THE PSEUDOPRESSURE OF SATURATED STEAM

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1. Definition and use of the pseudopressure

In a porous medium containing steam and immobile water, the equation for the flow of (saturated) steam is (Grant 1978):

$$(H_s - H_w)\frac{dp_s}{dT} k \nabla \cdot \left[\frac{\rho}{\mu}\nabla p\right] + \rho_m C_m \frac{\partial p}{\partial t} = 0 \qquad (1)$$

and $\quad p = p(T)$.

This is a nonlinear diffusion equation, nonlinear because of the $\frac{\rho}{\mu}$ multiplying $\nabla \rho$, and because the coefficient of $\frac{\partial p}{\partial t}$ is also a function of T and hence P. It can be linearised in P, to obtain

$$\nabla^2 p = \frac{1}{\kappa_o}\frac{\partial p}{\partial t} \qquad (2)$$

where $\quad \kappa_o = \frac{k}{\rho_m C_m}\left[\frac{\rho_s}{\mu_s}\frac{dp_s}{dT}(H_s - H_w)\right]_o$

is evaluated at the assumed initial uniform state.

We can also write

$$m^* = \int\frac{\rho}{\mu}\, dp = \int\frac{dp}{\nu} \qquad (3)$$

so that

$$\nabla^2 m^* = \frac{1}{\kappa}\frac{\partial m^*}{\partial t} \qquad (4)$$

This can again be linearised by setting κ equal to its initial value κ_o:

$$\nabla^2 m^* = \frac{1}{\kappa_o}\frac{\partial m^*}{\partial t} \qquad (5)$$

The nonlinearity caused by the coefficient of ∇p has been removed, but the variation in κ has not been accounted for. However, it is nearly always better practice, with nonlinear diffusion equations, to represent the divergence terms exactly (here $\nabla^2 m$). The errors incurred by ignoring the variations in κ seem less important. A rough reason can be advanced. Both (5) and (2) are valid if the pressure changes are small. But (5) is also valid for steady pressure changes of any magnitude. If a bore is running for a long time, there is a quasi-steady region near the bore. The approximation (5) may better represent this, for large drawdowns, than (2).

It should be noted that the pseudopressure defined here does not have the dimensions of pressure, but $\rho p/\mu$ (= kg/m^{3}s). Conventional gas pseudopressures are differently defined. Such a pseudopressure for superheated steam is given by Atkinson and Mannon (1977).

The boundary conditions are also scaled. If we have two-dimensional flow to a bore producing at a rate q (m^3/sec) = W^* (kg/sec), the condition is

$$2\pi r\left(\frac{\kappa}{\mu}\frac{\partial p}{\partial r}\right)_{r=a} = q \tag{7}$$

this is

$$2\pi r\left(k\frac{\partial m^*}{\partial r}\right)_{r=a} = W^* \tag{8}$$

Thus, wh re the expres ion $q\mu$ ccurs, it is replaced by W^*. For example, in the standard 2-d solution

$$\Delta p = p - p_o = \frac{qu}{4\pi kh} E_1\left(\frac{a^2}{4\kappa t}\right) \tag{9}$$

we have, using the pseudopressure,

$$\Delta m^* = m^* - m_o^* = \frac{W^*}{4\pi kh} E_1\left(\frac{a^2}{4\kappa t}\right) . \tag{10}$$

Since it is usually mass flow rates that are specified, use of the pseudopressure usually makes some small saving of effort in looking up densities and viscosities.

2. Formulae for $m^*(p)$.

Use of the pseudopressure is convenient only if there is a simple expression for it as a function of pressure. Fortunately this is so. In any approximate formula for pseudo-pressure, we need its derivative dm^*/dp represented to some order of accuracy, for we always use differences $m^*(p_1) - m^*(p_2)$. Thus we approximate ρ/μ to some order of accuracy, and integrate to get m^*.

