THE EFFECTS OF A STEP CHANGE IN WATER FLOW ON AN INITIALLY LINEAR PROFILE OF TEMPERATURE

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In recent analyses of the hot-water system at Wairakei, New Zealand (Mercer, Pinder, and Donaldson, 1975) and the vapor-dominated system at Larderello, Italy (Petracco and Squarci, 1975), it has been suggested that large quantities of cold water are entering the reservoir by flowing down from the surface and then horizontally into the reservoir because of decreased reservoir pressures. It is also suggested that decreased reservoir pressures should increase these downward flows above their pre-exploitation levels. In order to estimate the effects of vertical flows on the temperature distribution, two idealized problems are analyzed in this paper. In both problems, the initial condition is a linear temperature increase with depth, and the flow starts at time equal to zero. In the first problem, the flow is through a semi-confining layer with the temperature fixed at the top and bottom of the layer. In the second problem, the flow is into a half-space with the surface temperature fixed.

The governing equation is conservation of energy in a porous medium (e.g., Bredehoeft and Papadopulos, 1965) which can be written in the form

$$\phi^* \frac{\partial T}{\partial t} + q\frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} \qquad (1)$$

where $\phi^* = (\phi\rho_w c_w + (1-\phi)\ \rho_r c_r)/\rho_w c_w$ combines the volumetric specific heats (ρc) of water and rock with the porosity ϕ, $\kappa - k_m/\rho_w c_w$ is an effective thermal diffusivity involving the thermal conductivity of the rock plus water and the volumetric specific heat of the water, and q is the seepage velocity. For the first problem of flow through a semi-confining bed of thickness ℓ the constant temperature at the top and bottom of the bed and the initial condition of constant gradient may be written as

$$T(0,t) = T_o \qquad (2a)$$

$$T(\ell,t) = T_1 \qquad (2b)$$

$$T(y,o) = T_0 + (T_1 - T_o)\ y/\ell. \qquad (2c)$$

The boundary and initial conditions for the half-space are written as

$$T(0,t) = T_o \qquad (3a)$$

$$\lim_{y\to\infty} \frac{\partial T}{\partial y} \text{ exists} \qquad (3b)$$

$$T(y,o) = T_o + Gy \qquad (3c)$$

where (3b) insures that there is no perturbation to the gradient at infinity and G is the temperature gradient at time equal zero.

The solution to equations (1) and (2) is obtained by changing to dimensionless variables that reduce the problem to homogeneous boundary and initial conditions. The form of the differential equation is then modified by the transformation $\tilde{T} = T^* \exp(q''y'')$ to an inhomogeneous equation but with no linear gradient term. The reduced problem is solved by classical techniques (see e.g., Berg and McGregor, 1964) to give

$$y = y''\ell \qquad t = \phi^*\ell^2 t''/\kappa \qquad q = 2\kappa q''/\ell \tag{4a}$$

$$\frac{T-T_o}{T_1-T_o} = y''-4q'' \quad e^{q''y''} \sum_{n=1}^{\infty} \frac{n\pi}{\lambda_n^2} (1-e^{-q''}\cos n\pi)(1-e^{-\lambda_n t''})\sin n\pi y'' \tag{4b}$$

$$\lambda_n = q''^2 + (n\pi)^2. \tag{4c}$$

The location of a fluid particle that started at the origin (y=o) at t=o may be written in terms of dimensionless variables as

$$y''_c = 2\phi^* \, q''t''/\phi. \tag{5}$$

The solution to equations (1) and (3) is obtained by a similar transformation to a homogeneous problem. The form of the differential equation is modified by the transformation $T' = T^* \exp(x'-t')$ as suggested by Brenner (1962) and the equations are solved by obtaining an ordinary differential equation by Laplace transforms, solving it, and using the inversion given in Carslaw and Jaeger (1959, p. 496). The solution may be written as

$$y = 2\kappa y'/q \qquad t = 4\phi^*\kappa t'/q^2 \tag{6a}$$

$$\frac{T-T_o}{\frac{2\kappa}{q} G} = (t' - \frac{y'}{2}) \text{ erfc} \left[\frac{y'/2-t'}{t'^{\frac{1}{2}}} \right] + (t' + \frac{y'}{2}) e^{2y'} \text{ erfc} \left[\frac{y'/2 + t'}{t'^{\frac{1}{2}}} \right] - 2t' + y' \tag{6b}$$

$$y'_c = \frac{2\phi^*}{\phi} t'. \tag{7}$$

Some sample solutions are presented in Fig. 1 for the semi-confining layer. The values for infinite time are obtained from Bredehoeft and Papadopulos (1965) formula $(T-T_o)/(T_1-T_o) = (\exp(2q'' y/\ell)-1)/(\exp(2q'')-1)$ as it is easier to evaluate. The solution is presented in terms of dimensionless variables for a flow rate $q'' = 1$ (top) and 2.5 (bottom) with the location of a fluid particle that started at the origin at t=o marked with a horizontal line. Choosing a layer thickness of 100 m, diffusivity of 23 m^2/yr, $\phi = 0.2$, and ϕ^* - 0.68, the dimensionless flow rates correspond to seepage velocities of 0.46 and 1.2 m/yr and the inset table shows the correspondence between physical and dimensionless time. The figures show that times greater than 60 years are required to reach the steady state solution. For a layer that is 10 m thick, this time is reduced to 0.6 year (while the velocities are 1/10 the values in Fig. 1).

