LARGE-SCALE GEOTHERMAL FIELD PARAMETERS AND CONVECTION THEORY

R. A. Wooding
Applied Mathematics Division
D.S.I.R.
Wellington, New Zealand

The question of the depth reached by groundwater in natural recharge to a geothermal field is of interest for geothermal development, since it can affect the nature of the recharge regime during withdrawal, and the volume of water within reach during exploitation. Also, useful inferences may be drawn about the large-scale permeability of the system if the groundwater flow regime is understood.

Evidence for the presence of thermal convection in the groundwater now appears to be well-established, although topographic effects may also be important (Studt and Thompson 1969, Healy and Hochstein 1973). Two regions which serve particularly well as illustrations are (1) the Imperial Valley of Southern California and (2) the Taupo Volcanic Zone of New Zealand. Both exhibit a number of quite well-defined zones of anomalously high heat flow (geothermal fields), separated by distances of 10 to 15 Km, the intervening areas usually having very low heat flow. At Imperial Valley, the fairly permeable sands in which convection is likely to occur are overlain by sediments of low permeability, roughly 0.6 Km in thickness, and thermal conductivity alone without appreciable convection, commonly occurs in these upper layers (Palmer, Howard and Lande 1975). In the case of (2), the heat flow in areas surrounding geothermal fields is depressed practically to zero, and this has been interpreted by Studt and Thompson as being due to downflowing recharge water from precipitation. The water issuing naturally from geothermal fields is predominantly meteoric, but the residence times in the groundwater stage appear to be very long.

It follows that the upper boundary conditions of the two cases must be significantly different. In (1) the upper flow boundary is practically impermeable while, in (2), flow through the upper boundary is almost unimpeded. Idealized conditions which correspond approximately to these cases were introduced by Lapwood (1948); these will be designated as boundary conditions 1 and 2 respectively.

Lapwood calculated critical Rayleigh numbers ($R = R_c$) for neutral stability in a horizontal layer of uniform isotropic porous material, heated from below to maintain a constant temperature difference between the two boundaries. Fluid properties and thermal conductivity of the saturated medium were assumed constant. Although the stability approach does not yield heat-flux Nusselt numbers for convection at supercritical Rayleigh numbers, it provides a useful prediction of the most likely aspect ratio--horizontal wavelength to layer depth--of convection cells under finite amplitude conditions provided that $R - R_c$ is small in comparison with R_c. Also, the approach is convenient for studying the influence of changing fluid or medium properties; many cases can be treated quickly, and likely combinations of parameters may be selected for more detailed study at higher Rayleigh numbers.

Magnitudes of Convection Parameters

Several factors indicate that R/R_c is not very large in the two geothermal zones discussed above. It is likely that heat enters the system by conduction through rock layers from quite shallow, perhaps magmatic, sources. If convection were not present, a thermal anomaly would still exist, with a different spatial distribution, and probably with a heat flow several times normal. The presence of convection will enhance the heat flow, but probably by a factor of order 2, rather than 10. (From a practical point of view, perhaps the most important function of convection is to redistribute and concentrate the heat flow.) A low Nusselt number will be associated with only moderate values of R/R_c.

In round numbers, a $1000^{o}C$ magma body at a depth of about 5 Km would give rise to a conduction heat flow of 5-10 heat flow units (1 h.f.u. being the world average). If convection were present in the upper part of the 5 Km layer, giving rise to an overall Nusselt number of 2, this would account for the heat flow observed in, for example, the Taupo Volcanic Zone.

A low value of Rayleigh number appears to be consistent with estimated physical parameters, average values from the upper part of the Wairakei field (McNabb, Grant and Robinson 1975). Assuming vertical permeability $K \simeq 7 \times 10^{-11}$ cm^2, cold water viscosity $\mu_o \simeq 10^{-2}$ poise, thermal conductivity $K \simeq 3 \times 10^{-3}$ c.g.s. units, liquid density contrast $\Delta\rho \simeq 0.2$, it is found that

$$R/L - kg\Delta\rho/\kappa\mu_o \qquad (1)$$

$$\simeq 50 \text{ per Km depth.}$$

Here the depth L of the permeable layer is unknown, but it is suggested that it is not more than about 3 Km. It is important to establish whether the convection theory is consistent with this shallow depth of groundwater penetration and the observed 10-15 Km separation of geothermal fields.

Extensions of the Theory

The matrix permeability K and the fluid viscosity μ are involved only through the ratio K/μ--the "mobility"--but in practice this function may be quite complex. This has led to various extensions of Lapwood's work.

