MECHANISM OF RESERVOIR TESTING

Gunnar Bodvarsson
School of Oceanography
Oregon State University
Corvallis, Oregon 97331

(1) Introduction

In evaluating geothermal resources we are primarily interested in data on the distribution of temperature and fluid conductivity within the reservoir, the total volume of the productive formations, recharge characteristics and chemical quality of the thermal fluids. While geophysical exploration by surface methods may furnish some data on the temperature field and give indications as to the reservoir volume, they furnish practically no information on the fluid conductivity and production characteristics. Such information will generally have to be obtained by tests performed within the reservoir, primarily by production tests on sufficiently deep wells. Reservcir testing is therefore one of the most important tasks in a general exploration program.

In principle, reservoir testing has much in common with conventional geophysical exploration. Although the physical fields applied are to some extent different, we face the same type of selection between controlled and natural drives, forward and inverse problem setting, etc. The basic philosophy (Bodvarsson,, 1966) is quite similar.

In the present paper, we will discuss some fundamentals of the theory of reservoir testing where the fluid conductivity field is the primary target. The emphasis is on local and global aspects of the forward approach to the case of liquid saturated (dominated) Darcy type formaticns. Both controlled and natural driving pressure or strain fields are to be considered and particular emphasis will be placed on the situation resulting from the effects of a free liquid surface at the top of the reservoir.

(2) Relations governing the pressure field in Darcy type formations

Let $p(t,P)$ be the pressure field at time t and at the point P in a Darcy type domain B with the boundary surface Σ. Consider a general setting where the permeability k is a linear matrix operator and the kinematic viscosity of the fluid ν is also taken to be variable. It is convenient to introduce the fluid conductivity operator $c = k/\nu$ and express Darcy's law

$$\vec{q} = -c\nabla p \tag{1}$$

where $\vec{q}$ is the mass flow density. Moreover, let ρ be the fluid density, s the capacitivity or storage coefficient of the formation and f be a source density. Combining (1) with the equation for the conservation of mass, we obtain the diffusion equation for the pressure field

$$\rho s \partial_t p + \Pi(c)p = f \tag{2}$$

where $\Pi(c) = -\nabla(c\nabla)$ is a generalized Laplacian operator. Appropriate boundary conditions that may be of the Dirichlet, Neumann, mixed or more complex convolution type, have *to* be adjoined to equation (2). The case of a homogeneous/ **isotropic/isothermal** formation results in the simplication $\Pi(c) = c\Pi = -c\nabla^2$ where c is a constant. Moreover, stationary pressure fields **satisfy** the potential equation

$$\Pi(c)p = f. \tag{3}$$

The eigenfunctions $u_n(P)$ of $\Pi(c)$ in B satisfy the equations

$$\Pi(c)u_n = \lambda_n u_n, \quad n = 1,2, \ldots \tag{4}$$

where the constants λ_n are the eigenvalues and the boundary conditions on Σ are homogeneous of the same type as those satisfied by $p(t,P)$ in (2) and (3).

(3) Types of solutions

It is of interest to consider some general expressions for the solutions of equations (2) above. The key to the equation is the causal impluse response or Green's function $G(t,P,Q)$ which represents the pressure response of the causal system to an instantaneous injection of a unit mass of fluid at $t = 0+$ at the source point Q. This function satisfies the same boundary conditions as the **eigenfuctions** $u_n(P)$. Solutions to (2) in the case of a general source density $f(t,P)$, non-causal initial values and general boundary conditions can then be expressed in terms of integrals over the Green's function (Duff and **Naylor,** 1966).

Two fundamental types of expressions for the Green's function are available. First, in the case of simple layered domains B with a boundary Σ composed of a few plane faces, $G(t,P,Q)$ can be expressed as a sum (or integral) over the fundamental source function

$$p = (8\rho s)^{-1}(\pi at)^{-3/2}\exp(-r_{PQ}^2/4at)U_+(t) \tag{5}$$

and its images. The symbol $U_+(t)$ is the causal unit step function and r_{PQ} is the distance from Q to **P.** Whenever applicable, **sums** of this type represent the most elementary local and/or global expressions for $G(t,P,Q)$.

Second, the Green's function can be expanded in a series (in integral) over the eigenfunctions of $\Pi(c)$. **If** ρ and s are constant, then

$$G(t,P,Q,) = (1/\rho s)\sum_n u_n(P)u_n(Q)\exp(-\lambda_n t/\rho s). \tag{6}$$

The series expansion (6) is of a more general applicability than solutions of the type based on the fundamental source function (5). However, because of quite poor convergence properties, (6) is largely of a more global long-term relevance. **It** is less suited for the computation of local values. The formal link between the two types of solution (5) and (6) is provided by the Poisson **summation** formula (Zemanian, 1965).

