ANALYSIS OF A "SLUG TEST" OR DRILL STEM TEST FOR LINEAR FLOW

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INTRODUCTION

The objective of this work is to provide a means for analyzing pressure transients from drill stem tests (DSTs) in fractured wells dominated by linear flow in the formation. The consequent partial differential equations have been solved by numerical inversion of the Laplace transform. A DST consists of isolating an open stretch of borehole with packers and connecting it instantaneously with the atmosphere by means of a drill-string. In water-dominated reservoirs, at the end of the test a level usually stabilizes in the string and the well does not produce spontaneously. The method can be applied only to the "analysis of DSTs in which the fluid influx into the drill pipe tends to kill the flow, giving only a partial DST recovery."[1] The analysis is performed by type-curve matching, and leads to the determination of the initial reservoir pressure, flow characteristics of the formation, and wellbore damage. These parameters allow the prediction of early-time pressure drawdown due to a constant flowrate.

Methods already exist in the literature[1] for analyzing these pressure transients in the presence of radial flow in the formation. In the fractured reservoirs found in some geothermal fields, linear flow models seem more appropriate than radial models.[2]

DESCRIPTION OF THE MODEL AND RESULTS

Figure 1 presents the assumed physical model. It consists of:

a) A homogeneous one-dimensional rock formation in which the pressure propagates according to the diffusion equation:

$$\frac{\partial^2 v(x,t)}{\partial x^2} = \frac{1}{\eta}\frac{\partial v(x,t)}{\partial t} \tag{1}$$

where: $v(x,t)$ = pressure drawdown in the formation
x = distance from well
t = time
η = formation diffusivity, $k/\phi\mu c$

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Equation 1 describes fluid flow in a porous medium according to Darcy's law and the continuity equation, and assuming the fluid has a small constant compressibility.[3]

b) A concentrated flow resistance, R, that produces a pressure drawdown proportional to the flowrate through it. This could, for example, be caused by a layer of mud on the walls of the bore. The following equation therefore holds:

$$Rq(t) = u(t) - v(o,t) \tag{2}$$

where $q(t)$ = instantaneous volumetric flowrate
$u(t)$ = pressure drawdown at bottomhole

c) The fluid produced by the formation is stored in the drillstring, causing a back-pressure on the formation proportional to the fluid stored. Therefore, we have:

$$- C \frac{du(t)}{dt} = q(t) \tag{3}$$

where C is a constant:

$$= \frac{\text{internal cross-section of drill pipe}}{\text{specific weight of fluid}} = \frac{A}{\gamma}$$

The partial differential equation (Eq. 1), along with the boundary conditions (Eqs. 2 and 3), Eq. A6 (in the Appendix), and with the initial conditions:

$$v(x,o) = 0$$

$$u(o) = V$$

$$x > 0$$

was solved by means of the Laplace transform, as in Carslaw and Jaeger.[4] The solution is:

$$\frac{\overline{u}(p)}{V} = \frac{\sigma\sqrt{p} + 1}{\sqrt{p}(\sigma p+\sqrt{p}+1)} \tag{4}$$

where $\overline{u}(p)$ is the Laplace transform of $u(t)$ and p its parameter, while σ is a dimensionless constant:

$$\sigma = \beta CR$$

where $\beta = \left(\frac{Sk}{\mu C}\right)^2 \frac{1}{\eta}$

See the Appendix for further details.

Unfortunately, the analytical inversion of Eq. 4 is not straightforward, so the numerical method given in Eq. 5 is used. The results are given in Tables 1 and 2 for different values of σ, and summarized in the type-curves 1, 2, and 3. The computer code is also included for the numerical inversion of Eq. 4.

USING THE TYPE-CURVES

a) If the initial reservoir pressure is unknown, type-curves 1 and 2 should be used. These show:

$$\frac{V-u(t)}{V} = 1 - \frac{u(t)}{V}$$

as a function of $t_D = \beta t$ on a log-log graph. The pressure recovery, $V-u(t)$, should be plotted against time, t, on log-log tracing paper of the same scale. The field data graph should be shifted without rotation over the type-curves until the best match is obtained. V can be estimated; hence, initial reservoir pressure from the vertical match.

From the horizontal match, we can estimate the value of the constant:

$$\beta = \frac{t_D}{t} = \left(\frac{Sk}{\mu C}\right)^2 \cdot \frac{1}{\eta}$$

From the selected type-curve, we obtain:

$$\sigma = \beta C R$$

3) If the initial reservoir pressure is known or was estimated as described in a), type-curve 3 should be used, since it permits a better evaluation of the parameters. In this case, semilog tracing paper is used, plotting $u(t)/V$ versus time, t. Once again, the scales should be the same as those of the type-curve.

