A SEMI-ANALYTICAL APPROACH TO GEOTHERMAL RESERVOIR PERFORMANCE PREDICTION

S. K. Sanyal, M. Sengul and H. T. Meidav
Geonomics, Inc.
3165-7 Adeline Street
Berkeley, CA 94703

This paper presents a simplified analytic treatment of the problem of fluid flow and heat transfer in a hot water reservoir. A multi-layered reservoir is considered, with a circular array of producing wells surrounded by a concentric, circular array of injection wells. Complete injection of produced water, and hence an eventual steady state, is assumed for the flow system. A temperature gradient is assumed in the radial direction. The rock properties are allowed to vary from layer to layer, but are considered uniform within a particular layer. The heat transfer problem is handled by a modification of the solution to the problem of heat extraction from fractured dry rocks proposed by Gringarten, et al. (1975). The reservoir is represented as a vertical stack of horizontal layers, with permeable and impermeable layers alternating. The pressure distributions in various layers are calculated by spatial superposition of the continuous line source solution for the given geometry, with average fluid and rock properties within the system. This approach can yield results such as the breakthrough time of injected water in each layer, pressure distribution in space and time and the temperature of the produced water over time. In a study of the Heber geothermal reservoir in the Imperial Valley of California such results have shown reasonably close agreement with the results from computer simulation.

Many hot water geothermal reservoirs display a closed temperature anomaly, i.e., the temperature of the reservoir is highest near the center and gradually declines towards the periphery. For such reservoirs a logical development plan is to produce hot water from the central part of the reservoir through an array or cluster of production wells. The heat is extracted from the produced water for power generation, and the cooled water is injected into the cooler marginal areas of the reservoir through an array of injection wells. This paper presents a semi-analytic method for analyzing the heat and fluid flow characteristics of such a system.

HEAT FLOW ANALYSIS

The objective of this analysis is to be able to forecast the outlet temperature which, together with the fluid production rate at the production wells, determines the heat flow rate.

Physical and Mathematical Model. As shown in Figure 1a, the reservoir consists of thin sand and shale layers with differing thickness, permeability and porosity for each sand layer, and with shale layers all having the same thickness and assumed to be located between these sand layers. Cold water is injected through the injection wells located on a circle with radius R_2 and hot water is produced at the production wells located on a circle with radius R_1 as shown in Figure 1b. Initially the reservoir temperature increases linearly from the injection wells to the production wells.

The mathematical model is based upon Figure 2 where the relevant information concerning the heat flow for a sand layer is represented. Z_E is the distance from the bottom of the shale layer to the no heat flow boundary within it. If the average water flow rate for all the sand layers is the same and the thickness of the sand layers and the shale layers is constant then Z_E will be half of the shale thickness.

The following assumptions are made in simplifying the physical model:

1. The sand layers and the shale layers are homogeneous and isotropic.

2. The density, heat capacity, and thermal conductivity of water, of the solid matrix of the sand layer, and of the shale layer are constant. Further, the density, specific heat, and thermal conductivity of the shale and of the solid matrix of the sand layer are the same.

3. The water temperature T_w is only a function of radial coordinate, r, and time, t, and does not vary with the vertical coordinate, z.

4. Heat conduction in the radial direction in both sand and shale layers is negligible.

5. Initially, both the sand and shale layers are at the same temperature at any given r. Taking the temperature gradient in the r direction into account, the initial temperature distribution at any given r is given by the initial rock temperature T_{ro} at the point of production minus the product of the temperature gradient, a, and the distance from the production well.

Heat flow for a single layer, shown in Figure 2, is governed by two differential equations

$$\frac{h}{2}\rho_1 C_1 \frac{\partial T_w(r,t)}{\partial t} + \frac{h}{2}\rho_w C_w V_w \frac{\partial T_w(r,t)}{\partial r} = k_R \frac{\partial T_R(r,z,t)}{\partial z}\bigg|_{z=h/2} \qquad (1)$$

where $\rho_1 C_1 = (1-\phi)\rho_R C_R + \phi\rho_w C_w$

and

$$\frac{\partial^2 T_R(r,z,t)}{\partial z^2} = \frac{1}{D_R} \frac{\partial T_R(r,z,t)}{\partial t} \tag{2}$$

$T_w(r,t)$ and $T_R(r,z,t)$ are water and rock temperatures respectively.