Using upper boundary conditions of type 1, Kassoy and Zebib (1976) have considered the case of temperature-dependent viscosity, noting that, for water, μ may change by an order of magnitude over the range of temperatures encountered in geothermal applications. On the other hand, Ribando, Torrance and Turcotte (1976) treated viscosity as constant, and carried out numerical calculations of finite-amplitude convection both for the Lapwood system and for permeability decreasing exponentially with depth.

A peculiar effect observed recently in silica-water systems (H. J. Ramey, Jr., pers. comm.) is that the permeability appears to decrease with rising temperature, perhaps by a factor of 2 or more in a range of a few hundred degrees centigrade. Although an explanation is not forthcoming at this time of writing, it is interesting to note that silica polymerizes in aqueous solution to form a gel--a property which has been studied in connection with the formation of scale (Marsh, Klein and Vermeulen 1975). Thus the phenomenon may be equivalent to an increase of effective viscosity with temperature, partially counteracting the usual viscosity decrease associated with pure water. For purposes of calculation, this can be incorporated into the assumed temperature-viscosity law.

The Permeability Problem

Permeable media encountered in geothermal areas depart greatly from the simple homogeneous isotropic systems frequently considered in the laboratory and in theory. The Taupo Volcanic Zone exhibits many such complications, in particular the layering produced by a sequence of many thin volcanic deposits, varying in degrees of welding, brecciation, etc., and perhaps interspersed with thin sedimentary lenses, the occasional existence of highly permeable, weathered horizons between successive deposits, and the presence of numerous near-vertical faults trending along the Zone. On the large scale, a fracture-dominated system still appears to be well represented by a Darcy-type flow law, but the permeability is likely to be non-isotropic (H. J. Ramey, Jr., pers. comm.).

Borehole data on which large-scale permeability might be estimated is inadequate, generally because detailed information on fractures and permeable horizons is missed. However, zones of drill circulation loss are recorded, and can give a useful indication of fractures encountered. For the deepest borehole in the Wairakei geothermal field (Bore 121, 2265 metres) circulation losses are encountered frequently down to 1000 m, but only a few cases are noted at greater depths (1680 m and 2250 m, G. Grindley and P. Browne, pers. comm.). This indication of fewer permeable fractures at the greater depths is in accord with the observed hydrothermal alteration (P. Browne, pers. comm.), which implies a lesser through-flow of water. However, there are no other bores of comparable depth at Wairakei to supplement these limited observations.

Attempts to estimate the vertical and horizontal components of large-scale permeability in the area of the Wairakei field (McNabb, Grant and Robinson 1975) indicate that the horizontal permeability could have been anything up to 10 times as great. A contrast as high as this would be consistent with a layered system having very permeable horizons. The vertical faulting could be less important, as there are indications that permeability varies to a lesser extent with horizontal direction.

Stability Analysis from Convection Theory

The basic equations of thermal convection of a variable-viscosity fluid in a saturated medium have been given elsewhere (e.g., Wooding 1975).

A simple, but relevant generalization to anisotropic permeability is realized by assuming horizontal stratification, so that one principal axis of the permeability tensor is vertical and the other two are horizontal. Let γ_1, γ_2 be the ratios of the vertical component of permeability to the two horizontal components. These ratios will be assumed constant although the individual components of permeability may vary with depth.

Suitable scales for the convection problem are the length L (layer depth), the thermal diffusivity κ and the velocity $R\kappa/L$, where R is the Rayleigh number defined in (1). The time scale is $EL^2/R\kappa$, where E is the ratio of the heat capacity of the saturated medium to that of the fluid (Wooding 1957). Also, $\Delta\rho$ is an appropriate density scale.

If z is the dimensionless upward vertical coordinate, the dimensionless density profile corresponding to steady conduction of heat from below is equal to z. Any small perturbation Θ (x, y, z, t) of this profile will give rise to a perturbation velocity field; if w (x, y, z, t) is the vertical component of velocity, let

$$(\theta, w) = (\theta_1 (z), w_1 (z)) e^{\lambda\tau} \sin \alpha x \sin \beta y \qquad (2)$$

where τ is dimensionless time and α, β are dimensionless wave numbers. Then the linearized equations give, for the z-dependent functions Θ_1, w_1,

$$D(\sigma D)w_1 - (\alpha^2/\gamma_1 + \beta^2/\gamma_2)(\sigma w_1 + \theta_1) = 0 \qquad (3)$$

$$w_1 = \frac{1}{R} (D^2 - \alpha^2 - \beta^2 - \lambda R) \theta_1 \qquad (4)$$

where $D \equiv d/dz$ and $\sigma = (\nu/\kappa)/(\nu/\kappa)_o$, ($\nu = \mu/\rho$), the suffix o referring to values at the upper boundary. The boundary conditions 1 and 2 give

$$\theta_1 = w_1 = 0 \text{ at } z = 0 \qquad (5)$$

and 1) $\quad \theta_1 = w_1 = 0$ at $z = 1$ $\qquad$ (6a)

2) $\quad \theta_1 = Dw_1 = 0$ at $z = 1$ $\qquad$ (6b)

where 1) refers to an impermeable upper boundary and 2) to a boundary which is permeable (giving constant pressure).