In this case, we can obtain the match only by shifting the data horizontally over the type-curve, obtaining:

$$\beta = \frac{t_D}{t} = \left(\frac{Sk}{\mu C}\right)^2 \cdot \frac{1}{\eta}$$

and:

$$\sigma = \beta\ R$$

In both cases a) and b), having evaluated σ and β and knowing $C = A/\gamma$, we obtain R and the group $(Sk/\mu)^2 \cdot 1/\eta$, which controls linear flow.

From the above parameters, it is possible to forecast the early-time pressure transient caused by a constant flowrate. In fact, from Eq. 6:

$$v(o,t) = \frac{2q\sqrt{\eta}}{\sqrt{\pi}Sk\mu} \cdot \sqrt{t} = \frac{2}{\sqrt{\pi}} \cdot \frac{q}{C\sqrt{\beta}} \cdot \sqrt{t} \qquad (5)$$

Hence:

$$u(t) = Rq + \frac{2}{\sqrt{\pi}} \cdot \frac{q}{c\sqrt{\beta}} \cdot \sqrt{t} = q\beta c \left(\sigma + \frac{2}{\sqrt{\pi}} \sqrt{\beta t}\right) \qquad (6)$$

All the parameters in Eq. 6 are either known or determined by this analysis. Where R is too high, i.e., where σ is large in comparison to $2/\sqrt{\pi}\ \sqrt{\beta t}$, stimulation could prove useful.

ACKNOWLEDGEMENTS

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NOMENCLATURE

A = internal cross-section of drill pipe, m^2

c = effective compressibility, Pa^{-1}

$C = \frac{A}{\gamma}$ = fluid storage constant in drill string, $m^4 kg^{-1} s^2$

$D(p)$ = function independent of x, Pa

$E(p)$ = function independent of x, Pa

k = formation permeability, m^2

p = Laplace transform parameter

$q(t)$ = volumetric flowrate from formation to drill pipe, $m^3 s^{-1}$

R = concentrated flow resistance, $m^{-4} kg\ s^{-1}$

S = fracture surface crossed by flowrate q. If fluid flows from opposite directions, this surface should be doubled, m^2

t = time from when bottomhole valve is opened, seconds

$t_D = \beta t$ = dimensionless time

$u(t)$ = pressure drawdown in wellbore, Pa

$\overline{u}(p)$ = Laplace transform of u(t), Pa

$v(x,t)$ = pressure drawdown in formation, Pa

$\overline{v}(x,p)$ = Laplace transform of v(x,t), Pa

$V=u(o)$ = maximum pressure drawdown in wellbore at beginning of test, Pa

x = distance from fracture (see Fig. 1), m

$\alpha = \frac{\beta p}{\eta} = \frac{\sigma}{\varepsilon}\sqrt{p}$ = a constant with respect to x and t, m^{-1}

$\beta = \frac{Sk}{\mu C}^2 \cdot \frac{1}{\eta} = \frac{t_D}{t}$ = a constant, s^{-1}

γ = specific weight of fluid produced, $m^{-2} kg\ s^{-2}$

$\varepsilon = \frac{SkR}{\mu}$ = formation thickness equivalent to concentrated resistance, m

μ = fluid viscosity, Pa(s)

σ = βCR = a constant

$\eta = \frac{k}{\phi \mu C}$ = formation diffusivity, $m^2 s^{-1}$

ϕ = porosity

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APPENDIX

SOLUTION OF THE PARTIAL DIFFERENTIAL EQUATION

Introducing dimensionless time, $t_D = \beta t$, Eq. 1 becomes:

$$\frac{\partial^2 v(x,t_D)}{\partial x^2} = \frac{\beta}{\eta} \frac{\partial v(x,t_D)}{\partial t_D} \qquad \text{(A-1)}$$

Laplace transforming and remembering that $v(x,o) = 0$, we have:

$$\frac{\partial^2 \bar{v}(x,p)}{\partial x^2} = \frac{\beta p}{\eta} \cdot \bar{v}(x,p) \qquad \text{(A-2)}$$

where $\bar{v}(x,p)$ is the Laplace transform of $v(x,t_D)$ and p its parameter.

