

SIMULATION OF GEOTHERMAL RESERVOIRS INCLUDING CHANGES
IN POROSITY AND PERMEABILITY DUE TO SILICA-WATER REACTIONS

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Introduction

Changes in porosity and permeability due to water-rock reactions may affect a geothermal reservoir in two important ways. One is the possibility of **clogging** of the pore space in the vicinity of a reinjection well. The second concerns the **long** term evolution of a geothermal system, for example, the development of a self-sealing cap rock. Two major types of reactions that may affect the porosity and permeability in the reservoir are dissolution-precipitation reactions and alteration reactions. Silica precipitation is the major self-sealing process in hot-water geothermal reservoirs (White, 1973). The purpose of this work is to develop a mathematical model which can simulate the changes in porosity and permeability resulting from water-rock reactions in a geothermal reservoir.

The model being developed describes the flow of hot water in an axially symmetric porous medium. The model is a vertical cross-section in r - z coordinates. For simplicity, only a single dissolved chemical species is being modeled. The fluid is single-phase water and only dissolution and precipitation of quartz are considered. The kinetics for silica are those developed by Pimstidt and Rarnes (1979) and Rimstidt (1979).

The Basic Equations

The set of basic partial differential equations that describes a hot water geothermal system are the fluid mass balance (continuity equation), the fluid momentum balance (Darcy's Law), the thermal energy balance, and the dissolved species mass balance equations. Additional conditions needed to fully describe the physics and chemistry are an equation of state (that is, density as a known function of pressure, temperature, and concentration), constitutive relations for fluid and rock, chemical rate equations, and initial and boundary conditions. In addition, the rate of change of porosity and permeability as a function of the reaction must be derived.

The final set of partial differential equations is formulated in terms of fluid pressure, temperature, and concentration. Since exact solutions are too idealized, an approximate method is necessary. The method used is a numerical method in which the spatial derivatives are

approximated using a Calerkin finite-element method and the temporal derivatives are approximated using a finite-difference method.

Changes in Porosity and Permeability

To model changes in porosity and permeability it is necessary that changes in porosity be expressed in terms of bulk pore volume changes in the porous medium due to dissolution or precipitation. Furthermore, it is assumed that the changes in permeability can be directly related to changes in porosity by some empirical or theoretical relationship between permeability and porosity. The relationship used is an empirical one from Pearson (1976),

$$k = 10^{(13.614 \phi - 1.8126)} \times 9.87 \times 10^{-12} \quad (1)$$

where ϕ is porosity (volume fraction), and k is permeability (square centimeters).

To obtain changes in porosity in terms of the reaction kinetics, consider the solute mass balance equation:

$$\frac{\partial}{\partial t}(\phi \rho_w C) + \nabla \cdot (\rho_w C \bar{u}) - \nabla \cdot (\rho_w E \nabla C) - q_s C_s + q_{Re} = 0 \quad (2)$$

Assuming that the only reactive component is quartz and that the changes in rock mass are due to dissolution and precipitation of quartz, a rock mass balance equation is written

$$\frac{\partial}{\partial t}[(1-\phi) \rho_r C_r] = q_{Re} \quad (3)$$

In equations (2) and (3), ρ_r and ρ_w are the densities of the rock and water, respectively. C_r and C are concentrations (mass fraction) of silica in the rock and water, respectively. \bar{u} is the Darcy velocity and E is the dispersion coefficient. q_s is the mass flux (grams per second per cubic centimeters) due to a well source and C_s is the concentration of the source fluid. The term q_{Re} is the net rate of precipitation. The term q_{Re} may also be thought of as the time rate of loss of dissolved silica mass per unit volume by precipitation. The subscript Re refers to the part of the subscripted expression which is due to the reaction.

