ANALYSIS OF WELL TESTS WITH VARIABLE DISCHARGE

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The conventional methods of well tests analysis usually assume a constant rate of discharge of the producing well. The procedure involves matching a log-log plot of test data (drawdown versus time) to analytic or semi-analytic solutions that are based on a model of the production well as a line source of constant strength in an infinite reservoir. However, variable discharge well test conditions may arise under a variety of conditions, such as existing well-field production schedules, step-drawdown tests, and influence of the pumping water level on the production rate. It is very desirable to have the capability to reliably interpret data from the tests. In fact, the present study was prompted by a set of recent geothermal well test data in which due to various mechanical problems, the flow rate during the first 70 hours of production varied widely and could not effectively be treated as a mean constant rate. The present paper reports the development of a general technique of analyzing well tests with variable flow rates.

Method of Analysis

The variable flow is approximated by a series of sequential straight line segments of arbitrary length and slope (Figure 1). The pressure response of each linearly varying production pulse at any time is derived analytically in terms of the well-known exponential integral. The change in pressure head at time t and distance r from the producing well that is caused by a production pulse n, occurring between τ_n and τ_{n+1} is given by

$$\Delta h_n(t,r) = \frac{1}{4\pi kH} \int_{\tau_n}^{\tau_{n+1}} Q_n(\tau) \frac{e^{-\frac{r^2}{4\alpha(t-\tau)}}}{t-\tau} d\tau$$

If we define

$$x_1 = \frac{1}{4\pi kH}$$

$$x_2 = \frac{r^2}{4\alpha} = \frac{\mu\phi CHr^2}{4kH}$$

and the linear flow rate is given by

$$Q_n(\tau) = A_n + B_n(\tau - \tau_n)$$

then

$$\Delta h_n(t,r) = x_1 \int_{\tau_n}^{\tau_{n+1}} \left[A_n + B_n(\tau - \tau_n)\right] \frac{e^{-\frac{x_2}{t-\tau}}}{t-\tau}\, d\tau$$

The result of the integration is given by

$$\Delta h_n(t,r) = x_1 \left\{ \left[A_n + B_n(\tau_n + x_2)\right]\left[W(u_n) - W(u_{n+1})\right] - \left[B_n \tau_n e^{-u_n} - \tau_{n+1} e^{-u_{n+1}}\right] \right\}$$

where

$$u_n = \frac{x_2}{t-\tau_n}$$

$$u_{n+1} = \frac{x_2}{t-\tau_{n+1}}$$

and $W(u)$ is the well function, which is related to the exponential integral $Ei(x)$ by

$$W(u) = -Ei(-u)$$

The total pressure drop as a function of time is obtained by a superposition of the reservoir responses attributable to each production pulse:

$$\Delta h(t,r) = \sum \Delta h_n(t,r)$$

To account for the influence of one linear boundary, a third parameter is defined as

$$x_3 = \frac{\mu\phi C H r_i^2}{4kH}$$

where r_i is the image well distance, and the pressure drop is then given by

$$\Delta h(t,r) = \sum \Delta h_n(t,r) \pm \Delta h_n\ t, r_i$$

where the position or negative sign indicated an impermeable or fully leaky boundary respectively.

These equations are used to calculate pressure drawdown values that correspond to a specific flow rate variation and a given set of reservoir parameters. Thus with a set of initial guess values of reservoir parameters a multiparameter least-square-fit computer routine is employed to compare well-test data with predicted values in a search for the best set of reservoir parameters. Input data for the program are A_n and τ_n, coordinate points on the production history record, Δh and t, coordinate points on the well-test pressure drawdown record, and a few program control parameters.

Corresponding to optimal values of x_1, x_2, and x_3 obtained, the program assigns values respectively to transmissivity, kH; storativity, ϕCH; and the image well distance r_i. Both interference and production tests can be analyzed. In the latter case, work is in progress to account for the influence of wellbore storage and skin effects, also in a parametric fashion.

Applications

The method has been applied to data from seven well tests to evaluate its utility (see Table 1). Three of the analyses involve theoretically generated well-test data and four analyses involve field data.

The field data were from two well tests conducted at the East Mesa Geothermal field in southern California and from two well tests conducted at the Raft River in southern Idaho. The three theoretical cases involve well test data calculated assuming: (1) constant discharge; (2) variable discharge in steps, and (3) exponentially decaying discharge. Three of the four field tests were constant discharge interference tests, two of which indicated the presence of a boundary. The last remaining field test involved a discharge rate with a very wide fluctuation.