Combining Eqs. 2 and 3, we obtain the first boundary condition at $x = 0$:

$$C \frac{du(t)}{dt} + \frac{u(t)-v(o,t)}{R} = 0 \qquad \text{(A-3)}$$

Introducing dimensionless time, t_D, and the constant $\sigma = \beta CR$ into Eq. A-3, we get:

$$\sigma \frac{du(t_D)}{dt_D} + u(t_D) - v(o,t_D) = 0 \qquad \text{(A-4)}$$

Laplace transforming and defining the initial pressure disturbance, $V = u(o)$, we obtain:

$$(\sigma p+1) \cdot \bar{u}(p) - \bar{v}(o,p) = \sigma V \qquad \text{(A-5)}$$

where $\bar{u}(p)$ is the Laplace transform of $u(t_D)$.

The second boundary condition at $x = 0$ derives from Darcy's law:

$$q(t) = \frac{Sk}{\mu} \left(\frac{\partial v(x,t)}{\partial x} \right)_{x=0} \qquad \text{(A-6)}$$

which, combined with Eq. 2, becomes:

$$\frac{u(t)-v(o,t)}{R} + \frac{Sk}{\mu} \left(\frac{\partial v(x,t)}{\partial x} \right)_{x=0} = 0 \qquad \text{(A-7)}$$

Defining $\varepsilon = \frac{SkR}{\mu}$, introducing dimensionless time, t_D, and Laplace transforming Eq. A-7, we have:

$$\overline{u}(p) - \overline{v}(o,p) + \varepsilon \left(\frac{\partial v(x,p)}{\partial x} \right)_{x=0} = 0 \qquad \text{(A-8)}$$

Now we must solve Eq. A-2 with Boundary conditions (Eqs. A-5 and A-8). The general solution of Eq. A-2 is:

$$\overline{v}(x,p) = D(p)\ e^{-\alpha x} + E(p)\ e^{+\alpha x} \qquad \text{(A-9)}$$

with $\alpha = \sqrt{\frac{\beta p}{\eta}} = \frac{\sigma}{\varepsilon} \sqrt{p}$ and $D(p)$ and $E(p)$ two functions non x-dependent.

Since $v(x,t)$, and hence $v(x,p)$ are limited for $x \to \infty$, $E(p)$ must be nil and the solution we are looking for is:

$$\overline{v}(x,p) = D(p)\ e^{-\alpha x} \qquad \text{(A-10)}$$

Substituting Eq. A-10 into Eqs. A-5 and A-8, we get the expression for $D(p)$ and $\overline{u}(p)$:

$$D(p) = \frac{V}{\sqrt{p}(\sigma p + \sqrt{p} + 1)} \qquad \text{(A-11)}$$

$$\overline{u}(p) = D(p) \cdot (\sigma\sqrt{p} + 1) \qquad \text{(A-12)}$$

Hence:

$$\frac{\overline{u}(p)}{V} + \frac{\sigma\sqrt{p} + 1}{\sqrt{p}(\sigma p + \sqrt{p} + 1)} \qquad \text{(A-13)}$$

