SIMULATION OF HEAT TRANSPORT IN FRACTURED, SINGLE-PHASE GEOTHERMAL RESERVOIRS

William G. Gray, Kevin O'Neill and George F. Pinder
Water Resources Program
Department of Civil Engineering
Princeton University
Princeton, N. J. 08540

Although many geothermal reservoirs depend upon fracture permeability to obtain adequate mass flows, relatively little research effort has been directed toward fractured reservoir simulation. This paper outlines the mathematical apparatus necessary to develop a numerical simulator for a fractured, single-phase geothermal reservoir. It is assumed that the fracturing is extensive and well-distributed (though not necessarily uniform) so that it is reasonable to consider a superficial discharge through the fractures as well as the pores. While mass and heat transport are of course coupled in a system of this kind, we have subdivided the ensuing discussion into mass flow and heat flow for clarity of presentation.

Mass Flow Equation

Analytical solutions for the pressure distributions in porous blocks of various shapes and sizes show that the pressure in the interior of a typical block reaches 95% of the value of an initial "step" input imposed on the block surface in a time which is very short relative to the length of time typically required for overall, macroscopic system changes. In addition, recent modeling analyses and examination of pertinent field data by Closmann (1975) support the point of view that for most purposes one may consider both pore and fracture flow fields to be characterized by a single pressure variable. A net flow of mass may exist between one flow regime and the other, but this will be such as to maintain the near equality of pressure. Application of accepted space-averaging techniques (Gray and Lee, 1976) to a point mass balance equation provides the following mass conservation equation:

$$\frac{\partial}{\partial t}(\rho_w \varepsilon_w) = \nabla\cdot[\rho_w \underset{\sim}{v}_w] + S_m = \nabla\cdot[\rho_f \underset{\sim}{v}_f + \rho_p \underset{\sim}{v}_p] + S_m \qquad (1)$$

where ρ_w is the averaged density of all (pore plus fracture) water,

ρ_f is the density of fracture water,

ρ_p is the density of pore water,

ε_w is the void fraction occupied by all water,

v_f is the superficial discharge through the fracture (vector),

$\tilde{v}_p$ is the superficial discharge through the pores (vector), and

S_m is the mass source or sink strength, that is, mass entering or leaving per unit time per unit volume of total medium.

The lefthand side of (1) may be expanded as

$$\frac{\partial}{\partial t}(\rho_w \varepsilon_w) = \rho_w \frac{\partial \varepsilon_w}{\partial t} + \varepsilon_w \frac{\partial \rho_w}{\partial t} \tag{2}$$

$$= \rho_w \alpha_p \frac{\partial p}{\partial t} + \rho_w \alpha_T \frac{\partial T_w}{\partial t} + \varepsilon_w \rho_w \beta_p \frac{\partial p}{\partial t} + \varepsilon_w \rho_w \beta_T \frac{\partial T_w}{\partial t}$$

where ε_f is the void fraction of the fractures,

ε_p is the void fraction of the pores,

p is the incremental fluid pressure,

T_f is the local average fluid temperature in the fractures,

T_{pm} is the local average temperature of the porous medium, and

T_w is the locally averaged temperature of all water defined as

$$T_w = \varepsilon_f T_f + \varepsilon_p T_{pm} \tag{3}$$

The parameters α_p, α_T, β_p and β_T are empirical coefficients defined through the relations:

$$\frac{\partial \varepsilon_w}{\partial t} = \alpha_p \frac{\partial p}{\partial t} + \alpha_T \frac{\partial T_w}{\partial t} \tag{4a}$$

$$\frac{\partial \rho_w}{\partial t} = \rho_w \beta_p \frac{\partial p}{\partial t} + \rho_w \beta_T \frac{\partial T_w}{\partial t} \tag{4b}$$

Superficial fracture and pore discharges may be expressed in terms of incremental pressure gradients, as

$$\underset{\sim}{v}_f = -\left(\frac{\underset{\approx}{k}}{\mu}\right)_f \cdot \underset{\sim}{\nabla} p \tag{5a}$$

$$\underset{\sim}{v}_p = -\left(\frac{\underset{\approx}{k}}{\mu}\right)_p \cdot \underset{\sim}{\nabla} p \tag{5b}$$

where μ is the fluid viscosity,

$\underset{\approx}{k}_f$ is the fracture permeability (tensor), and

$\underset{\approx}{k}_D$ is the pore permeability (tensor)

Under certain conditions $\underset{\approx}{k}_f$ may be considered to be a function of $\underset{\sim}{v}_f$.