The left hand side of equation (3) can be simplified, assuming that ρ_r and C_r are constant;

$$\frac{\partial}{\partial t} [(1-\phi) \rho_r C_r] = -\rho_r C_r \left. \frac{\partial \phi}{\partial t} \right|_{Re} \quad (4)$$

Therefore, an expression for $\left. \frac{\partial \phi}{\partial t} \right|_{Re}$ can be obtained, which is

$$\left. \frac{\partial \phi}{\partial t} \right|_{Re} = -\frac{1}{\rho_r C_r} q_{Re} \quad (5)$$

The term q_{Re} is evaluated by incorporating the kinetic rate equation for quartz which describes the time rate of change of silica concentration in the fluid as a function of concentration and temperature. This equation is (Rimstidt and Barnes, 1979; Rimstidt, 1979)

$$\frac{\partial m}{\partial t} = \frac{A}{M} [k_+ - k_- m] \quad (6)$$

where m is molality of dissolved silica, A/M is the ratio of interfacial area to mass of water, and k_+ and k_- are the rate constants for dissolution and precipitation, respectively. The rate constants are exponential functions of temperature.

Numerical Method

The numerical method used to solve for pressure, temperature, and concentration is based on the finite-element method. The essential features of this method are:

- 1) The spatial derivatives are approximated using a modified Galerkin finite-element method in which upstream weighting of the convective terms is used.
- 2) The temporal derivatives are approximated using a backward (implicit) finite difference in time.
- 3) Concentration, temperature, and pressure are solved for sequentially in a manner similar to that outlined by Coats et al. (1974) for finite difference methods. Solving for concentration, temperature, and pressure in that order allows the coefficient matrix for the spatial derivatives to be symmetric. The method also requires lumping of the time derivative matrices.

- 4) The convective terms in the temperature and concentration equations are evaluated at the new time level using a linearized semi-implicit method. This allows reasonably large time steps without changing the symmetry of the matrices.
- 5) The reaction term in the concentration equation is treated semi-implicitly.

Examples

The model has been tested against some analytical solutions. The solution for pressure has been compared to the Theis solution from groundwater hydrology (Theis, 1935). The solution for temperature has been compared to an analytical solution to the problem of hot fluid injection into an aquifer, including the effect of conductive heat loss through the confining beds (Avdonin, 1964). A modification of the Avdonin solution without leakage was used to test the solution of the concentration equation. All three finite element solutions compared favorably to the analytical solutions. The differential rate equation for quartz was integrated analytically over time at constant temperature. This was compared to computed concentrations for conditions of no flow and constant temperature. The accuracy of the concentrations computed by the finite element model under these conditions is dependent on the size of the time step.

The relationship between the temperature and the concentration solutions and how this effects the reaction rate for quartz dissolution-precipitation was studied through a series of one-dimensional problems. In these problems, the fronts are caused by the injection of hot water into a cooler aquifer, with the initial fluids saturated with silica. The results suggest that the temperature solution controls the rate of the quartz precipitation-dissolution reaction in two basic ways.

First, the reaction rate is rapid enough that the curvature of the concentration front basically follows that of the temperature front. This implies that thermal dispersion (conductivity) of the reservoir is relatively more important than silica dispersion in controlling the reaction. Precipitation tends to occur on the upstream side of the fronts where hot water mixes with cold water. This results in decreased porosity and permeability in the vicinity of the well.

The second way in which the temperature solution controls the reaction rate is through the exponential temperature dependence of the rate constants. The result is that the reaction rate is more sensitive to changes in concentration and temperature at higher temperatures.

We are presently in the process of designing two-dimensional problems with the purpose of modeling self-sealing in a reservoir.

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References

- Avdonin, N. A., 1964, Some formulas for calculating the temperature field of a stratum subject to thermal injection. *Neft'i Gaz*, 7 (3), 37-41.
- Coats, K. H., George, W. D., Chu, C., and Marcum, R. E., 1974, Three-dimensional simulation of steamflooding. *Soc. Pet. Eng. J.*, 14 (6), 573-592.
- Pearson, R. O., 1976, Planning and design of additional East Mesa geothermal test facilities (Phase 1B), Volume 1 - Final Report, SAN/1140-1/1, ERDA.
- Rimstidt, J. D., 1979, in preparation. The kinetics of silica-water reactions. Ph.D. Thesis, The Pennsylvania State University.
- Rimstidt, J. D. and Barnes, H. L., 1979, Kinetics of silica-water reactions, submitted to *Geochim. Cosmochim. Acta*.
- Theis, Charles V., 1935, The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Eos Trans. Am. Geophys. Union*, 16, 519-524.
- White, Donald E., 1973, Characteristics of geothermal resources, in Geothermal Energy, edited by Paul Kruger and Carel Otte, pp. 69-94. Stanford University Press, Stanford, California.