TABLE 1

t_D	$\frac{u}{V}$		
	$\sigma = 0$	$\sigma = 0.01$	$\sigma = 0.1$
0.15849D-05	0.99858D+00	0.99986D+00	
0.25119D-05	0.99821D+00	0.99977D+00	
0.39811D-05	0.99775D+00	0.99966D+00	
0.63096D-05	0.99717D+00	0.99947D+00	
0.10000D-04	0.99644D+00	0.99920D+00	
0.15849D-04	0.99553D+00	0.99879D+00	
0.25119D-04	0.99437D+00	0.99820D+00	
0.39811D-04	0.99292D+00	0.99734D+00	
0.63096D-04	0.99110D+00	0.99614D+00	
0.10000D-03	0.98882D+00	0.99446D+00	
0.15849D-03	0.98595D+00	0.99218D+00	0.99856D+00
0.25119D-03	0.98237D+00	0.98911D+00	0.99776D+00
0.39811D-03	0.97788D+00	0.98508D+00	0.99655D+00
0.63096D-03	0.97228D+00	0.97984D+00	0.99473D+00
0.10000D-02	0.96529D+00	0.97312D+00	0.99200D+00
0.15849D-02	0.95662D+00	0.96462D+00	0.98797D+00
0.25119D-02	0.94587D+00	0.95394D+00	0.98212D+00
0.39811D-02	0.93260D+00	0.94064D+00	0.97378D+00
0.63096D-02	0.91632D+00	0.92423D+00	0.96213D+00
0.10000D-01	0.89646D+00	0.90414D+00	0.94623D+00
0.15849D-01	0.87241D+00	0.87976D+00	0.92507D+00
0.25119D-01	0.84358D+00	0.85050D+00	0.89766D+00
0.39811D-01	0.80940D+00	0.81580D+00	0.86318D+00
0.63096D-01	0.76946D+00	0.77525D+00	0.82108D+00
0.10000D+00	0.72358D+00	0.72867D+00	0.77124D+00
0.15849D+00	0.67191D+00	0.67626D+00	0.71413D+00
0.25119D+00	0.61508D+00	0.61867D+00	0.65083D+00
0.39811D+00	0.55425D+00	0.55709D+00	0.58306D+00
0.63096D+00	0.49107D+00	0.49322D+00	0.51311D+00
0.10000D+01	0.42758D+00	0.42913D+00	0.44353D+00
0.15849D+01	0.36597D+00	0.36703D+00	0.37687D+00
0.25119D+01	0.30822D+00	0.30891D+00	0.31526D+00
0.39811D+01	0.25590D+00	0.25632D+00	0.26021D+00
0.63096D+01	0.20993D+00	0.21017D+00	0.21244D+00
0.10000D+02	0.17058D+00	0.17072D+00	0.17199D+00
0.15849D+02	0.13762D+00	0.13769D+00	0.13838D+00
0.25119D+02	0.11045D+00	0.11049D+00	0.11086D+00
0.39811D+02	0.88335D-01	0.88356D-01	0.88545D-01
0.63096D+02	0.70477D-01	0.70488D-01	0.70585D-01
0.10000D+03	0.56141D-01	0.56147D-01	0.56196D-01
0.15849D+03	0.44675D-01	0.44678D-01	0.44703D-01
0.25119D+03	0.35528D-01	0.35529D-01	0.35541D-01
0.39811D+03	0.28241D-01	0.28242D-01	0.28248D-01
0.63096D+03	0.22443D-01	0.22443D-01	0.22447D-01
0.10000D+04	0.17832D-01	0.17833D-01	0.17834D-01
0.15849D+04	0.14167D-01	0.14167D-01	0.14168D-01
0.25119D+04	0.11255D-01	0.11255D-01	0.11255D-01
0.39811D+04	0.89407D-02	0.89408D-02	0.89409D-02
0.63096D+04	0.71022D-02	0.71021D-02	0.71023D-02
0.10000D+05	0.56416D-02	0.56416D-02	0.56417D-02

TABLE 2

t_D	$\frac{u}{V}$		
	$\sigma = 1$	$\sigma = 10$	$\sigma = 100$
0.15849D-01	0.98564D+00	0.99843D+00	0.99984D+00
0.25119D-01	0.97785D+00	0.99752D+00	0.99975D+00
0.39811D-01	0.96608D+00	0.99609D+00	0.99960D+00
0.63096D-01	0.94857D+00	0.99383D+00	0.99937D+00
0.10000D+00	0.92300D+00	0.99028D+00	0.99900D+00
0.15849D+00	0.88661D+00	0.98473D+00	0.99842D+00
0.25119D+00	0.83651D+00	0.97609D+00	0.99750D+00
0.39811D+00	0.77052D+00	0.96273D+00	0.99605D+00
0.63096D+00	0.68838D+00	0.94226D+00	0.99375D+00
0.10000D+01	0.59324D+00	0.91136D+00	0.99012D+00
0.15849D+01	0.49217D+00	0.86566D+00	0.98442D+00
0.25119D+01	0.39474D+00	0.80016D+00	0.97548D+00
0.39811D+01	0.30943D+00	0.71043D+00	0.96154D+00
0.63096D+01	0.24036D+00	0.59544D+00	0.93997D+00
0.10000D+02	0.18692D+00	0.46164D+00	0.90699D+00
0.15849D+02	0.14611D+00	0.32581D+00	0.85751D+00
0.25119D+02	0.11480D+00	0.21091D+00	0.78539D+00
0.39811D+02	0.90538D-01	0.13256D+00	0.68487D+00
0.63096D+02	0.71589D-01	0.88025D-01	0.55405D+00
0.10000D+03	0.56701D-01	0.63390D-01	0.40078D+00
0.15849D+03	0.44956D-01	0.47935D-01	0.24765D+00
0.25119D+03	0.35669D-01	0.37070D-01	0.12663D+00
0.39811D+03	0.28312D-01	0.28989D-01	0.57518D-01
0.63096D+03	0.22479D-01	0.22810D-01	0.30073D-01
0.10000D+04	0.17850D-01	0.18014D-01	0.20304D-01
0.15849D+04	0.14176D-01	0.14258D-01	0.15221D-01
0.25119D+04	0.11260D-01	0.11300D-01	0.11750D-01
0.39811D+04	0.89430D-02	0.89632D-02	0.91795D-02
0.63096D+04	0.71033D-02	0.71134D-02	0.72188D-02
0.10000D+05	0.56423D-02	0.56473D-02	0.56993D-02

