AN EVALUATION OF JAMES' EMPIRICAL FORMULAE FOR THE DETERMINATION OF TWO-PHASE FLOW CHARACTERISTICS IN GEOTHERMAL WELLS

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Introduction

One of the most economical and simple methods of determination of two-phase flow parameters in geothermal well testing is the so-called James' method [1,2]. The method consists of the measurements of lip pressure (p), and the flow rate of water (w) by a conventional weir. The stagnation enthalpy (h_o) is then determined from a plot showing h_o versus $w/p^{0.96}$ which is empirically determined by James [1,2]. The mass flow rate is then determined from the following empirical formula

$$G = 11,400 \frac{P^{0.96}}{h_0^{1.102}} \tag{1}$$

where G is the total mass flow rate in $lb_m/sec\text{-}ft^2$, p is the lip pressure in psia, and h_0 is the specific enthalpy in BTU/lb_m. The above relation is empirically determined for discharge pressure up to 64 psia and pipe diamters up to 8". For pipe diamters smaller than 0.2", it has been suggested that the value of 11,400 be replaced by 12,800. In view of the widespread use of the James' method, it is important to assess its accuracy and range of applicability.

Two-Phase Critical Flow Theory

In this paper we shall compare the wellbore discharge characteristics obtained from James' empirical formulae to those predicted by Fauske's two-phase critical flow theory [3]. Fauske suggested that in two-phase flow the maximum discharge rate may not necessarily be accomplished by a shock front. He proposed that at the critical flow condition the absolute value of the pressure gradient at a given location is maximum but finite for a given flow rate or quality, i.e.,

$$(dp/dz)_{G,x} = \text{maximum and finite}, \tag{2}$$

where z is the coordinate along the streamwise direction, and x the quality of the saturated mixture.

Under the assumptions of (i) annular flow pattern, (ii) two-phases in thermal equilibrium, (iii) negligible frictional loss, and (iv) one-dimensional steady flow, Fauske [3] obtained the following analytical expression €or the critical flow rate of a saturated mixture:

$$G_{critical} = \left[g_c k/Q\right]^{1/2} \tag{3}$$

where $Q = -[(1-x+kx) \times (dv_g/dp) + (v_g(1+2kx-2x) + v_f(2xk-2k-2xk^2+k^2)) dx/dp]$,

$g_c = 32.2 \dfrac{lb_m\text{-}ft}{lb_f\text{-}sec^2}$, and $k = (v_g/v_f)^{1/2}$ with v_g and v_f denoting the specific volume of the saturated vapor and liquid respectively. Thus, the critical flow rate can be calculated from Eq.(3) if the steam quality and the lip pressure are known.

The corresponding stagnation enthalpy can be determined from

$$h_0 = h_f(1-x) + h_g x + (G^2/2g_c)((x^3 v_g^2/Rg^2) + (1-x)^3 v_f^2/(1-Rg)^2)/J , \tag{4}$$

where R_g is the gas void fraction which is related to steam quality [4]. In comparison with experimental data, Levy [4] found, however, that using Eq.(4) for the computation of h_0 would lead to under-prediction of the mass flow rate. For this reason we shall compute the stagnation enthalpy on the basis of a homogeneous model, i.e.,

$$h_0 = h_f(1-x) + h_g x + G^2 v_h^2/2g_c J , \tag{5}$$

where $v_h = v_f(1-x) + v_g x$ and $J = 778$ ft-lb_m/BTU .

The weir flow rate is then determined from

$$w = G(1 - x) . \tag{6}$$

Results and Discussion

For a given set of values of lip pressure and steam quality and with the data of saturated steam-water properties [5], Eq.(3) can be used for the computation of total mass flow rate G . The stagnation enthalpy and the weir flow rate are then determined from Eqs.(5) and (6). The results of the computations for the lip pressure from 14.7 psia to 200 psia for geothermal well testing applications are plotted in Figs. 1 and 2. When the lip pressure and the weir flow rate are measured in a geothermal well test, the stagnation enthalpy of the reservoir, the steam quality at the well head, and the total mass flow rate can easily be determined from these plots.

To assess the accuracy of the James' method, calculations were carried out for five different sets of lip pressure and weir flow rate using James' empirical formulae and Fauske's theoretical prediction (i.e., Figs. 1 and 2). The results for total mass flow rate, the stagnation enthalpy, and the steam quality are tabulated in Table 1 for comparison. It is shown that the results based on the two methods differ within 8%.