The temperatures must also satisfy the following initial and boundary conditions:

$$T_R(r,z,t) = T_w(r,t) = T_{RO} - a(R_2-r), \quad t<r/v \tag{3}$$

$$T_R(R_2,z,t) = T_w(R_2,t) = T_{RO} \qquad t<0 \tag{4}$$

$$T_R(R_2,z,t) = T_w(R_2,t) - T_{wo} \qquad t\geq 0 \tag{5}$$

$$T_w(r,t) = T_R(r,Z_E,t) \qquad \text{for all } r \text{ and } t \tag{6}$$

$$\partial T_R(r,z,t)/\partial z\Big|_{z=Z_E} = 0 \tag{7}$$

For a single layer, taking Z_E at infinity, Lauwerier (1955) gave a solution for the above problem in Cartesian coordinates. In order to use Lauwerier's solution the shale layers separating the sand layers should be thicker than they are assumed to be in this study. Carslaw and Jaeger (1959) gave the solution to the same problem as Lauwerier except that they considered a single fracture instead of a porous sand layer. Recently, Gringarten et al. (1975) gave the solution for the mathematical problem above in Cartesian coordinates, but they solved the problem for an infinite series of parallel, equidistant fractures of uniform thickness rather than for sand layers. Gringarten gave the solution, dimensionless temperature $T_{wD}(r,t_D)$, in the form of a graph as a function of two dimensionless numbers, given in our notation as follows:

$$Z_{ED} = (\rho_w C_w/k_R)(\overline{q}/r)Z_E \tag{8}$$

$$t'_D = [(\rho_w C_w)^2/k_R\rho_R C_R](\overline{q}/r)^2 t' \tag{9}$$

where $t' = t-(R_2-R_1)/\overline{v}_w$. The second term $(R_2-R_1)/\overline{v}_w$ is the breakthrough time, i.e., the time taken by the injected water to arrive at the production well. The dimensionless temperature, T_wD, is given by:

$$T_{wD} = [T_{RO}-T_w(r,t)]/(T_{RO}-T_{wo}) \qquad (10)$$

For given values of Z_{ED} and t'_D, T_{wD} is read from the graph.

Defining an average flow rate, $\bar{q}$, per sand layer per unit length, one can use the solution given by Gringarten et al. in the analysis of the problem at hand. This application is summarized in the following section.

Application of Gringarten's Solution. In order to use the solution given in graphical form to find the produced water temperature, one needs to determine the dimensionless numbers given by Eqs. 8 and 9. Assuming the thermal properties of the water and shale are known, still to be found are the values of the breakthrough time, $(R_2-R_1)/v_w$, and the ratio between the average flow rate, $\bar{q}$, and the distance $r = R_2-R_1$.

Given the total injection rate Q, the average flow rate, $\bar{q}$, is given by the expression:

$$\bar{q} = Q/\pi(R_1+R_2) \qquad (11)$$

Based on the relative magnitude of the $(kh)_i$ product of each layer, the average flow rate for each layer is found as:

$$\bar{q}_i = \bar{q}(kh)_i / \sum_{i=1}^{m} (kh)_i, \quad i=1,2,\ldots,m \qquad (12)$$

where $(kh)_i$ is the product of permeability and thickness of the i^{th} layer, and m is the number of sand layers.

Dividing the rate, $\bar{q}_i$, for a sand layer by the product of its thickness and porosity $(h\phi)_i$, the average velocity in the layer is obtained:

$$\bar{V}_{wi} = \bar{q}_i/(h\phi)_i, \quad i=1,2,\ldots,m \qquad (13)$$

Using these values of $\bar{q}_i$, and $\bar{v}_{wi}$ together with $r = R_2-R_1$ in Eqs. 8 and 9, the values of Z_{ED} and the breakthrough time, $(R_2-R_1)/\bar{v}_{wi}$, are found.

To obtain the water outlet temperatures for each layer at different times, now the task is to determine the dimensionless time, t'_D, which is taken as zero for $t<(R_2-R_1)/\bar{v}_{wi}$. However, in the application of Gringarten's solution to the problem under investigation one faces two problems:

(1) as pointed out earlier, Gringarten's solution assumes that all the flow rates are the same and therefore that Z_{EDi}'s are the same, whereas here they are different for each layer, and
(2) Gringarten assumes that there is no temperature gradient along the fracture, whereas there is a temperature gradient along the sand layer in the present problem.