COMPUTER CODE

```
C    QUESTO PROGRAMMA INVERTE LA TRASFORMATA DI LAPLACE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON G(50),V(50),H(25),M
      N=18
      T=0.01
      WRITE(6,2)N
    2 FORMAT(7X,'T',15X,'FA',7X,'N=',I2)
   10 T=T*10.**(1./5.)
      CALL LINV(T,FA,N)
      WRITE(6,1)T,FA
    1  FORMAT(2E15.5)
      IF(T.LT.10000.) GO TO 10
      STOP
      END
C
      FUNCTION P(ARG)
C    QUESTA E' LA TRASFORMATA DI LAPLACE CHE DEVE ESSERE  ANTITRASFORMATA
C   DALLA SUBROUTINE LINV
      IMPLICIT REAL*8(A-H,O-Z)
      SIGMA=100.
      RQ=DSQRT(ARG)
      P=(SIGMA*RQ+1.)/(RQ*(SIGMA*ARG+RQ+1.))
      RETURN
      END
C
      SUBROUTINE LINV(T,FA,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON G(50), V(50), H(25),  M
      DLOGTW = .6931471805599453
      IF(M.EQ.N)GO TO 100
```

```
C     CALCULATE V-ARRAY
      M=N
      G(1)=1.
       NH=N/2
      DO 5 I=2,N
    5 G(I)=G(I-1)*I
      H(1)=2./G(NH-1)
      DO 10 I=2,NH
      FI=I
      IF (I.EQ.NH) GO TO 8
      H(I)=FI**NH*G(2*I)/(G(NH-I)*G(I)*G(I-1))
      GO TO 10
    8 H(I)=FI**NH*G(2*I)/(G(I)*G(I-1))
   10 CONTINUE
      SN=2*(NH-NH/2*2)-1
      DO 50 I=1,N
      V(I)=0.
      K1=(I+1)/2
      K2=I
      IF(K2.GT.NH)K2=NH
      DO 40 K=K1,K2
      IF (2*K-I.EQ.0) GO TO 37
      IF (I.EQ.K) GO TO 38
      V(I)=V(I)+H(K)/(G(I-K)*G(2*K-I))
      GO TO 40
   37 V(I)=V(I)+H(K)/(G(I-K))
      GO TO 40
   38 V(I)=V(I)+H(K)/G(2*K-I)
   40 CONTINUE
      V(I)=SN*V(I)
      SN=-SN
   50 CONTINUE
  100 FA=0.
      A = DLOGTW/T
      DO 110 I=1,N
      ARG=I*A
  110 FA=FA+V(I)*P(ARG)
      FA=A*FA
      RETURN
      END
```