A different type of solution of (2) that is of interest in the present context can be obtained by operational methods. Limiting ourselves to the pure initial value problem with $p(0,P) = p_0(P)$ in the case of an infinite domain, we can, since ρ, s and $\Pi(c)$ are independent of t, formally express the solution of the homogeneous form of (2) as

$$p = \exp[-t\Pi(c)/\rho s]p_0 \tag{7}$$

where the exponential operator is to be interpreted as a Taylor series in the operator $\Pi(c)$

$$\exp[-t\Pi(c)/\rho s] = 1 - [t\Pi(c)/\rho s] + (1/2)[t\Pi(c)/\rho s]^2 - \ldots \tag{8}$$

The series represents an iteration process where the convergence is limited to (properly defined) small values of t. The practical applicability is therefore fundamentally different from (6). Moreover, it is of considerable interest that rather general situations with regard to $\Pi(c)$ can be admitted in (7) and (8).

Three simple but fundamental physical parameters are associated with processes governed by the diffusion equation (2). First, the local diffusivity $a = c/\rho s$. Second, the skin depth $d = (2c/\rho s\omega)^{\frac{1}{2}}$ which is a measure of the penetration of a wave of angular frequency ω (Bodvarsson, 1970). Finally, the relaxation time $t_0 = 1/ak^2$ which is a measure of the attenuation in time of a one-dimensional wave like pressure field of wave number k. The time t_0 is the time during which the wave amplitude decreases from unity to $1/e$. This parameter is obtained by inserting a solution of the form $\exp[(-t/t_0)+ kx)]$ into the one-dimensional form of (2).

(4) Effects of a free liquid surface

The presence of a free liquid surface in a reservoir requires the introduction of a rather complex surface boundary condition. Let Σ now represent the free liquid surface at equilibrium and Ω be the free surface in a perturbed state. The boundary R is a surface of constant pressure which without loss of generality can be taken to vanish. The free surface condition (Lamb, 1932) is then expressed

$$Dp/Dt|_{p=0} = 0 \tag{9}$$

where D/Dt is the material derivative. This is an essentially non-linear condition which leads to a much more complex problem setting. Losing the principle of superposition the construction of solutions to the forward problem becomes a difficult task.

Bodvarsson (1977a) has shown that when R deviates only little from c, (9) can be simplified and linearized. For this purpose we place a rectangular coordinate system with the z-axis vertically down such that the (x,y) plane coincides with Σ. Moreover, let the amplitude of Ω relative to Σ be u and the scale of the undulation of Ω be L. Then provided $|u/L \ll 1$, the condition (9) can be replaced by the approximation

$$(1/w)\partial_t p - \partial_z p = 0, \tag{10}$$

where $w = cg/\phi$ is a new parameter, namely, the free sinking velocity of the pore liquid under gravity (g = acceleration of gravity). Under these circumstances, the solution of the forward problem is (obtained by constructing a solution to (2) which satisfies (10) at the free surface and appropriate conditions at other sections of the reservoir boundary.

The presence of a first order derivative with respect to time in the free-surface condition (10) obviously leads to an additional relaxation process analog to the purely diffusive phenomena associated with the first order time derivative in the basic equation (2). As we shall conclude below, the individual time scales of the two phenomena are, however, quite different.

For the sake of brevity, we shall limit the present discussion to the simplest but practically quite relevant case of the semi-infinite liquid saturated homogeneous, isotropic and isothermal half-space. To consider the pure free-surface related phenomena, we eliminate pressure field diffusion by neglecting the compressibility of the liquid/rock system. In this setting we can combine the potential equation **(3)** and the surface condition (10) in one single equation confined to the Σ plane (Bodvarsson, 1978a), which expressed in terms of the fluid surface amplitude $u(t,x,y) = p/\rho g$ takes the form

$$(1/w)\partial_t u + \Pi_2^{\frac{1}{2}} u = f/\rho g c \tag{11}$$

where $\Pi_2^{\frac{1}{2}} = (-\partial_{xx}-\partial_{yy})^{\frac{1}{2}}$ is the square root of the two-dimensional Laplacian and f is an appropriately defined source density. To obtain the pressure field in the space $z>0$, ths boundary values derived from (11) have to be continued into the lower half-space on the basis of standard potential theoretical methods. The fractional order of the Laplacian in (11) is quite unusual, but the operator is well defined and poses no mathematical problems. For a further discussion of such operators in a slightly different setting, we refer to a paper by Bodvarsson (1977b).