Results from Stability Analysis

When the ratio ν/κ is constant ($\sigma = 1$), (3) to (6) can be solved analytically, and would include the case where the decrease in kinematic viscosity with depth (due to rising temperature) is balanced by the decrease of permeability with depth--a reasonable approximation to reality.

Figure 1 is a plot of wavenumber α_m and minimum Rayleigh number R_m, for given values of the permeability ratio γ_1, assuming that $\beta = 0$. The curves 1 and 2 correspond to boundary conditions 1 and 2. For any given value of γ_1, the system is more unstable with boundary conditions 2 than with boundary conditions 1. However, the curves 1 and 2 are quite similar in position and shape, and situations involving boundary conditions intermediate between 1 and 2 might be inferred readily. For this reason equal values of γ_1 on the two curves are joined by broken lines. Curves 1 and 2 tend to the same value of R_m as γ_1, and α_m tend to zero; i.e., as the horizontal-to-vertical permeability ratio increases, the permeability of the upper boundary to fluid flow becomes less significant.

The reduction of α_m with decreasing γ_1 (increasing anisotrophy) is substantial. If, for example, $\gamma_1 \simeq 0.1$--a possible value according to McNabb, Grant and Robinson (1975)--α_m is likely to be in the range 1.4 to 1.8, which corresponds to a horizontal wavelength to layer depth ratio of 4 to 5.2 for hexagonal cells. If this can be extrapolated to finite-amplitude convection in a geothermal zone, a 3 Km depth of groundwater flow would lead to a field spacing of 12 to 15.6 Km, which is plausible when compared with observation.

When σ varies with z, the equations (3) ff. have been solved numerically. Surprisingly, the wavenumber of greatest instability, α_m, is relatively insensitive to variations of viscosity and permeability with depth, even when these approach an order of magnitude. This suggests that if other, unsuspected, factors are not present, the observed field geometry is most strongly influenced by anisotropic permeability.

When the medium also exhibits anisotrophy in the horizontal, it is necessary to consider three-dimensional instability in more detail. Contours of R_c have been plotted as a function of wavenumbers α and β. When the horizontal permeability in the x-direction exceeds that in the y-direction, R_c has a minimum (R_m) at $\beta = 0$ and $\alpha = \alpha_m$. This shows that the most unstable small disturbance consists of two-dimensional rolls with axes at right angles to the direction of maximum permeability. It does not follow, however, that such rolls will be observed at finite amplitudes when $R > R_m$. For example, the effect of variable viscosity may be to impose three-dimensional convection cells upon the system.

A more detailed discussion of these results is given elsewhere (Wooding 1976).

References

Healy, J. and M. P. Hochstein, 1973, Horizontal flow in hydrothermal systems; J. Hydrol. N.Z. 12, 71-82.

Kassoy, D. R. and A. Zebib, 1976, Variable viscosity effects on the onset of convection in porous media; Phys. Fluids 18, 1649-51.

Lapwood, E. R., 1948, Convection of a fluid in a porous medium; Proc. Camb. Phil. Soc. 44, 508-21.

McNabb, A., Grant, M. A. and J. L. Robinson, 1975, Permeability estimates; AMD Tech. Rep no. 34.

Marsh, A. R., III, Klein, G. and T. Vermeulen, 1975, Polymerization kinetics and equilibria of silicic acid in aqueous systems; Lawrence Berkeley Lab. Rep. LBL-4415.

Palmer, T. D., Howard, J. H. and D. P. Lande, 1975, Geothermal development of the Salton Trough, California and Mexico. Lawrence Livermore Lab. Rep. UCRL-51775.

Ribando, R. J., Torrance, K. E. and D. L. Turcotte, 1976, Numerical models for hydrothermal circulation in the oceanic crust; J. Geophys. Res. 81, 3007-12.

Studt, F. E. and G. E. K. Thompson, 1969, Geothermal heat flow in the North Island of New Zealand; N.Z. J. Geol. Geophys. 12, 673-83.