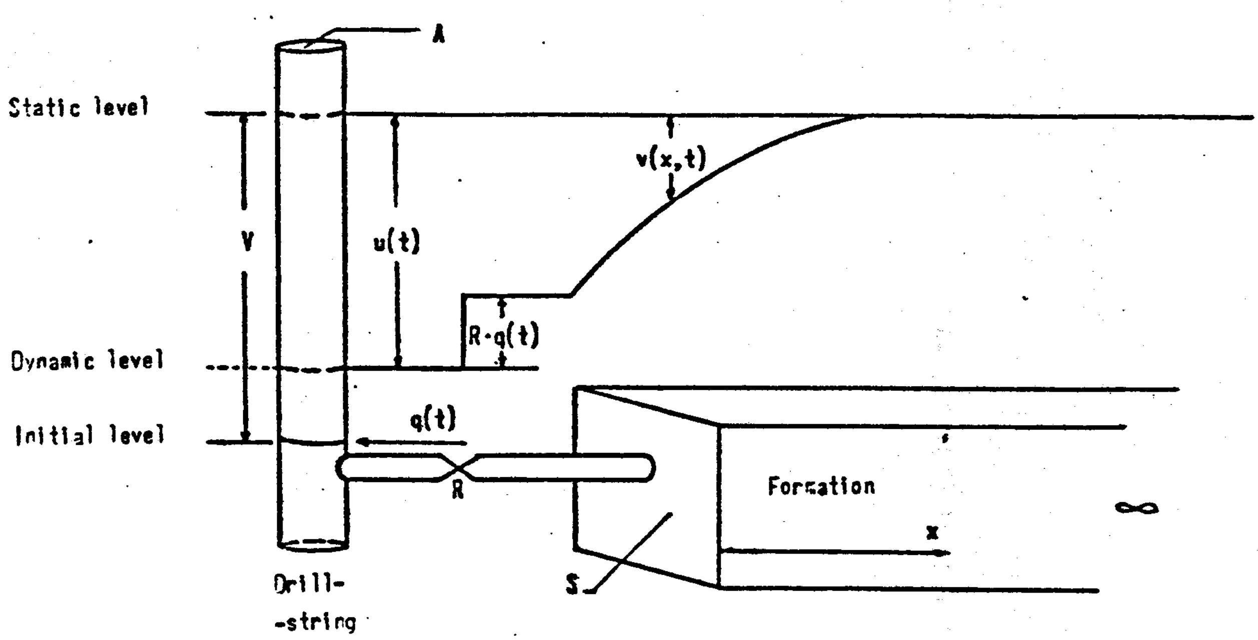
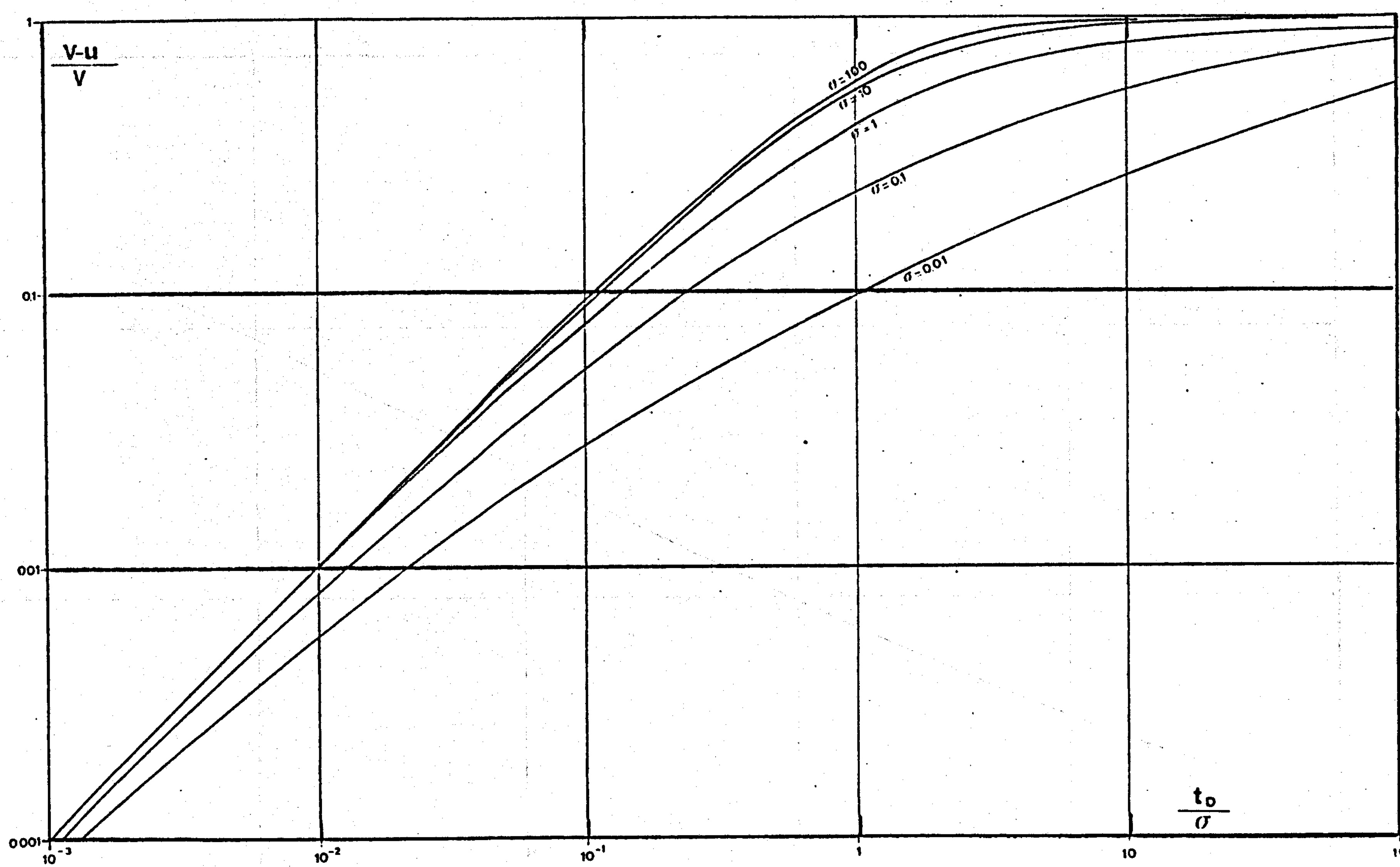