References

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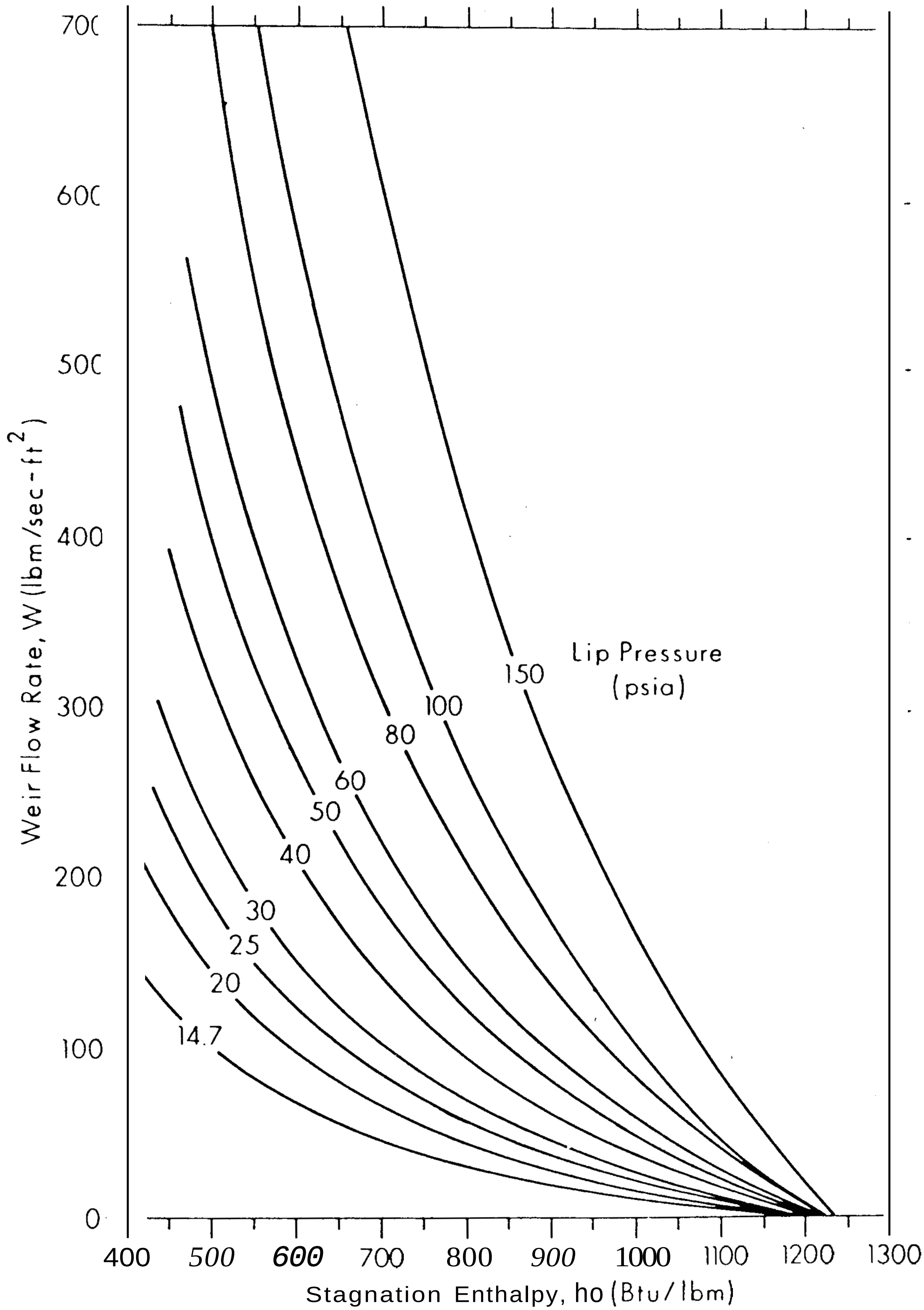

Fig. 1. Weir Flow Rate vs. Stagnation Enthalpy at Selected Values of Lip Pressure According to Fauske's Theory