Pertaining to the first of these problems, if the values of Z_{EDi}'s for the layers expected to have significant outlet temperature drops all fall in a narrow range, then the errors introduced by the variance of Z_{ED} from layer to layer can be considered acceptable for engineering purposes. As for the second problem, to relax the assumption of no temperature gradient in the radial direction and to include the effect of this temperature gradient in the solution, all of the layers can be divided into several concentric sections. Initially all the sections are assumed to be at a uniform temperature which is given by their median temperature, and the temperature gradient in each of these sections is neglected. Also the area weighted average flow rate, $\overline{q}$, is calculated for each section of every sand layer and thus the same is done for $\overline{V}_{wi}$.

In finding the outlet temperature history of a layer at $t = t_1<t_2<...<t_m$, one first calculates the outlet temperature at the production end of the first section, which will have the shortest breakthrough time. Using Gringarten's solution, the outlet temperature of this section is obtained at time t_2 with time interval $\Delta t_1 = t_1,t_2$. If during this first time period Δt_1, not only the first section but the next section (or sections) breaks through, then first the outlet temperature of this second section is found. The average of this outlet temperature is used as the injection temperature for the first section to find its outlet temperature by superposition.

The above procedure is repeated for all the layers and their outlet temperature histories are found. Taking the density and the specific heat of water constant, the average bottom hole water outlet temperature history is found by the following expression:

$$T(t)_{av} = \sum_{i=1}^{m} (\overline{q}_i/\overline{q}) T_{wi}(R_1, t) \qquad (14)$$

FLUID FLOW ANALYSIS

The objective in this section is to be able to predict the pressure behavior of production and injection wells again for the system shown in Figure 1. In order to get a better understanding of the fluid flow characteristics of the system, both early and late pressure histories will be investigated.

The following assumptions are made in order to simplify the physics of the problem:

(1) the reservoir is infinitely large, compared to the well bore radius
(2) homogeneous and isotropic medium
(3) formation has a uniform thickness
(4) porosity, permeability, and viscosity are constant (independent of pressure and temperature)

Further, for simplicity, the radii of the production and injection wells are taken to be the same, and the production and injection wells are assumed to have constant production and injection rates and show no skin effect.

The solution giving the pressure history of a production well producing at a constant rate located in a reservoir for which the above assumptions hold is the continuous line source solution given as follows:

$$\frac{kh}{141.3 q_p \mu B_w}[P_i - P(r,t)] = P_D(r_D, t_D) = \frac{1}{2} Ei(-\frac{r_D^2}{4t_D}) \qquad (15)$$

where $r_D = r/r_w$, $t_D = 0.000264\ kt/\phi\mu c_t r_w^2$, and

$$-Ei(-x) = \int_0^\infty \frac{e^{-u}}{u} du \text{ (exponential integral)}$$

For an injection well the production rate, q_p, in Eq. 15 will be replaced by the injection rate with negative sign $(-q_{in})$.

In an infinite reservoir where there are N production and M injection wells, the pressure history of any given production or injection well can be found through the solution given above with spatial superposition if the injection and production rates are constant with time or are a step function of time.

Taking q_p as the production rate for all the production wells and q_{in} as the injection rate for all the injection wells, the pressure history of a production well will be given by:

$$P_p(r,t) = P_i - \left[\frac{141.3\mu B_w q_p}{kh}\right]\sum_{i=1}^{N} -\frac{1}{2}E_i(-\frac{r_{Di}^2}{4t_D})$$
$$+ \left[\frac{141.3\mu B_w q_{in}}{kh}\right]\sum_{j=1}^{M} -\frac{1}{2}E_i(-\frac{r_{Dj}^2}{4t_D}) \qquad (16)$$

A similar expression will be obtained for the injection pressure history, P_{in} (r,t), for an injection well.

To find $p_p(r,t)$ for a given value of dimensionless time t_D one needs to find the dimensionless radial distance r_{Di} to all of the production wells and r_{Dj} for all of the injection wells, and then to find all of the values of the exponential integral for all of the arguments $(r^2_{Di}/4t_D)$, i = 1,..., N, and $(r^2_{Dj}/4t_D)$, j = 1,...,M.

For the system under investigation the values of r_{Di} and r_{Dj} can be evaluated through the following expressions (see Figure 3):

$$r_{Di} = 1, \; i = 1 \tag{17}$$

$$r_{Di} = \frac{R_1}{r_w} \sqrt{2[1-\cos \frac{2\pi(i-1)}{N}]}, \; i = 2,\ldots,N \tag{18}$$

$$r_{Dj} = \frac{1}{r_w}\sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos \frac{2\pi(j-1)}{M}}, \; j = 1,2,\ldots M \tag{19}$$

A computer program is developed and production and injection pressure histories are computed for various values of the variables affecting the fluid flow characteristics of the system.