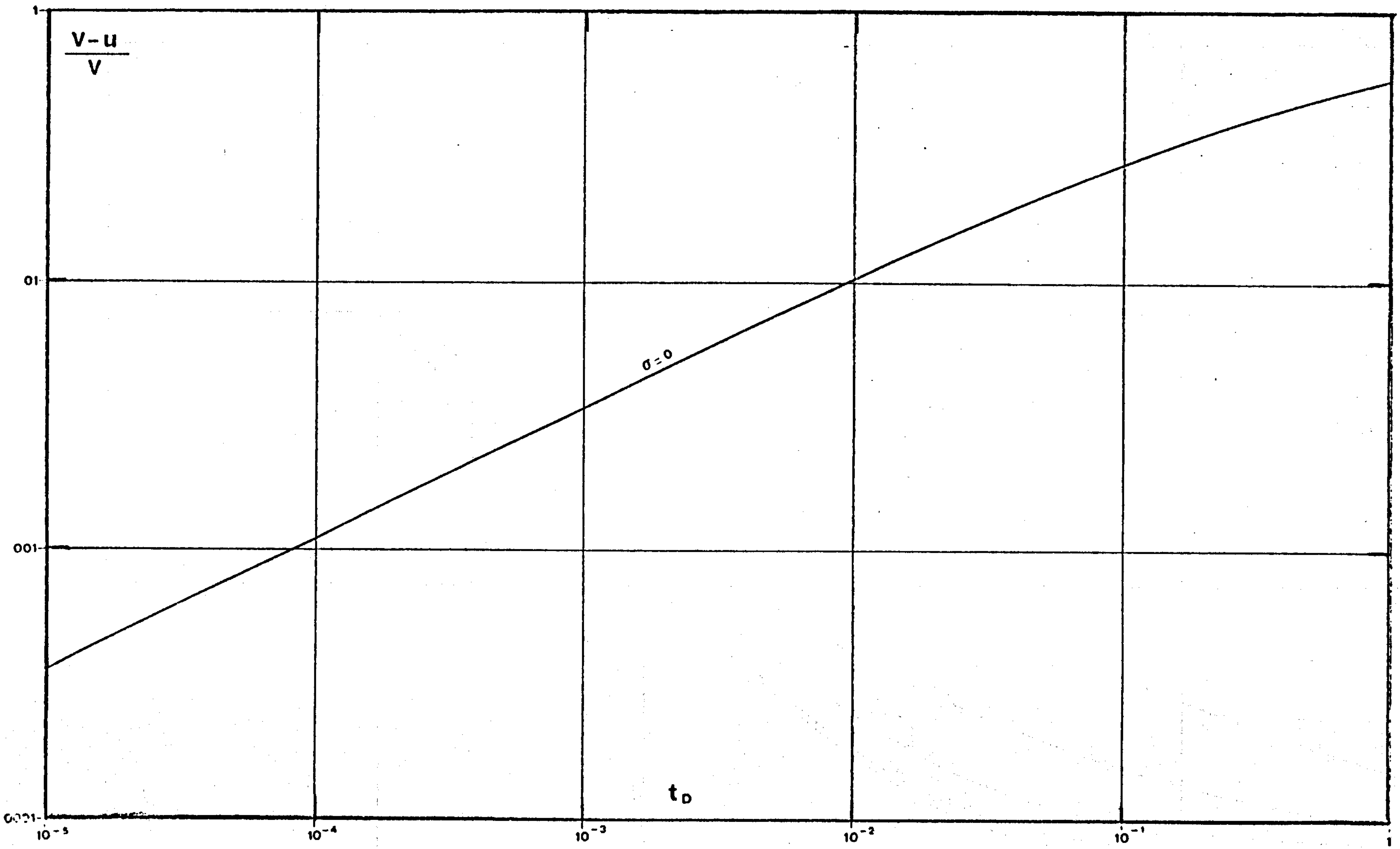

