Optimal Timing of Geothermal Energy Extraction

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Introduction

This paper is concerned with the optimal time to commence extraction of energy from a hot-water geothermal reservoir. The economic models that we have presented in the past have the common characteristics that the extraction program starts immediately (see [1] and [2]). Based on this assumption, we determined optimal extraction strategies and planning horizons such that the present values of total profits were maximized. In this study we relax the requirement that extraction be undertaken immediately, seeking instead the delay in starting time that along with the other decision variables maximizes the present value of total profits over the economic life of the reservoir. Of course, optimal starting time, economic life of the reservoir, optimal extraction rate, and optimal injection temperature are interrelated, and therefore, we analyze their effect on the overall planning strategy simultaneously.

Physical Assumptions

Our economic model is based on a production-reinjection well doublet (Gringarten-Sauty model [3]) where the aquifer is assumed to be saturated and homogeneous and is bounded top and bottom by impermeable aquicludes. The initial equilibrium temperature of the aquifer is T_o. After τ years from the start of pumping, the temperature declines below T_o, and this temperature is denoted by T_o^t, showing the dependence of the temperature on time. Our economic model can easily accommodate other hydrothermal models. However, the Gringarten-Sauty model allows for expressing T_o^t as a function of energy extraction rate (see Trang et al. [4]), and this functional relationship simplifies our analysis of the economic model.

Hot water is pumped from the aquifer, run through a heat exchanger and reinjected to the aquifer. Low-pressure steam is generated on the other side of the heat exchanger. We will assume that steam can be sold at the cost of the least expensive alternative to produce this steam.

The Economic Model

We will assume that the real value of energy increases with time in an exponential manner. Extraction commences at time u and continues for L years at the rate of Q, and the brine is reinjected in the aquifer at the temperature of T_i. We are interested in maximizing the total discounted profits. In other words, we like to:

$$\underset{u,\ L,\ Q,\ T_i}{\text{Maximize}} \ \Pi(u, L, Q, T_i) = (1-\eta)\int_u^{\tau+u} a\ P_o e^{rt} Q c_f \rho_f (T_o - T_i) e^{-it} dt$$
$$+ (1-\eta)\int_{\tau+u}^{L+u} a\ P_o e^{rt} Q c_f \rho_f (T_o^t - T_i) e^{-it} dt \qquad (1)$$
$$-C(u, L, Q, T_i),$$

subject to:

$$u, L, Q \geq 0$$

$$T_i \geq T_s$$

$$T_o^t - T_i \geq \delta$$

where

P_o = price (value) of energy at time zero,

r = rate of increase of real energy price,

η = royalty for geothermal lease paid as a fraction of revenues,

Q = extraction rate (m^3/hr)

c_f = specific heat of the fluid (cal/gr°C),

ρ_f = fluid density (gr/cm^3),

i = discount rate,

u = starting time (years),

τ = breakthrough time (years),

L = extraction period (years),

T_i = injection temperature (°C),

T_o^t = production temperature as a function of time (°C) for given extraction rate,

T_s = steam temperature (°C).

C( ) = total cost as a function of decision variables,

and a is a conversion factor to yield revenues in dollars/year.

The cost function includes capital costs for wells and equipment, operating and maintenance costs, rents and salaries, and termination costs. C is, of course, a function of our four decision variables.

The constraints simply imply that the injection temperature (which is the same as the heat exchanger outlet temperature) should remain above the steam temperature, and that the difference between the inlet and outlet temperatures of the heat exchanger should not fall below a certain level, δ (we are ignoring heat losses in surface pipes).

Optimization

Denoting the sum of the first two expressions in (1) as revenues $R(u, L, Q, T_i)$, we show that if total revenues associated with immediate extraction (R) can be computed (as done in [2]), then the analogous revenue when extraction delay is incorporated is just:

$$R(u, L, Q, T_i) = e^{(r-i)u} R \qquad (2)$$

Likewise, for costs we show that total costs when delay is considered may be written as:

$$C(u, L, Q, T_i) = C_1 e^{-iu} + C_2 e^{(r-i)u} \qquad (3)$$

where

and

C_1 = total extraction costs less pumping energy costs

C_2 = pumping energy costs.

We also show that the optimal injection temperature T_i can be expressed as a function of Q and L. These results enable us to show that for each Q and L, the optimal starting time is either equal to zero or is given by

$$u^* = -\frac{1}{r}\ln\left[\frac{(i-r)B}{iH}\right] \qquad (4)$$

where

$B = R-C_2$

and

$H = C_1$-(annual pre-exploition rent/i).

In other words, depending on the value of the parameters involved, the profit maximizing entrepreneur should either start extraction immediately or wait for a time of u* years (as given by equation 4) before commencing extraction.

An interesting result which greatly facilitates the computation of the optimal vector (Q*, L*, u*) is our result that for each L, either the optimal extraction rate is the Q that maximizes B^i/H^{i-r}, or Q_o, the optimal extraction rate when extraction is immediate, depending on whether the ratio (i-r)B/iH falls between 0 and 1 or not. Using this result, we have developed an algorithm that finds the global maximum efficiently.

Results

The optimization is conducted with a particular set of data which to our best judgment reflects the current value of pertinent costs. The geohydrological data have generally been chosen in mid-range of values associated with known hot-water geothermal resources.

We note that the computer program developed for this study can be readily utilized for decision making under a different set of conditions. Geohydrological and economic data are inputs to the program, and the cost subroutine can be easily modified to accommodate the particular costs involved in the exploitation of each individual field.

In our computations we allowed interest rate i to vary from 4% to 15% and r, the real energy value growth rate, from 1% to 3%.

As expected, optimal profits decrease as i increases and increase as r increases. Also, the optimal starting time increases with r and decreases with i. In other words, when the rate of increase in value of energy is high, the profit maximizing entrepreneur postpones the onset of extraction, while he prefers to start extraction immediately if the value of energy is not expected to rise as fast.

The optimal pumping rate increases with i and decreases with r. Thus, as r is increased, the optimal decision is to extract heat more slowly and leaving more heat for the future when the value is higher. An interesting result is the fact that even when extraction is postponed, the optimal extraction rate is approximately the same as the optimal rate when extraction is immediate.

The economic lives L^* are nonincreasing in i and nondecreasing in r (with L^* taking predominantly the value of the assumed well life). Thus, when future profits are discounted more heavily, the entrepreneur tends to start extraction sooner and pumps the energy faster over a shorter period of time compared to when the discount rate is not as high.

References

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