Consider the attenuation of a wave formed pressure field of the form $\exp[-(t/t_0)+ikx]$ where t_0 is the relaxation time and k is the wave number. Inserting into equation (11), we find that $t_0=1/wk$. Comparing this result with the case of the purely diffusive pressure field we find that the ratio of the free surface/diffusion relaxation times is $ak^2/wk=k\phi/g\rho s$. The assumptions of waves of lengths 10 to 10^3 meters and porosities of 10^{-2} to 10^{-1}, results in ratios ranging from about 10^2 to 10^5. The relaxation times of diffusion phenomena are therefore orders of magnitude shorter than for free-surface Phenomena of a comparable spatial scale. As a result, we can conclude that in most cases of practical relevance, the two phenomena can be separated and treated individually.

Some solutions of equations (11) of practical interest have been obtained by Bodvarsson (1977a). Confining ourselves again to the simple semi-infinite half-space, the most important result is given **by** the causal impluse-response function $G(t,S,Q,)$ which represents the response of the surface amplitude at the point $S = (x,y)$ in Σ and time $t \geq 0+$ to an instantaneous injection of a unit mass fluid at a point Q in the half-space at time $t=0+$. The system is assumed to be in equilibrium for $t<0$. Let $Q = (0,0,d)$, the resulting expression for the surface amplitude is

$$G(t,S,Q) = (1/2\pi\phi\rho)(wt+d)[x^2+y^2+(wt+d)^2]^{-3/2} U_+(t) \tag{12}$$

where $U_+(t)$ is the causal unit-step function. The impluse response is essentially the key to the solution of (11) for more general conditions. The pressure field in the half-space is obtained from (12) by a simple continuation technique where the singularity at Q has to be taken into consideration. The long term response of the surface amplitude to a periodic source function at Q is of particular interest in the present context. Let the mass flow injected at Q=(0,0,d) take the form $\exp(-i\omega t)$. The amplitude of the frequency response is then obtained by

$$F(S,Q,\omega) = \int_0^\infty G(S,Q,\tau)\exp(i\tau\omega)d\tau \qquad (13)$$

The present results on the dynamics of the free surface amplitude provide the basis for a technique of reservoir probing and testing which yields results on C and ϕ that are supplementary to the conventional well test techniques (see Bodvarsson and Zais, 1978, this volume).

(5) Testing with controlled signals

Local. Reservoir tests with controlled drive yielding mostly local parameter values include primarily the driving po nt tests which are usually referred to as pressure buildup and/or drawdown tests on single we ls. Pioneering work on the development of this technique has been carried out at Stanford, and there exists considerable literature (see e.g. Ramey, 1976). nterpretation is based on appropriate solutions of equation (2).

Interference. The spatial scale of well-to-well interference tests depends on the distances involved. Short distances tend to yield only local parameter values whereas long distances may lead to results of a more global nature. In the case of simple systems of sufficient extend, the interpretation is to be based on the following concentrated source unit-step responses obtained on the basis of equation (2) (Carslaw and Jaeger, 1959).

axi-symmetric two-dimension	point-symmetric three-diemsion	
$-(m/4\pi c)Ei(-F^{-1})$,	$(m/4\pi Cr)erfc(F^{-\frac{1}{2}})$	(14)

where $F = 4at/r^2$ is the Fourier number and m is the appropriately defined (constant) mass flow applied. Again, there is very considerable literature on the subject (Matthews and Russell, 1967; Earlougher, 1977) mostly emphasizing the two-dimensional axi-symmetric situation.

It is important to note that in the above expressions, the complementary error function (erfc) and the exponential integral (Ei) will in the interval $0<F<0.5$ yield very similar values when taken as functions of time at a fixed field point. The short-term well interference test is therefore largely "blind" with regard to the space dimensions involved. Although the value of the amplitude factor is observable, its structure depends critically on the space dimension and this data does therefore not convey any information unless strong assumptions are made with regard to the underlying model. Considerable caution is therefore called for in the interpretation of well interference data, and it would appear that too much confidence has been placed in the applicability of the axi-symmetric two-dimensional Theis-type solution.

Global. Tests of this nature can be carried out only when there is sufficient data on the global characteristics of' the pressure/flow field. An important case consists in the use of global free liquid surface data. A brief review of the theory involved has already been given in section (5) above. Forward solutions based on the type of Green's 'function expression given by (6) above are of particular relevance in global work.

(6) Testing with natural signals

Types of drive. Available natural driving strain or force fields are of the following LF (low-frequency) to ULF (ultra-low-frequency) types (period range in parenthesis): seismic strain (1 to 10^3s), hydroelastic oscillations and noise (two-phase flow, etc.)(10 to 10^5s), tidal strain (10^4 to 10^6s), atmospheric pressure variations (10^4 to 10^6s), precipitation load (10^4 to 10^6s) and seasonal water-level variations (10^6 to 10^8s).