A
Static level
V
u(t)
v(x,t)
R·q(t)
Dynamic level
q(t)
Initial level
R
Formation
∞
x
S
Drill-
-string

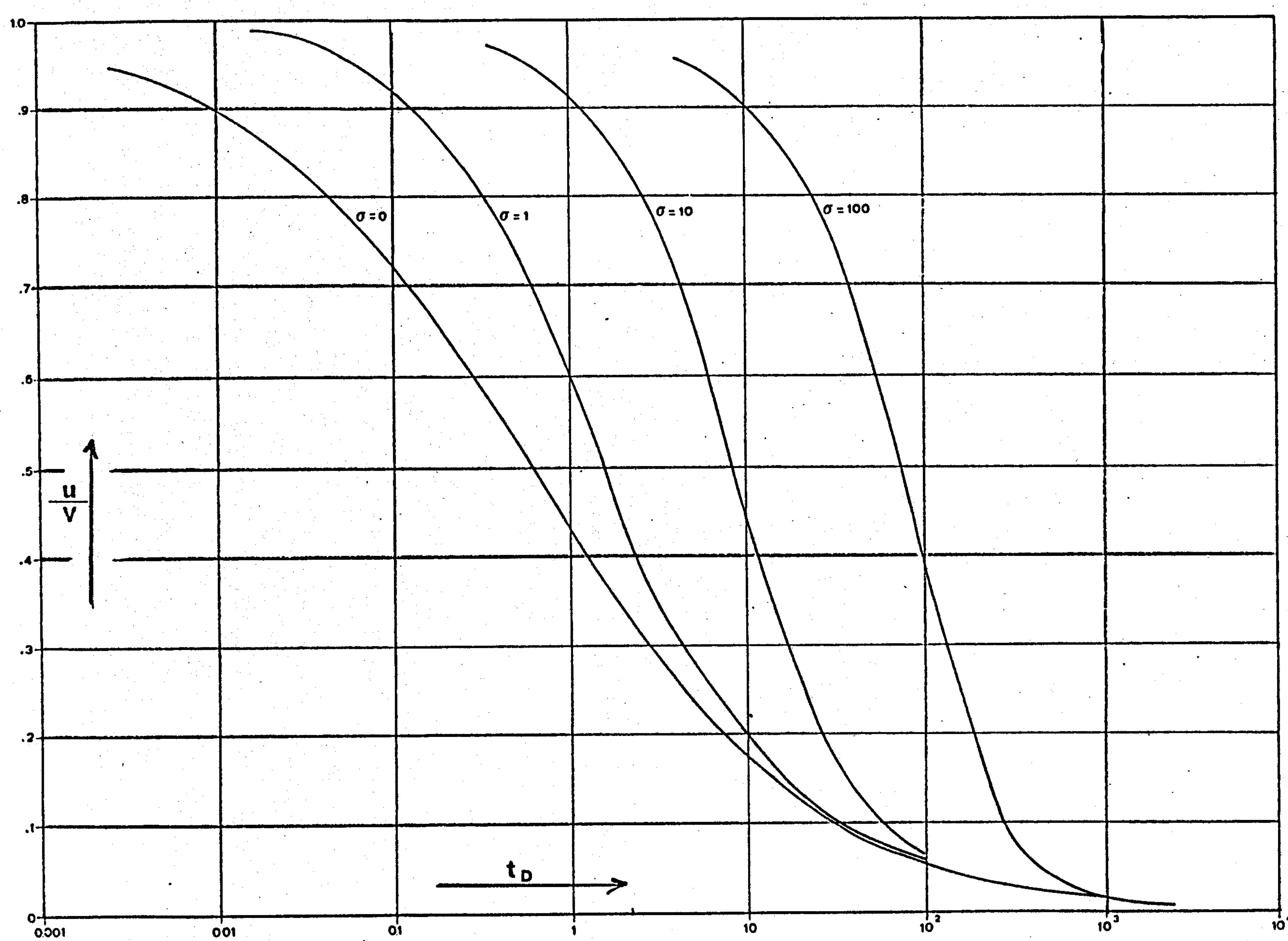
Fig. 1



Curva tipo n. 1



Curva tipo n. 2



Curva tipo n. 3