PROGRESS REPORT ON MULTIPHASE GEOTHERMAL MODELING

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Work over the past year has concentrated on three areas: 1) to implement a concept of vertical equilibrium in geothermal modeling, 2) to improve the matrix equation solution technique for both two- and three-dimensional models, and 3) to apply a vertical equilibrium, areal model to the Wairakei, New Zealand geothermal field.

At the last Stanford meeting, a concept of vertical equilibrium as applied in the petroleum industry was outlined (for example, Coats and others, 1967). That is, vertically averaged liquid saturations are related to pressure at some reference level by employing pseudo capillary pressure and pseudo relative permeability curves. For the geothermal problem, many thermodynamic properties are strongly dependent functions of pressure and enthalpy, and an analogous approach would require many pseudo functions. Instead, the concept of vertical equilibrium is used to vertically integrate the thermodynamic properties and relate them to vertically averaged pressure and enthalpy. This results in a quasi three-dimensional model that allows a finite-difference block to become two-phase as soon as the pressure at the top of the block drops below the saturation pressure. The normal procedure for determining thermodynamic properties on the basis of pressures and enthalpies at specified reference levels in the grid block (usually the center) can lead to significant errors for thick blocks. The implementation of this vertically averaging approach has been verified by comparing a vertical equilibrium, areal model with a three-dimensional model.

To improve the matrix equation solution technique in the two-dimensional (vertical equilibrium) model a sequential solution formulation outlined in Coats and others (1974) is used. Solving the enthalpy equation first, the Newton-Raphson iteration is used on only the accumulation terms in two symmetric matrix equations that are each N x N (N being the number of nodes). By imbedding the sequential solution in the linearized Newton-Raphson equations, decomposition of the two matrices is required only on the first "sequential" iteration. Subsequent sequential iterations require only the formulation of a new righthand side and back substitution. Each additional Newton-Raphson iteration requires the formulation of an updated lefthand side, one decomposition, and several back substitutions.

The work involved in solving the matrix equation includes the initial decomposition plus 3-5 back substitutions depending on the convergence criterion. Usually, the computation time for all back substitutions is less than the computation time for the one decomposition. The symmetric matrix equations are solved using Gauss-Doolittle decomposition that takes advantage of D4 ordering (Price and Coats, 1974). In this ordering the finite-difference blocks are numbered in alternating diagonals. This numbering schemes results in a matrix with the upper half already in upper triangular form, so that only the lower half needs to be decomposed.

To summarize, the current model for areal problems incorporates the concept of vertical equilibrium, includes gravity terms, and the equations are solved using Newton-Raphson iteration on the accumulation terms. The resulting matrix equations are solved sequentially using D4 ordering and Gauss-Doolittle decomposition.

For the three-dimensional problems slice successive over-relaxation (SSOR) is imbedded in the Newton-Raphson iteration. For a description of SSOR see Wattenbarger and Thurnau (1976) or for the more general case of block successive over-relaxation (BSOR) see Woo and Emanuel (1976). This method is similar to line successive over-relaxation (LSOR) in two dimensions for coupled equations, except that rather than solving each row implicitly, each vertical cross-section of the grid is solved implicitly. This results in a matrix equation for each slide in which the matrix contains five non-zero diagonals. Thus, of the seven non-zero diagonals in the total three-dimensional matrix equation, only two are treated explicitly. Each of these matrix equations are solved using the Gauss-Doolittle method with normal ordering. Since SSOR is imbedded in the Newton-Raphson iteration, only linearized equations are solved. Therefore, the matrix decomposition for each slide is required only on the first SSOR iteration of each Newton-Raphson iteration. On subsequent SSOR iterations only back substitution is necessary. In addition, since the SSOR is imbedded in the Newton-Raphson iteration, the convergence is obtained in only a few iterations.

The vertical equilibrium (areal) model was applied to the Wairakei, New Zealand geothermal field. It is commonly believed that the Wairakei field was completely single phase (water) prior to exploitation; however, our recent steady-state modeling indicates that large regions in the reservoir probably had a small steam cap prior to exploitation. Furthermore, transient simulations indicate that leakage into the reservoir is significant; that is, the Wairakei reservoir is not a closed system. The most difficult part of history matching at Wairakei is adjusting permeabilities in order to remove enough mass from storage (as opposed to leakage) and reproduce the observed pressure decline trends.

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