Fig. 2 shows the results for a half-space. Because of the non-dimensionalization (equations 6a), different values of y' and t' are required to obtain the same values of physical length when changing the flow rate. The top of Fig. 2 is for the same flow rate as the bottom of Fig. 1. The solution in Fig. 2 is useful in enabling the influence of the upper boundary condition to be studied without having the bottom boundary condition of the solution in Fig. 1 propagate upwards. The major region of curvature in the profiles is well behind the location of the fluid particle that started at y=o at t=o, and fairly modest velocities show easily measured temperature changes in only a few years. In the model for Wairakei of Mercer, Pinder, and Donaldson (1975), the area of downflow needed to supply the natural recharge appears from the temperature contours to be about 10 km^2 although it could be larger. The velocity needed to supply the natural recharge of 440 kg/sec is 1.5 m/yr, about the same as that in Fig. 2 (top). The velocity of 4.6 m/yr in Fig. 2 (bottom) is roughly that which would be required if the current production were to be obtained without recourse to removing stored water but as steady state flow (Bolton, 1970) with recharge over the same 10 km^2. These assumptions, if true, indicate that large temperature differences should be easily found in such an area of recharge.

For Larderello, the maximum value of recharge as suggested by a hydrologic study is 9×10^6 m^3/yr (Petracco and Squarci, 1975). If this were to be distributed over an area equivalent to the entire productive area (200 km^2 from Gabbro to Carboli), the seepage velocity would be 0.05 m/yr and the effects would be small for a 100 m thick confining bed. If the recharge area were restricted to 20 km^2, the flow corresponds to Fig. 1 (top) and the effect should be easily measurable. The magnitudes of the effects for the two cases considered suggest that monitoring temperatures in undisturbed wells on the margins of producing geothermal areas should give a measure of the change in the fairly local recharge. If the amount of total recharge is known, subtracting the localized recharge should give an estimate of the recharge derived from deep circulation that originates at large distances from the reservoir.

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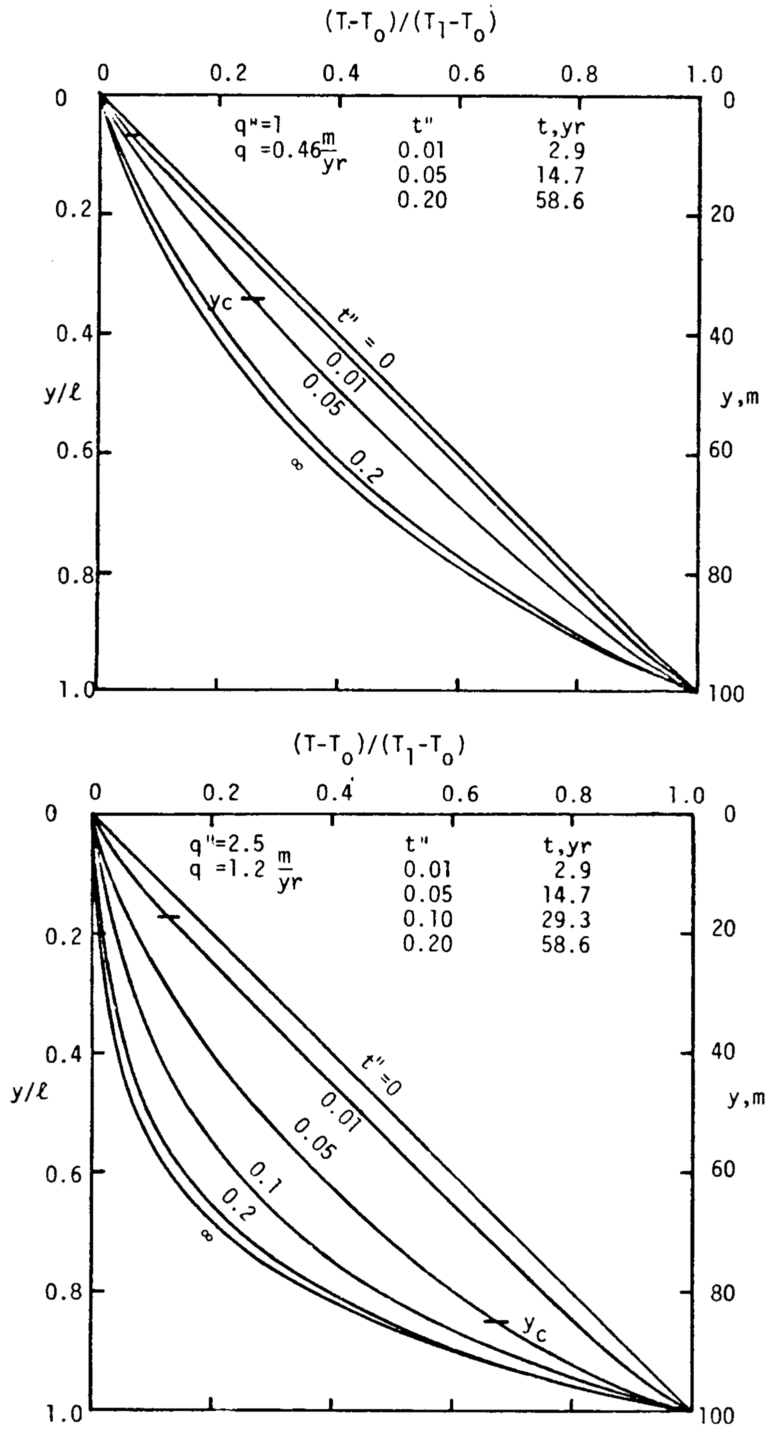