In all of these cases, a solution was possible and an unambiguous set of reservoir parameters was determined. In the three tests using generated data, known parameters are reasonably reproduced. The first three field interference tests analyses yielded reasonable parameters. Figures 2, 3 and 4 show respectively the theoretical case of exponentially decaying discharge rate, the East Mesa 31-1 constant discharge interference field tests analysis involving the detection of a barrier boundary, and the analysis of the Raft River #3 production test, in which the flow rate varied markedly. In the figures, the circles represent observed drawdown data and the squares represent the best-fit drawdown values. The close agreement with analytical and conventional results in cases where they are available indicates the validity of the method. Furthermore, the results of the RRGE 3 production test analysis indicate that the methods can successfully analyze variable flow field data.

Conclusions

This method will make it possible to do well test analyses when a constant discharge flow rate is difficult to maintain, and permit detection of boundaries even in situations where there is a markedly varying flow rate. Work is currently in progress to extend the analysis to the study short-term data.

Nomenclature

Δh	Pressure head drop
n	Designation of production segment
t	Time
r	Distance from well
k	permeability
H	Thickness of aquifer
τ	Time
Q	Flow rate
α	$k/(\phi\mu C)$
ϕ	Porosity
C	Total compressibility
r_i	Image well distance

TEST CASES	DISCHARGE		BARRIERS		TYPE TEST	
	Constant	Variable	None	Impermeable	Interference	Production
<u>Synthetic Data</u>						
Constant Discharge	●		●		●	
Stepped Discharge		●	●			●
Exponential Decay Discharge		●	●			●
<u>Field Data</u>						
Raft River No. 2	●			●	●	
East Mesa Well 31-1	●			●	●	
East Mesa Well 6-2	●		●		●	
Raft River No. 3		●	●			●

TABLE I. WELL TESTS ANALYZED.