Local. The most obvious applications of natural drive are to the local type of testing. For the sake of brevity, we will limit our attention to VLF and ULF test signals where the mass forces on liquid columns in boreholes can be ignored. Moreover, in the single borehole case the essential results are obtained by deriving the pressure amplitude in an open hole (free liquid surface) to a homogeneous harmonic formation dilatation drive of amplitude b and angular frequency ω. Responses to other types of signals can then be easily derived with the help of a Fourier transform analysis. Under some further plausible simplifying assumptions, the following essential results for the pressure amplitude p in a single borehole of cross section f were presented at the 1977 Stanford Symposium (Bodvarsson, 1977c and 1978a), namely,

$$p = p_o T/(1+T), \quad \text{where } p_o = \varepsilon b/s, \text{ and } T = -4i\pi g r_o c/\omega f = -iAS/\omega. \qquad (15)$$

Here, p_o is the static pressure amplitude, ε the formation matrix coefficient characterizing the relation between the imposed strain and the porosity, r_o the radius of the (spherical) well cavity, $A = 4\pi r_o c$ the admittance of the cavity and $S = dp/dm = g/f$ is the mass stiffness of the well. The first two relations in (15) are general, but the third one is obtained on the basis of the assumption that $d/r_o \gg 1$ where d is the skin depth of the formation at tidal frequencies.

The case of pressure oscillations of tidal nature in closed wel1-reservoir systems has been discussed recently by Arditty, Ramey and Nur (1978). By a proper definition of the well mass stiffness S, the relations (15) would also be applicable to systems of this type.

The application of seismic signals in reservoir testing offers interesting possibilities. Mainly because of vertical displacement oscillations, the forward theory for this case is more complex than the results given above and can therefore not be discussed in this brief note. The subject has been investigated by Bodvarsson (1970). Forward solutions based on the iteractive type of series given by (8) are of particular interest in the interpretation of local tests based on natural drive.

Interference and global. More global type interference tests can, in principle, be carried on the basis of natural pressure or strain signals. Because of the well/well pressure field scattering processes involved, the theory cannot be discussed within the framework of this short note.

(7) Fractured reservoirs

The discussion above has been devoted entirely to formations which are of the Darcy type or can be approximated by such media. Largely fractured reservoirs have not received any attention. The theory of such cases differs from the material presented above and will not be discussed here. It is of interest to note that the mechanism of pressure field propagation in fractures with elastic walls has been discussed by Bodvarsson (1978b).

References

Ardittty, P.C., Ramey, H.J., Jr., and A.M. Nur, 1978. Response of a closed well-reservoir system to stress induced by earth tides. SPE paper 7484 Houston, Texas.

Bodvarsson, G., 1966. Direct interpretation methods in applied geophysics. Geoexploration 4:113-138.

Bodvarsson, G., 1970. Confined fluids as strain meters. J. Geophys. Res. 75 (14):2711-2718.

Bodvarsson, G., 1977a. Unconfined aquifer flow with a linearized free surface condition. Jokull, 27:1977.

Bodvarsson, G., 1977b. An equation for gravity waves on deep water. Jokull, 27:1977.

Bodvarsson, G., 1977c. Interpretation of borehole tides and other elasto-mechanical oscillatory phenomena in geothermal systems. Workshop on Geothermal Reservoir Engineering, December, 1977, Stanford University, Stanford, California.

Bodvarsson, G., 1978a. Convection and thermoelastic effects in narrow vertical fracture spaces with emphasis on analytical technique. Final Report for U.S.G.S. pp. 1-111.

Bodvarsson, G., 1978b. Pressure field propagation and density currents. 2nd Invitational Well Testing Symposium, Lawrence Berkeley Laboratory, Berkeley California.

Bodvarsson, G., and E. Zais, 1978. A field example of free surface testing. 4th Stanford Workshop on Geothermal Reservoir Engineering,

Carslaw, H.S. and J.C. Jaeger, 1959. Conduction of Heat in Solids. 2nd ed., 496 pp., Oxford.

Duff, G.F.D. and D. Naylor, 1966. Differential Equations of Applied Mathematics. John Wiley and Sons, New York, 423 p.

Earlougher, R.C. Jr., 1977. Advances in Well Test Analysis. Society of Petroleum Engineers of AIME, New York, N.Y., and Dallas, Texas.

Lamb, H., 1932. Hydrodynamics. 6th ed., 738 pp., Dover Publications, New York, N.Y.

Matthews, C.S. and D.G. Russell, 1967. Pressure buildup and flow tests in wells. Society of Petroleum Engineers of AIME. New York, N.Y. and Dallas, Texas.

Ramey, H.J., Jr., 1976. Pressure transient analysis for geothermal wells. 2nd U.N. Symposium on the Development & Use of Geothermal Resources, San Francisco.

Zemanian, A.H., 1965. Distribution Theory and Transform Analysis. McGraw-Hill Book Company, New York, N.Y.