The following expression is accurate to 1% (130-240°C) 2% (100-260°C).

$$\frac{1}{\nu} = \frac{\rho}{\mu} = 4.793 \times 10^4 \, p^{6/7} \tag{11}$$

where p is in bars. Keeping p in bars from now on:

$$\frac{dm^*}{dp} = 10^5 \frac{\rho}{\mu} = 4.793 \times 10^4 \, p^{6/7}$$

$$m^* = 2.58 \times 10^9 \, p^{13/7} \tag{12}$$

For convenience, define

$$\boxed{m(p) = p^{13/7}} \tag{13}$$

$$= m^*/(2.58 \times 10^9) \tag{14}$$

3. Equations in field units

3.1. Units bar, tonne/hr, darcy-metre

It is simplest to work with these units, and the function

$$\boxed{m(p) = p^{13/7}} \quad .$$

The standard drawdown formula (10) now becomes:

$$m^* = 2.58 \times 10 \quad p^{13/7} = 2.58 \times 10^9 \; m$$

$$W^* = W/3.6$$

$$kh \rightarrow 10^{-12} \, kh$$

$$2.58 \times 10^9 \; \Delta m = \frac{W}{3.6} \; \frac{1}{4\pi \; 10^{-12} \, kh} \; E_1 \left(\frac{a^2}{4\kappa t}\right)$$

or $$\boxed{\Delta m = 8.57 \; \frac{W}{kh} \; E_1 \left(\frac{a^2}{4\kappa t}\right)} \quad . \qquad (15)$$

If a bore is running for time t, and then switched off for time Δt,

$$\Delta m = -8.57 \; \frac{W}{kh} \left\{ E_1 \left(\frac{a^2}{4\kappa \Delta t}\right) - E_1 \left(\frac{a^2}{4\kappa (t+\Delta t)}\right) \right\}$$

$$\simeq 8.57 \; \frac{W}{kh} \ln \left(\frac{t + \Delta t}{\Delta t}\right)$$

$$\simeq 19.7 \; \frac{W}{kh} \log_{10} \left(\frac{t + \Delta t}{\Delta t}\right) \qquad (16)$$

Then, if M is the slope of a plot of $p^{13/7}$ vs t, or $\frac{t+\Delta t}{\Delta t}$, on semilog paper (slope of M per cycle),

$$M = 19.7 \; W/kh$$

or $$\boxed{kh = 19.7 \; W/M} \quad . \qquad (17)$$

<u>Summary</u>

$$m(p) = p^{13/7}$$

$$\Delta m = m - m_o = 8.57 \; \frac{W}{kh} \; E_1 \left(\frac{a^2}{4\kappa t}\right)$$

$$kh = 19.7 \; W/M$$ (M = slope of m per cycle.)

3.2. Units kg/cm^2, tonne/hr, darcy-metre

It is simplest to redefine

$$\boxed{m(p) = p^{13/7}} \tag{18}$$

Since 1 kg/cm^2 = 0.981 bar, a factor of $(0.981)^{13/7}$ multiplies the formula for m^*

$$m^* = 2.49 \times 10^9 \, p^{13/7} = 2.49 \times 10^9 \, m .$$

Then the drawdown formula is

$$\boxed{\Delta m = 8.88 \frac{W}{kh} E_1 \left(\frac{a^2}{4\kappa t}\right)} \tag{19}$$

and $$\boxed{kh = 20.4 \; W/M} \; . \tag{20}$$

M = slope of a plot of $p^{13/7}$ vs t or $(t+\Delta t)/\Delta t$ on semilog paper (slope of M per cycle).

References

1. U.K. Steam Tables in S.I. Units, 1970.

2. Abramowitz & Stegun, "Handbook of Mathematical Functions", Dover, 1965.

3. Grant, "Two-phase Linear Geothermal Pressure Transients - A Comparison with Single-phase Transients", N.Z. Jl. Sci., Sept. 1978.

4. Mannon & Atkinson, "The Real Gas Pseudo-Pressure for Geothermal Steam", Third Workshop on Geothermal Reservoir Engineering, Stanford, 1977.

NOTATION

SI units are used unless otherwise specified.

t	time
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
T	temperature
p	pressure (also bar and **ksc)**
p_s	saturation pressure of steam
H	specific enthalpy
C	specific heat
ϕ	porosity
k	permeability (also darcy)
h	aquifer thickness
m^*	pseudopressure
m	$= p^{13/7}$ = scaled pseudopressure
W^*	mass flow
W	mass flow, tonne/hr
κ	diffusivity
M	slope of m vs $\log_{10} t$

$$E_1(x) = \int_x^\infty \frac{1}{u} e^{-u} \, du$$ (Abramowitz & Stegun 1965)

a	bore radius
r	radial distance

Suffices:

o	initial state
m	medium
s	steam