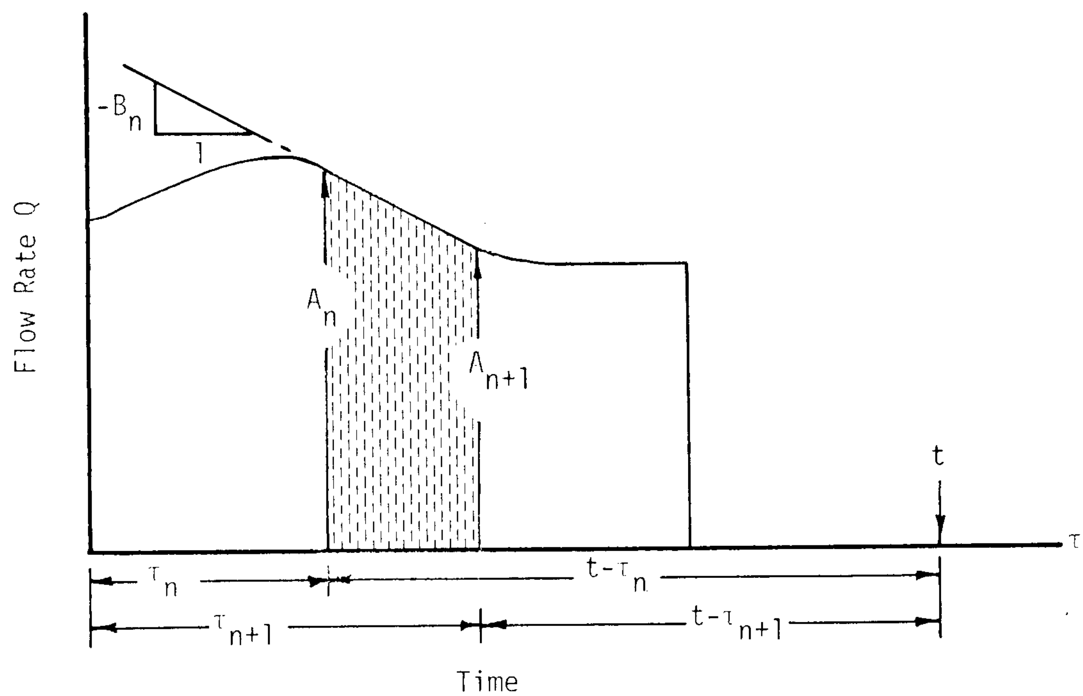


Figure 1. Time-Dependent Flow Rate

