FLUID FLOW THROUGH A LARGE VERTICAL CRACK IN THE EARTH'S CRUST

J. Weertman
Departments of Materials Science & Engineering and Geological Sciences
Northwestern University, Evanston, Ill. 60201
S. P. Chang
Engineering Sciences Department, Northwestern University

In this investigation, we are primarily concerned with modeling fluid flow through vertical cracks that were created for the purpose of extracting heat from hot, dry rock masses. The basic equation for the two-dimensional problem of fluid flow through a crack is presented and an approximate solution is found. The basic equation is a non-linear, Cauchy-singular integro-differential equation. Moderately simple formulae for the crack opening displacement and the effective pressure difference between the crack tips are derived. The results are valid for arbitrary vertical cracks, provided that the fluid injection and removal points are not placed too close to the crack tips. (A more complete treatment of this problem is given by us in a paper to appear in the Journal of Geophysical Research.)

The Basic Equation

Consider a vertical, liquid-filled, two-dimensional crack the center of which, at $y = 0$, is assumed to be at a depth below the earth's surface that is large compared with the half height L of the crack. Let $D(y)$ represent the crack opening displacement at the vertical distance y from the crack center. $D(y)$ is determined by the following nonlinear, Cauchy-singular integro-differential equation:

$$\frac{\mu}{2\pi(1-\nu)} \int_{-L}^{L} \frac{B(y')dy'}{y-y'} + \frac{\mu}{\pi(\lambda+2\mu)} \int_{-L}^{L} \frac{\tau(y')dy'}{y-y'} = -T(y) + P_0 - P_0' - (\rho-\rho')gy - \int_0^y P'_g(y)dy \tag{1}$$

where $B(y) = -dD(y)/dy$, P_0 is the overburden pressure within the rock mass at $y = 0$, P_0' is the hydrostatic pressure within the liquid at $y = 0$, $T(y)$ is any tensile or compressive tectonic stress component within the rock mass whose axis is perpendicular to the crack plane,

ρ is the density of rock and ρ' is the density of the liquid, g is the gravitational acceleration, P'_g is the component of the pressure gradient within the liquid that drives fluid flow, ν is Poisson's ratio, μ is the shear modulus and λ is the other Lamé constant, and $\tau(y)$, which is equal to $-P'_g(y)D(y)/2$, is the shear stress exerted parallel to the crack faces that is produced when fluid flows through the crack.

When the fluid flow is laminar and when the crack faces are nearly parallel to each other, the pressure gradient P'_g is equal to or very nearly equal to (Batchelor, 1967)

$$P'_g(y) = -12V\eta/D^3(y) \tag{2}$$

where η is the viscosity of the fluid and V is the volume of fluid that moves past the point y in unit time per unit length of crack. In the cases of interest to us, the fluid flow will always be laminar or not strongly turbulent.

It is unlikely that fluid used to extract geothermal energy from a vertical crack in a hot, dry rock mass would be injected exactly at the lower crack tip and removed exactly at the upper crack tip (or vice versa). A more realistic situation is one in which water is injected at $y = -L'$ and is removed at $y = L'$ where $L' < L$. In this situation, $P'_g = 0$ in the region $L' \leq |y| \leq L$. Thus, Eq. (1) can be written as (on inserting Eq. (2) into Eq. (1) and also using the relationship $2\tau(y) = D(y)P'_g(y)$).

$$\frac{\mu}{2\pi(1-\nu)} \int_{-L}^{L} \frac{B(y')dy'}{y-y'} = - \frac{6V\eta\mu}{\pi(\lambda+2\mu)} \int_{-L'}^{L'} \frac{dy'}{D^2(y')(y-y')}$$

$$-T(y) + P_0 - P'_0 - (\rho-\rho')gy + 12V\eta \int_0^y \frac{H(L'-|y|)dy}{D^3(y)} \tag{3}$$

where H is the Heaviside step function.

Solution by Linearization

An approximate solution of Eq. (3) may be obtained by setting up a perturbation scheme and solving, with increasing labor, the resulting equations. However, a reasonably good approximate solution can be found by using the following simple physical arguments. The crack opening displacement is not a rapidly varying quantity near the center of the crack. Furthermore, the integral terms on the right-hand side of Eq. (3) can be shown, a posteriori, to be relatively small in magnitude compared with the other terms for any reasonable values of V. Therefore, if the value of L' differs appreciably from the value of L, which we now assume it does, the crack opening displacement D(y) in the two integrals in question may be considered to have a constant value in a first approximation. It will be obvious later that there will be no need to carry the solution to a higher approximation in which D(y) is not considered to be a constant in the two integrals. We also assume that the tectonic stress T(y) is equal to a constant and is independent of the variable y.