Figure 1.--Temperature versus depth in semi-confining layer for several times at dimensionless flow rates of 1 (top) and 2.5 (bottom).

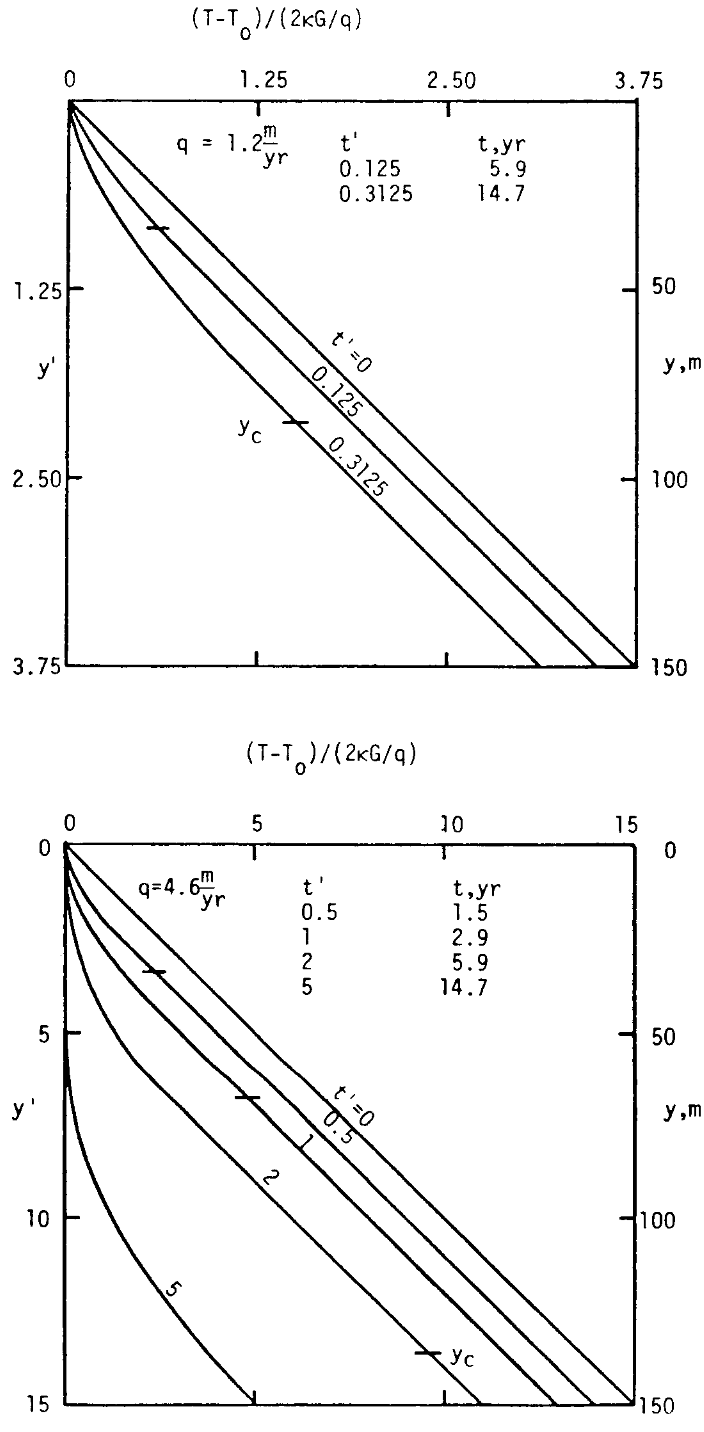

Figure 2.--Temperature versus depth for half-space for several times.