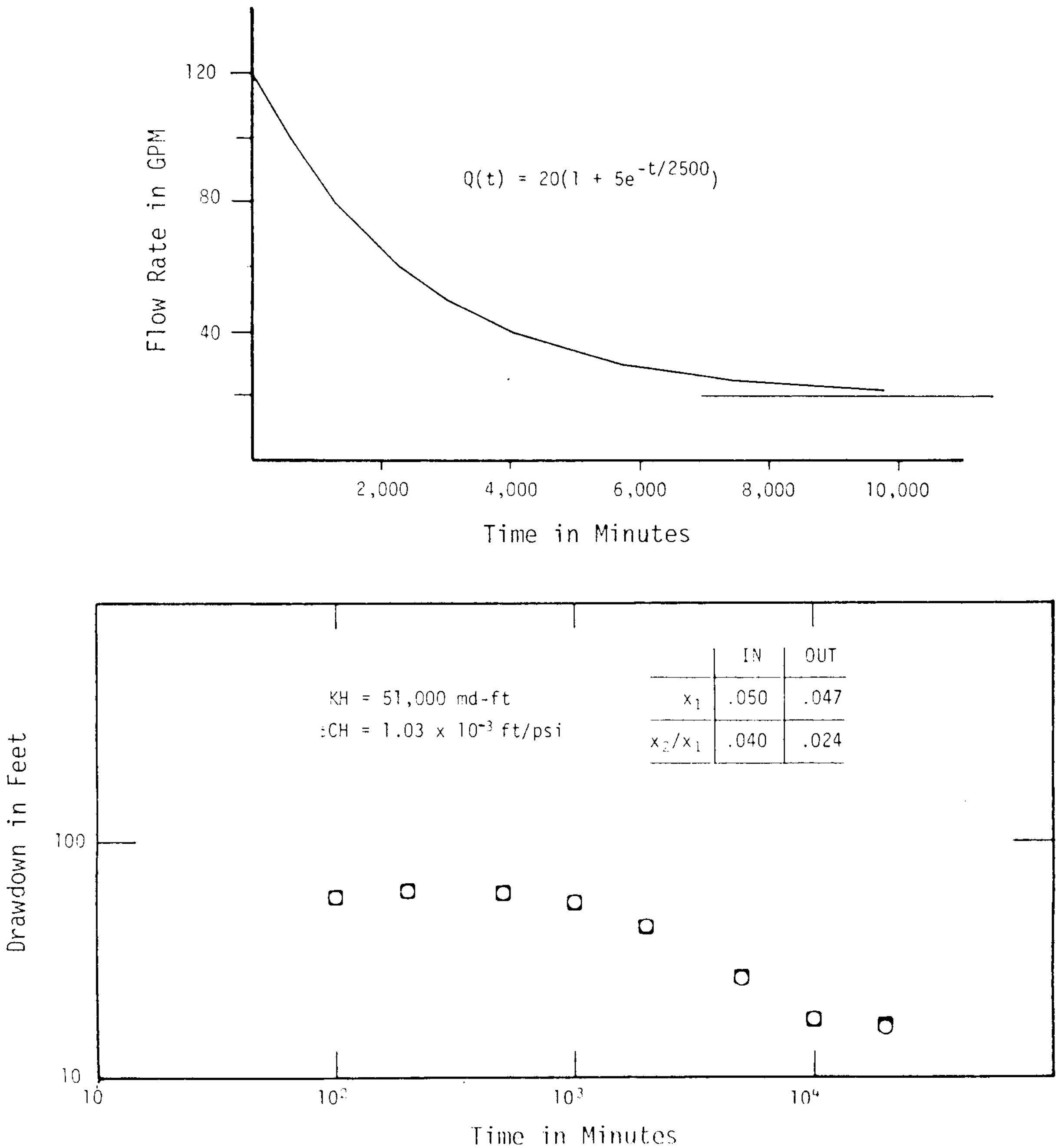
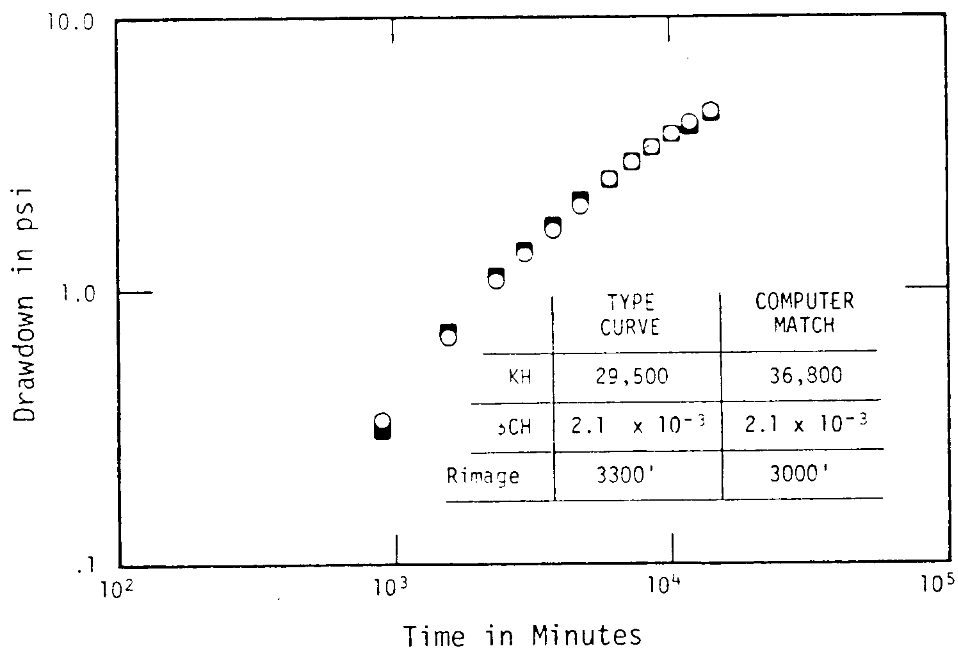