Under these assumptions the crack opening displacement D(y) is found to be

$$D(y) = \{(1-\nu)/\mu\}\{2(T-P_0+P_0') + (\rho-\rho')gy\}(L^2-y^2)^{\frac{1}{2}}$$

$$+ \left[\frac{12V\eta(1-\nu)}{\pi\mu D_c^{\,3}}\right]\Big[-(\pi\mu D_c/[\lambda+2\mu])\{(L'-y)H(L'-y) + [1-H(L'+y)](L'+y)\}$$

$$+ \{\pi-2\sin^{-1}(y/L)\}\{L'D_c\mu/[\lambda+2\mu]\} - 2y(L^2-y^2)^{\frac{1}{2}}\sin^{-1}(L'/L) \qquad (4)$$

$$- 2L'y\log|\{(L^2-y^2)^{\frac{1}{2}}+(L^2-L'^2)^{\frac{1}{2}}\}/\{(L^2-y^2)^{\frac{1}{2}}-(L^2-L'^2)^{\frac{1}{2}}\}|$$

$$+ (y^2+L'^2)\log|\{y(L^2-L'^2)^{\frac{1}{2}}+L'(L^2-y^2)^{\frac{1}{2}}\}/\{y(L^2-L'^2)^{\frac{1}{2}}-L'(L^2-y^2)^{\frac{1}{2}}\}|\Big]$$

where for $|y|>L$ the displacement $D(y) = 0$. The term D_c is equal to

$$D_c = (2L')^{-1}\int_{-L'}^{L'} D(y)dy = \{(1-\nu)(T-P_0+P_0')/\mu\}\{L^2-L'^2)^{\frac{1}{2}}$$

$$+ (L^2/L')\sin^{-1}(L'/L)\} \qquad (5)$$

and is not a function of V and η .

The crack opening displacement D(y) produces stress singularities at the crack tips. The tensile stress T' acting across the crack plane just ahead of the crack tip is equal to

$$T' = K/\sqrt{2\pi r} \tag{6}$$

where r(<<L) is the distance measured from a crack tip and K is the stress intensity factor which is defined as the limit

$$K_{\pm} = \pm \left[\{\mu/(1-\nu)\}\{(\pi/4L)(L^2-y^2)\}^{\frac{1}{2}}B(y)\right]_{y\to\pm L} \tag{7}$$

where the + sign is used in the limit of y - L (K at upper crack tip) and the - sign when y→ -L (K at lower tip). On substituting Eq. (5) into Eq. (8) the following values of the stress intensity factor are found:

$$K_{\pm} = (T-P_0+P_0')(\pi L)^{\frac{1}{2}} \pm (\pi L)^{\frac{1}{2}}\{[\rho-\rho')gL/2]$$

$$- (12V\eta/\pi D_c^3)[(L'/L)(L^2-L'^2)^{\frac{1}{2}} \tag{8}$$

$$+ L\sin^{-1}(L'/L) - \{\mu L'D_c/L(\lambda+2\mu)\}]\}.$$

For the situation in which V = 0, the longest possible crack half height that can exist without K_- taking on negative values for a given value of $(T-P_0+P_0')$ is equal to

$$L = 2(T-P_0+P_0')/g(\rho-\rho'). \tag{9}$$

A crack of this length has a displacement at y = 0 equal to

$$D(0) = (\rho-\rho')g(1-\nu)L^2/\mu. \tag{10}$$

The stress intensity factor is equal to

$$K_+ = (\rho-\rho')gL(\pi L)^{\frac{1}{2}} \tag{11}$$

when $K_- = 0$. If K_+ is equal to or larger than the critic value K_c for crack propagation, the crack will propagate to the upper surface by breaking rock open at the upper tip while simultaneously closing itself up at the lower tip (Weertman 1971a, 1971b, 1973; Secor and Pollard 1975).

Now consider the case in which V is not zero. Because $D_c \ll L$, if can be shown (Weertman and Chang) that the terms that contain the factor $\eta/(\lambda+2\mu)$ in Eqs. (4) and (8) can be dropped from these equations without introducing any appreciable error. It can further be shown that except for cracks with half heights smaller than 50 m the terms in Eqs. (5) and (8) that contain the expression $(12V\eta/\pi D_c^3)L$ are small in magnitude compared with the terms that contain the expression $(\rho-\rho')gL$ or $(T-P_0+P_0')$. Thus the crack profile and the stress intensity factor of a large crack with water flowing is essentially the same as that of a water-filled crack in which the fluid is stationary.

A Remark on Two Corrections

There are two corrections that can be made to our results. One of these is for the influence of the earth's surface. Another correction takes into account the force in the vertical direction produced at the crack walls by the fluid pressure because the crack walls are not vertical when the crack is filled with fluid. It can be shown (Weertman and Chang) that both of these corrections are negligibly small.

Conclusion

We conclude from this analysis that the crack profile and stress intensity factors of any large vertical fluid-filled crack for heat extraction purposes will not be changed appreciably when fluid is forced to flow through the crack at physically practical velocities.

References

Batchelor, G. K., An Introduction to Fluid Dynamics, Cambridge University Press, 1967.

Secor, D. T. and Pollard, D. D., On the Stability of Open Hydraulic Fractures on the Earth's Crust, Geophys. Res. Lett., 2, 510-513 (1975).

Weertman, J., Theory of Water-Filled Crevasses in Glaciers Applied to Vertical Magma Transport Beneath Ocean Ridges, J. Geophys. Res., 76, 1171-1183 (1971a).

Weertman, J., Velocity at Which Liquid Filled Cracks Move on the Earth's Crust or in Glaciers, J. Geophys. Res., 76, 8544-8553 (1971b).

Weertman, J., Ocean Ridges, Magma Filled Cracks and Mantle Plumes, Geofisica International, 13, 317-336 (1973).

Weertman, J. and Chang, S. P., Fluid Flow Through a Large Vertical Crack in the Earth's Crust, J. Geophys. Res. (in press).