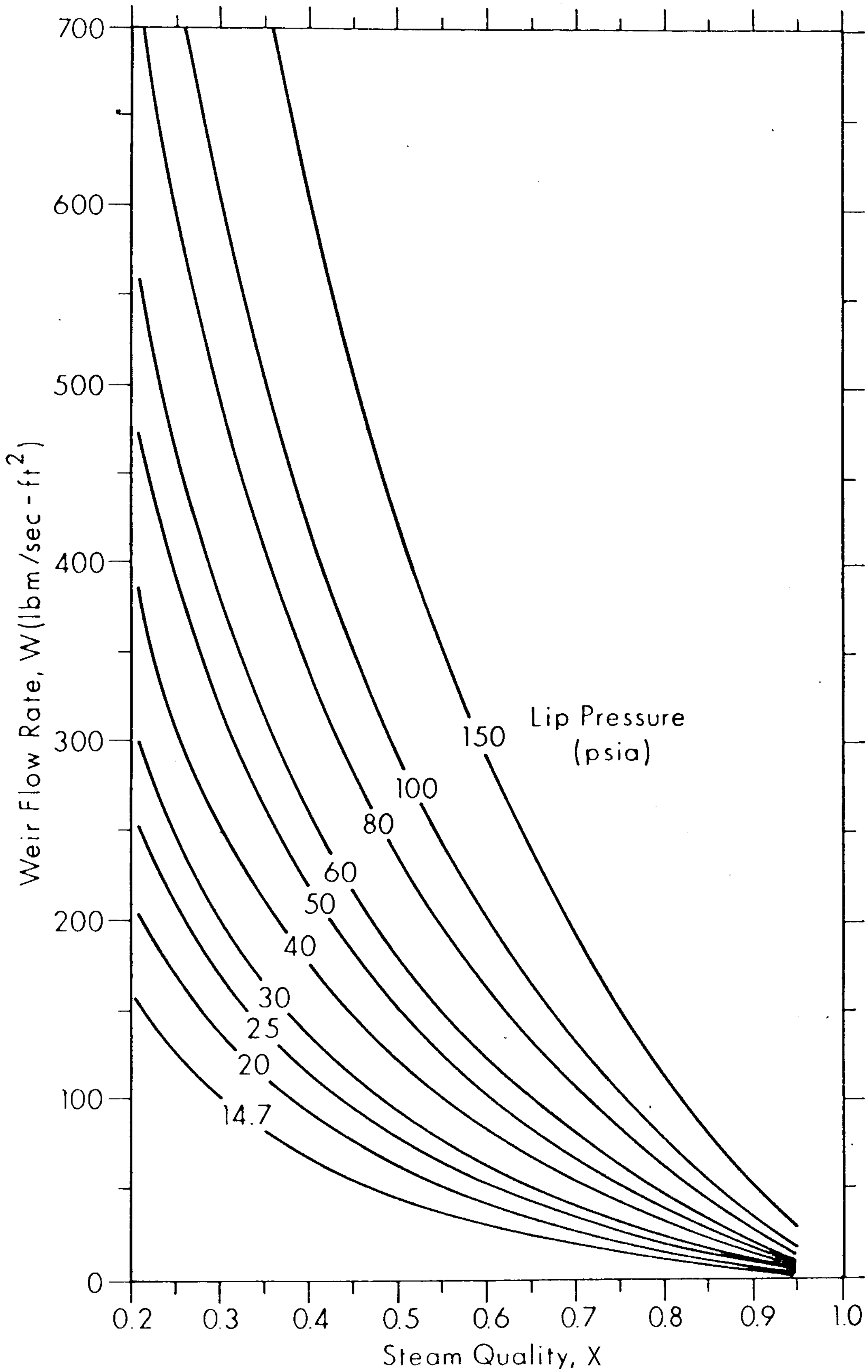

Fig. 2. Weir Flow Rate vs. Steam Quality at Selected Values of Lip Pressure According to Fauske's Theory.

Table 1. COMPARISON OF RESULTS BASED ON THE JAMES' METHOD AND FAUSKE'S ANALYTICAL MODEL

Case	P (psia)	w ($lb_m/sec\text{-}ft^2$)	h_o BTU/lb_m	G $lb_m/sec\text{-}ft^2$	x	Method
1	14.7	40	736.30	88.44	.54	Fauske (F)
			800.00	95.13	.58	James (J)
2	25.0	85.5	698.78	164.59	.48	F
			750.00	170.06	.50	J
3	60.0	226.0	697.79	403.39	.44	F
			715.00	415.42	.46	J
4	100.0	105.0	1004.52	419.84	.75	F
			985.00	476.60	.78	J
5	150.0	53.0	1148.87	523.41	.90	F
			1130.00	590.00	.90	J