NOMENCLATURE

B_w	=	formation volume factor for water
c_t	=	total system effective isothermal compressibility
C_R	=	specific heat of shale or the solid matrix of the sand layers
k	=	formation permeability
k_R	=	thermal conductivity of shale or the solid matrix of the sand layer.
P_D	=	dimensionless pressure
P_i	=	initial pressure
r_w	=	radius of the production or injection wells
T_{RO}	=	rock temperature at the point of injection
T_{wo}	=	water injection temperature
v_w	=	water velocity
ρ_R	=	density of shale or the solid matrix of the sand layers
ρ_w	=	density of water
μ	=	viscosity of water
ϕ	=	porosity
θ	=	angle

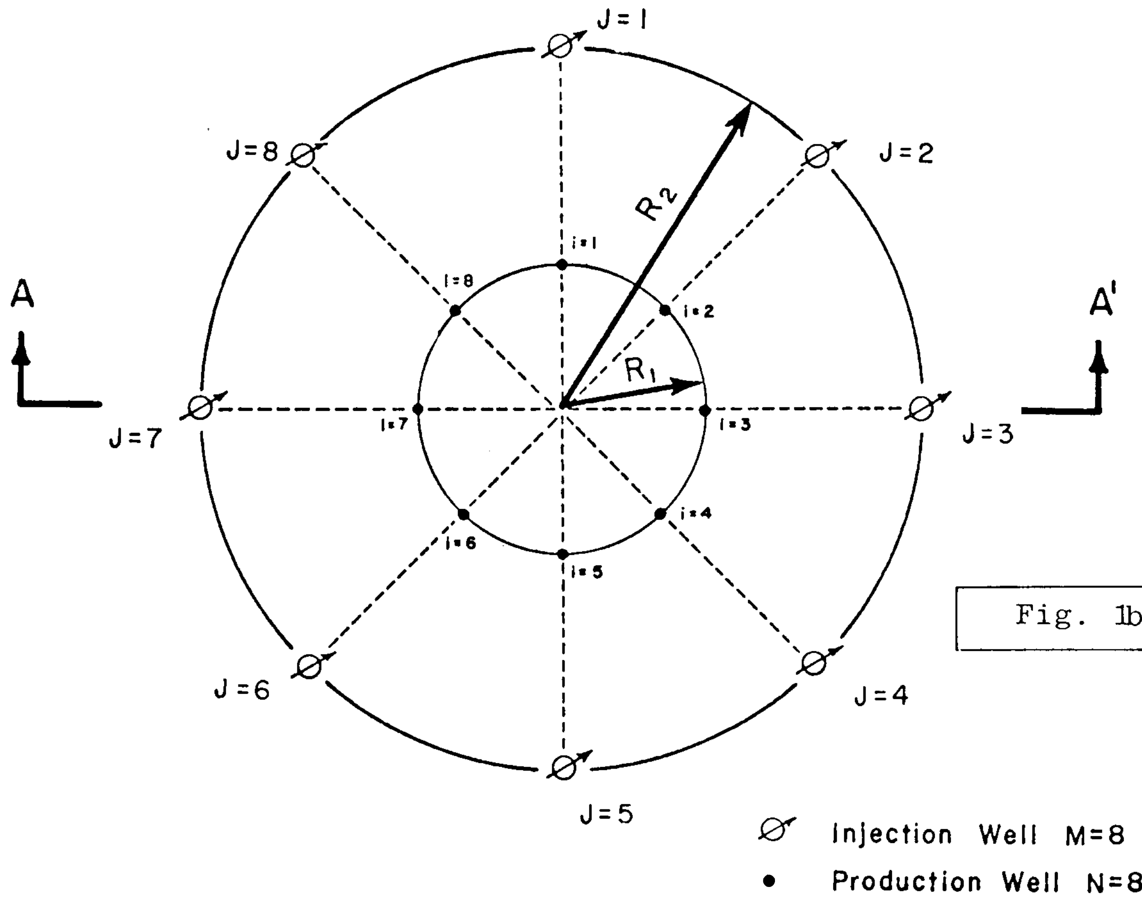

Fig. 1b

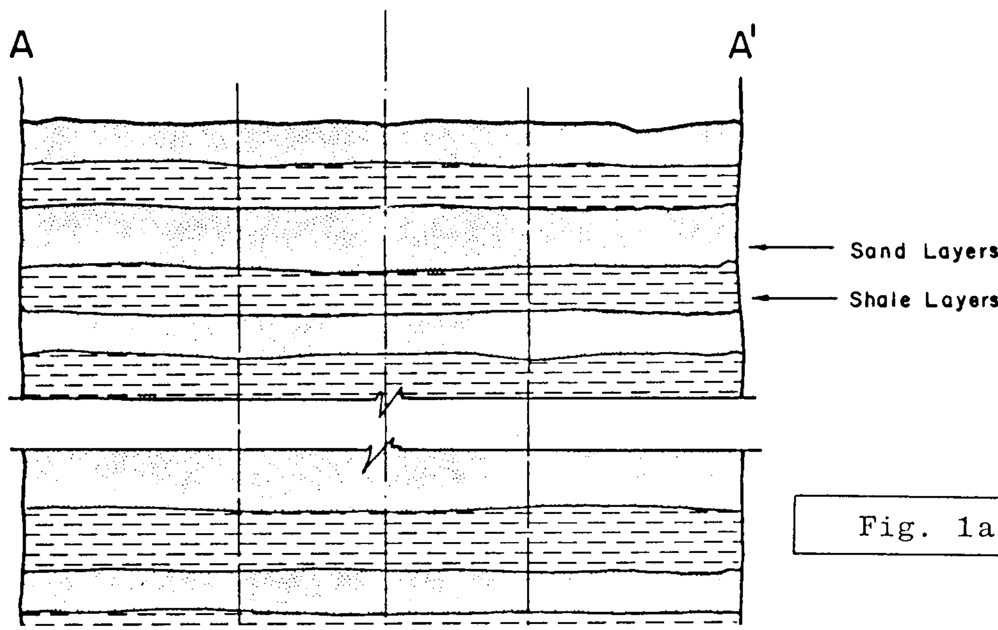

Fig. 1a

Figure 1. Physical system

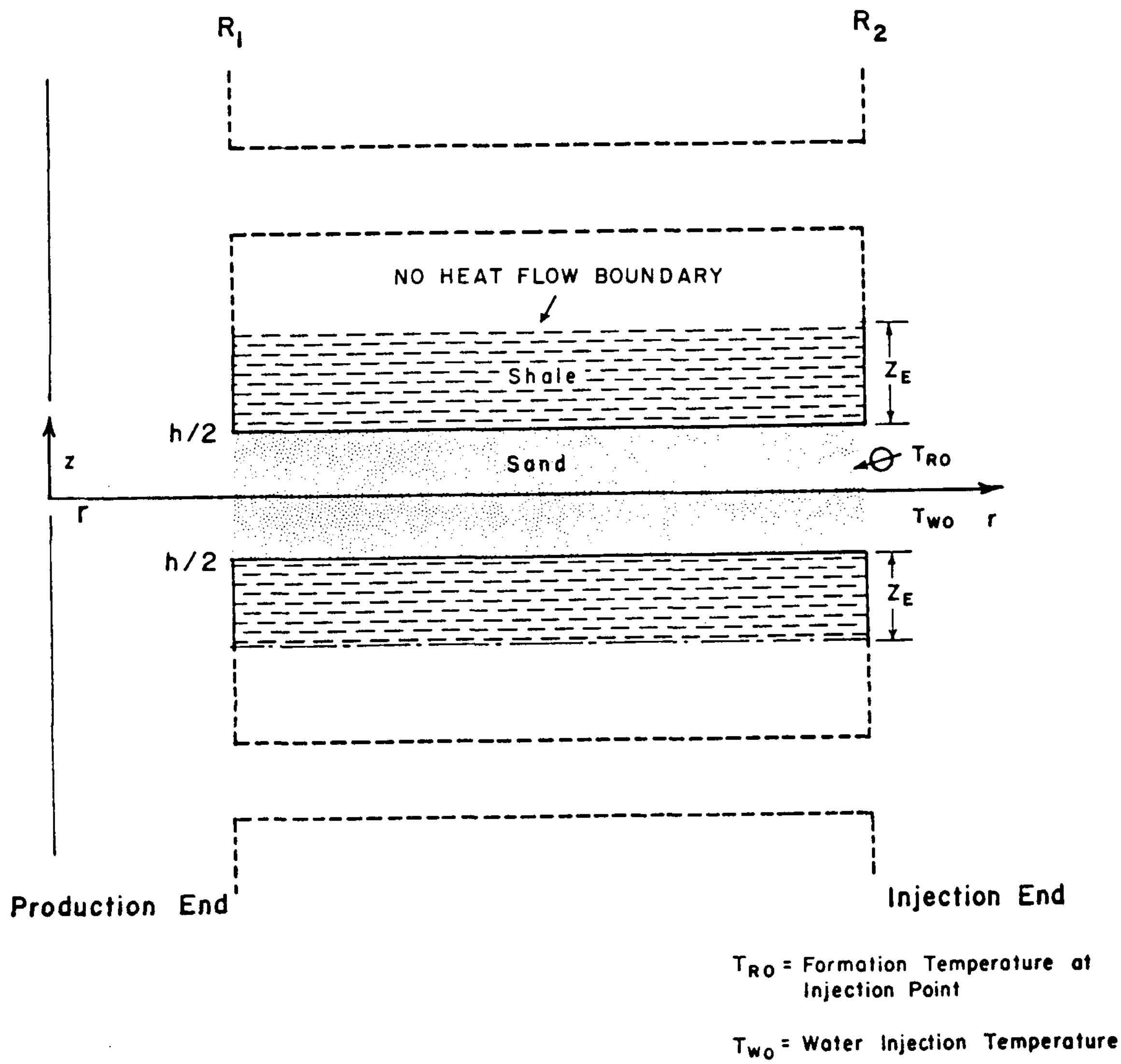

Figure 2. Mathematical model

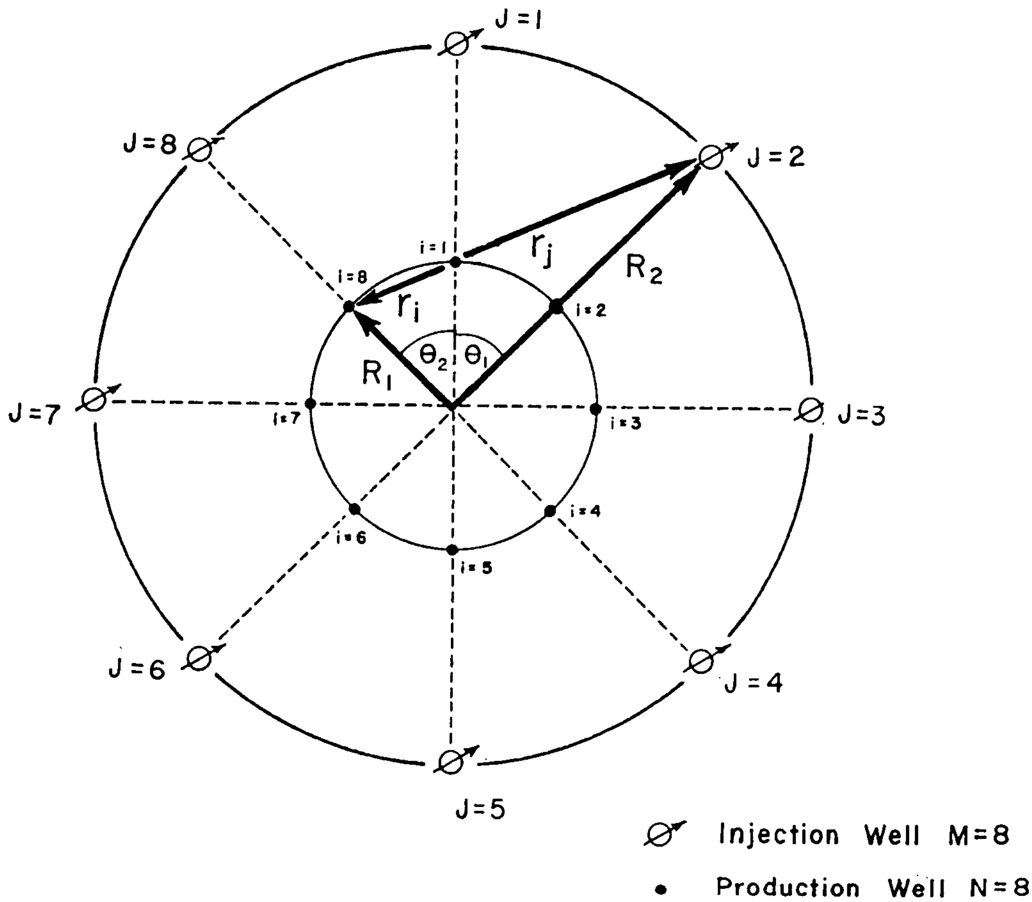

Figure 3. Explanation of symbols