Figure 2. Exponential Decay of Discharge



	TYPE CURVE	COMPUTER MATCH
KH	29,500	36,800
ϕCH	2.1×10^{-3}	2.1×10^{-3}
Rimage	3300'	3000'

Figure 3. East Mesa 31-1 Interference Test

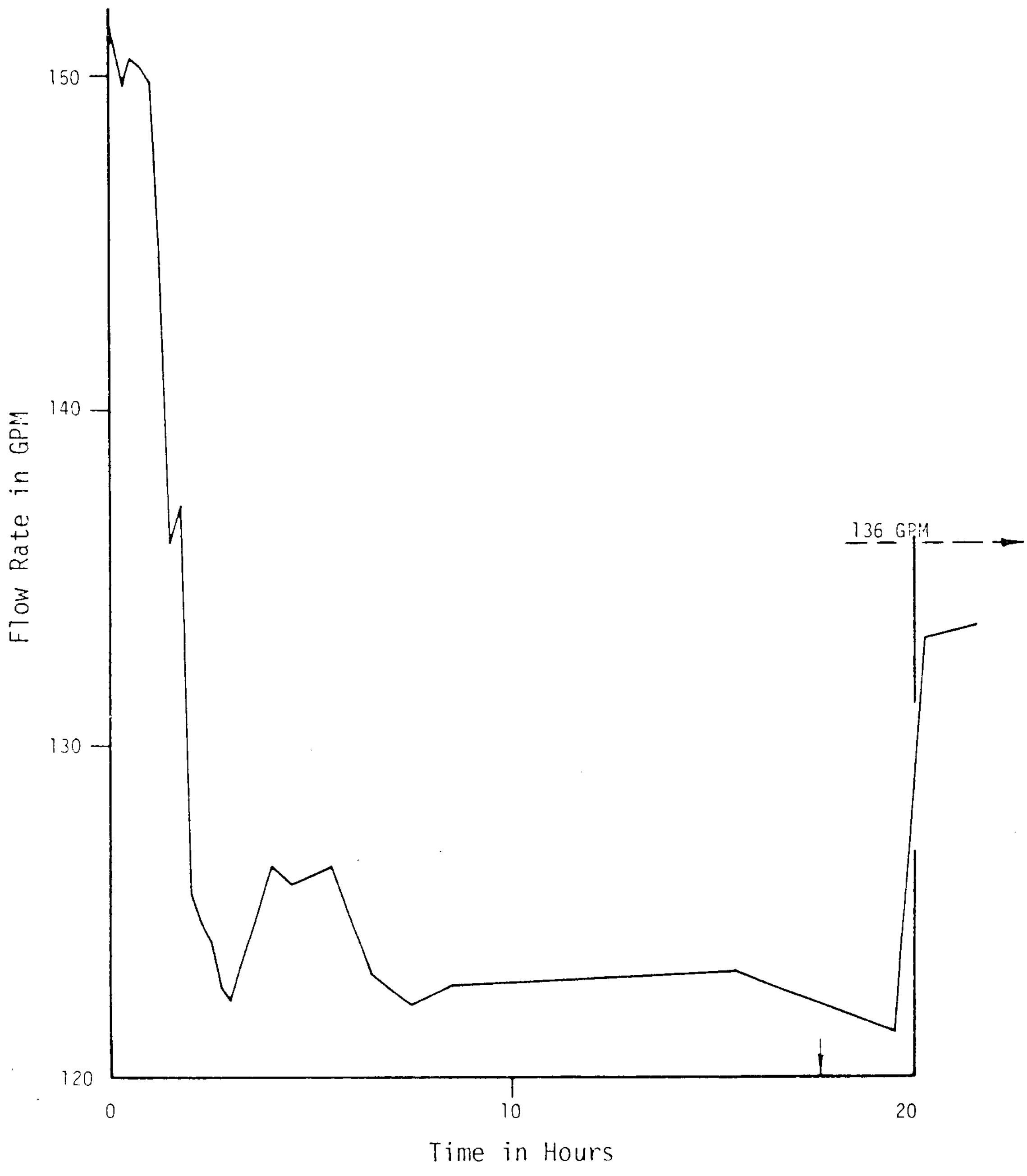


Figure 4a. RRGE 3 Production History

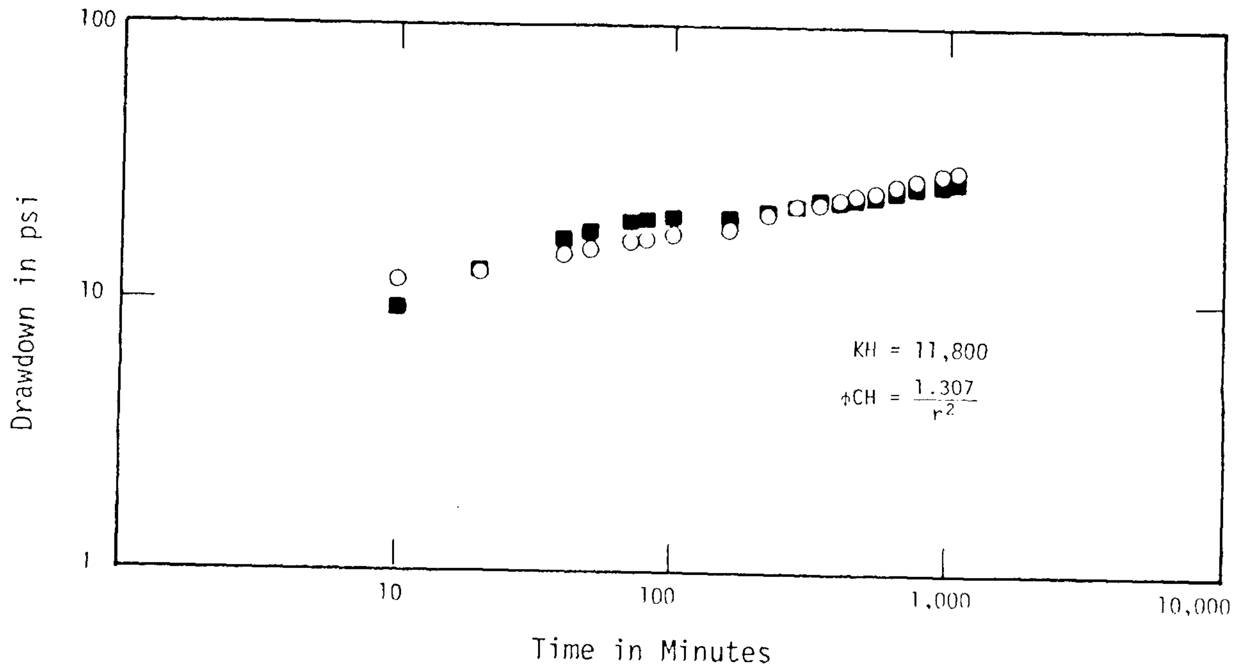


Figure 4b. RRGE 3 Production Test