Wooding, R. A., 1957, Steady state free thermal convection of liquid in a saturated permeable medium; J. Fluid Mech. 2, 273-85.

Wooding, R. A., 1975, Methods of solution of the equations for convection in porous media, with geothermal applications. In "Geothermal Reservoir Engineering," P. Kruger and H. J. Ramey, Jr., eds. Stanford Geothermal Program, Rep. SGP-TR-12, 206-12.

Wooding, R. A., 1976, Influence of anisotropy and variable viscosity upon convection in a heated saturated porous layer; AMD Tech. Rep. no. 55.

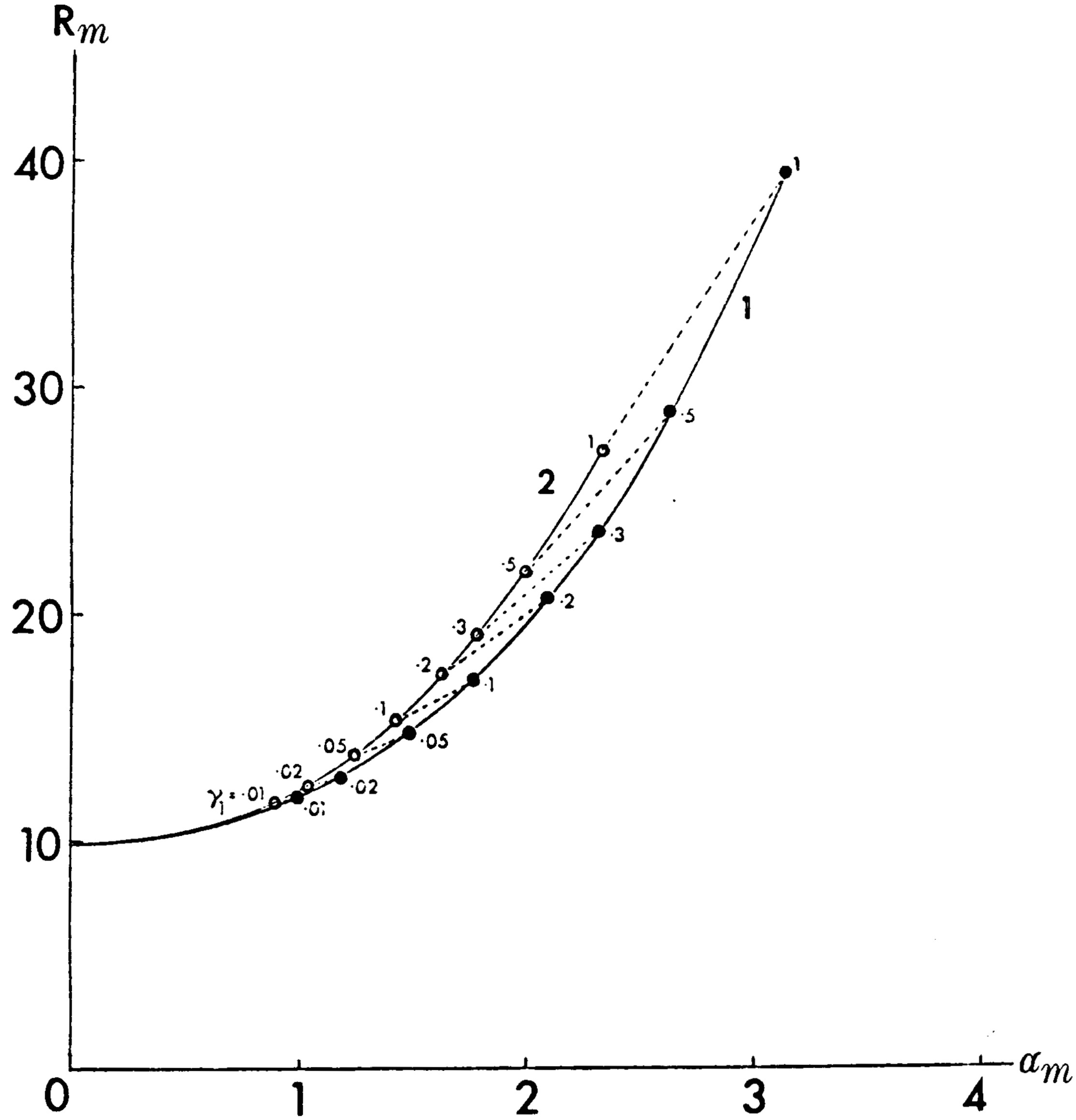

Figure 1. Minimum critical Rayleigh number R_m and the corresponding wavenumber α_m for various vertical-horizontal permeability ratios γ_1.