservoir properties are selected to correspond to conditions encountered at The Geysers geothermal field. The simulations indicate that liquid water residing in the fractures propagates into the block and is heated to the original reservoir temperature. Thermal energy is simultaneously conducted from the block by this liquid water. During the period of analysis there appears to be no significant difference in the general behavior of the parallel fracture and circular fracture models. It would be interesting and informative to extend the experiments described above until the blocks become water saturated. The role of fracture spacing (block size) also warrants investigation.

The simulation of the problems described herein required considerable computational effort. To achieve convergence of the numerical solution, relatively small time increments were required. In the Cartesian case, it was necessary to use $\Delta t \leq 2.0$ seconds for $\Delta x = 20$ cm. The radial model required smaller time increments as the simulation progressed. At 131 minutes, suitable parameters were $\Delta t \leq 0.25$ seconds for $\Delta x = 20$ cm.

ACKNOWLEDGEMENT

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REFERENCE

Shapiro, A.M.: "One-Dimensional, Finite Difference Model for Single- and Two-Phase Flow in Hydrothermal Reservoirs," Princeton University Water Resources Program Report No. 79-WR-10, 1979.

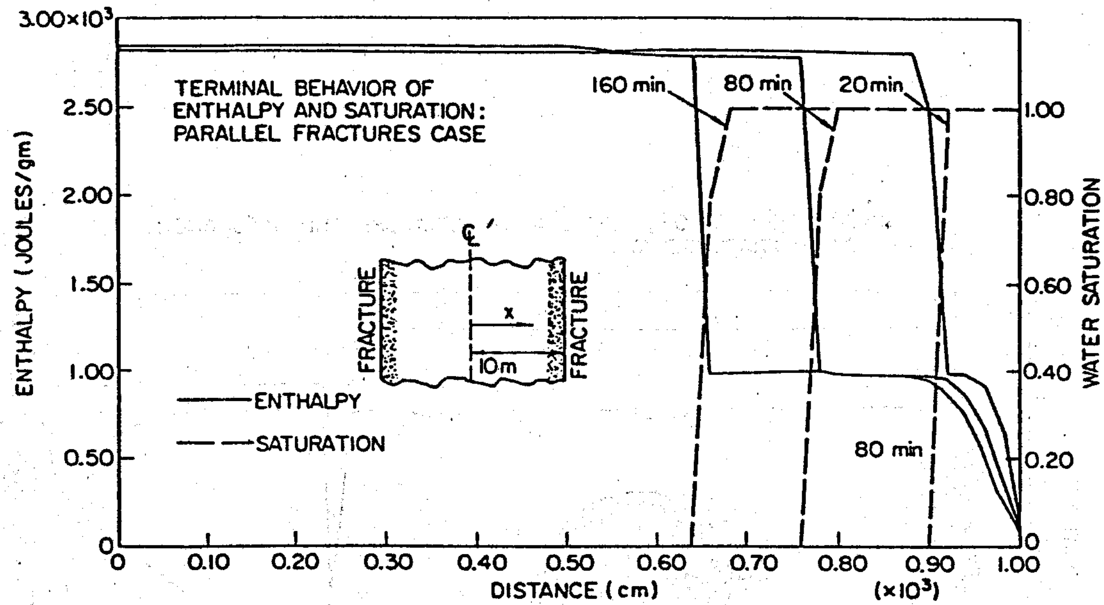

FIG. 1a: PROPAGATION OF ENTHALPY AND SATURATION FROM PARALLEL PLANAR FRACTURES INTO A POROUS MEDIUM BLOCK

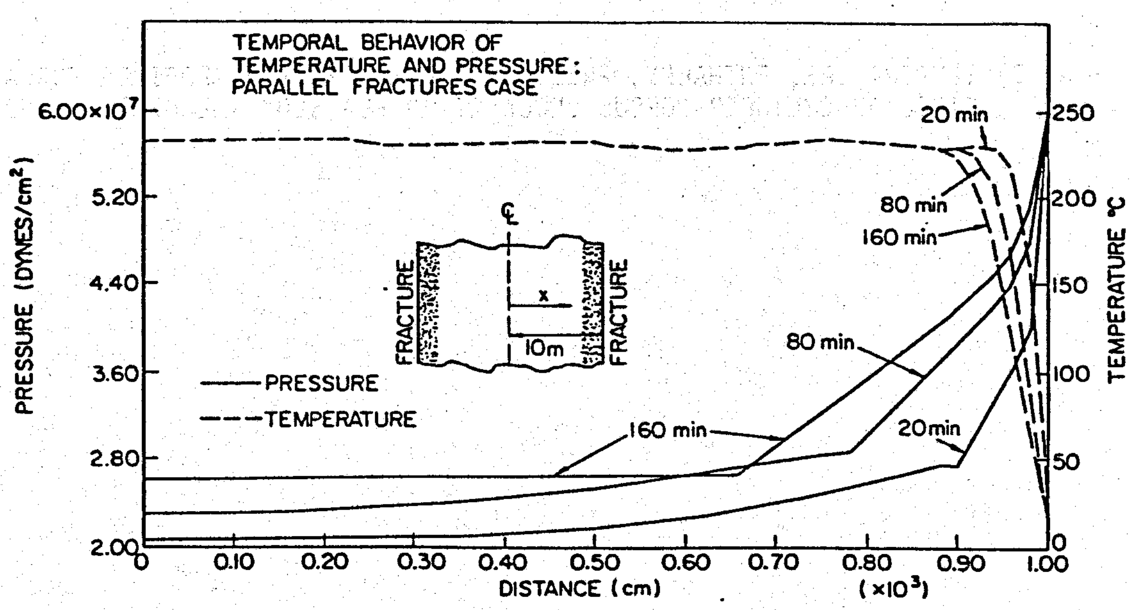

FIG. 1b: PROPAGATION OF TEMPERATURE AND PRESSURE FROM PARALLEL PLANAR FRACTURES INTO A POROUS MEDIUM BLOCK

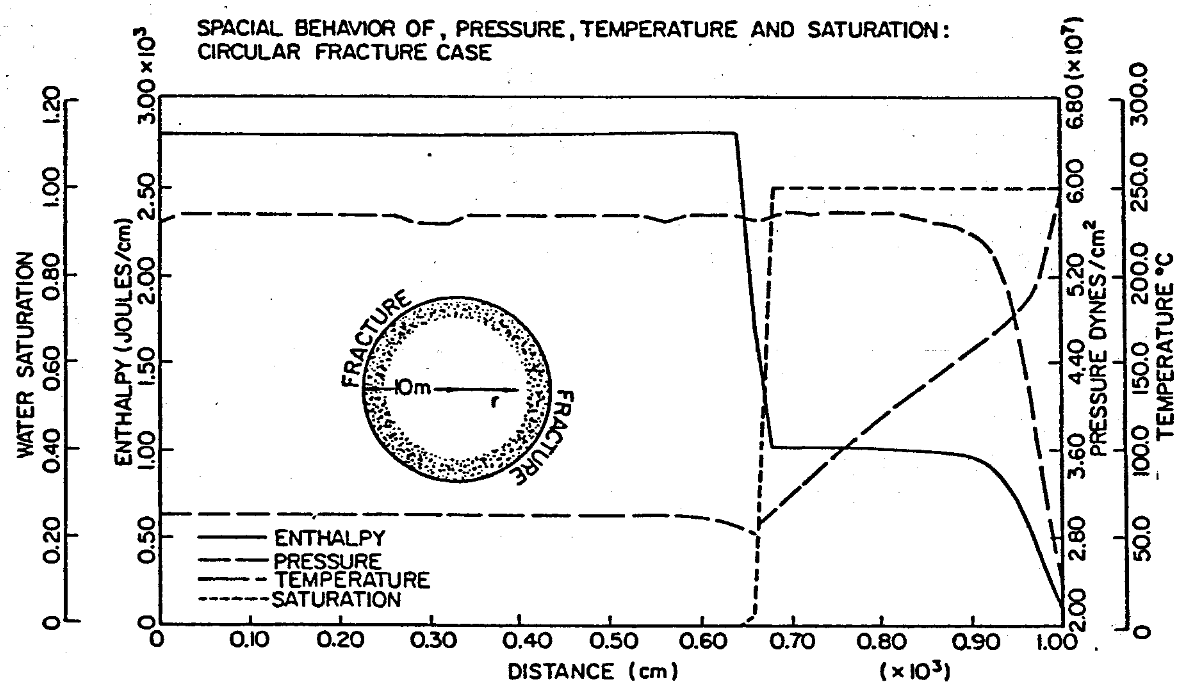

FIG. 2: TEMPERATURE, ENTHALPY, PRESSURE AND SATURATION PROFILES FOR A CIRCULAR CYLINDER POROUS BLOCK AT AN ELAPSED TIME OF 131 MINUTES