Substitution of equations (2) through (5) into (1) yields the following expression for the conservation of all fluid mass:

$$\rho_w(\alpha_p + \varepsilon_w\beta_p)\ \frac{\partial p}{\partial t} + \rho_w(\alpha_T + \varepsilon_w\beta_T)\ \frac{\partial T_w}{\partial t} \tag{6}$$

$$= \underset{\sim}{\nabla} \cdot \left[\left(\frac{\rho \underset{\approx}{k}}{\mu}\right)_f + \left(\frac{\rho \underset{\approx}{k}}{\mu}\right)_p\right] \cdot \nabla p + Sm$$

In addition to the explicit coupling of this equation to the temperature equations through the second term on the lefthand side, temperature dependence also enters implicitly through the changing value of μ .

Heat Flow

The governing equations for heat flow are provided by space averaging of conservation of energy equations written in terms of temperature. For the fracture system, this results in

$$\rho_f c \varepsilon_f\ \frac{\partial T_f}{\partial t} + \rho_f c \underset{\sim}{v}_t \cdot \underset{\sim}{\nabla} T_f - \underset{\sim}{\nabla} \cdot \underset{\approx}{D}_f \cdot \underset{\sim}{\nabla} T_f$$

$$= h(T_{pm} - T_f) + c\, S_{m,f}(T_{s,f} - T_f) \tag{7a}$$

and for the porous medium

$$(\rho c\varepsilon)_{pm} \frac{\partial T_{pm}}{\partial t} + \rho_p c v_{\sim p} \cdot \nabla T_{pm} - \nabla \cdot \underset{\sim}{\underset{\sim}{D}}_{pm} \cdot \nabla T_{pm}$$

$$= h(T_f - T_{pm}) + c\, S_{m,p}(T_{s,pm} - T_{pm}) \qquad (7b)$$

where $(\rho c\varepsilon)_{pm} \equiv \rho_p c \varepsilon_p + \rho_s c_s \varepsilon_s$,

ρ_s is the rock density,

c_s is the specific heat of the rock,

ε_s is the volume fraction of the rock,

c is the specific heat of water,

$\underset{\sim}{\underset{\sim}{D}}_f$ is the tensor coefficient of dispersion for the fractures,

$\underset{\sim}{\underset{\sim}{D}}_{pm}$ is the tensor coefficient of dispersion for the porous medium,

h is a porous medium-fracture heat transfer coefficient relating the time rate of heat transport between those regimes, per volume of the medium, to the temperature difference between the two. $T_{s,f}$ and $T_{s,pm}$ are source or sink temperatures of fracture and pore fluids, respectively. (For withdrawal, the sink temperature is the reservoir fluid temperature and the last terms in 7 vanish).

$S_{m,f}$ is the fracture mass source or sink strength,

$S_{m,p}$ is the pore mass source of sink strength, and

$S_m = S_{m,f} + S_{m,p}$ and the ratio of the two components can be determined using the permeabilities of the two systems.

The superficial velocities in (7) must, of course, be computed using the pressure field through equations (5) and (6). Equations (5), (6), and (7) provide five equations in the five dependent variables T_{pm}, T_f, p, $v_{\sim f}$ and $v_{\sim p}$. These equations have been solved successfully for a variety of hypothetical problems for which analytical solutions exist. The numerical simulator uses isoparametric Hermitean finite elements (Van Genuchten, et al, 1977) to solve in three space dimensions, and a time-centered difference scheme to solve in time.

Figures 1 and 2 show results for an additional fully coupled, one-dimensional, transient test case, subject to the following conditions:

at $x = 0$ $\quad T_{pm} = T_f = 40^{\circ}C$

$t > 0$ $\quad p = 0$

at $x = 100cm$ $\quad T_{pm} = T_f = 0$

$p = -1.0 \times 10^5$ dyne/cm^2

$\frac{1}{\mu} = 5.38 \times 10^2 + (T-150) \times 3.8 - (T-150)^3 \times 2.6 \times 10^{-5}$ cm.sec/g for $0<T<300^{\circ}C$

(Mercer et al, 1975)

$\varepsilon_f = 0.02$, $\varepsilon_p = 0.2$, $\alpha_p = 1.0 \times 10^{-10}$ cm^2/dyne, $\alpha_T = 0$

$\frac{k_f}{\varepsilon_f} = 10^{-7}$ cm^2, $\frac{k_{pm}}{\varepsilon_p} = 3.0 \times 10^{-8}$ cm^2, $\beta_p = 5.0 \times 10^{-11}$ cm^2/dyne

$\beta_T = 5.0 \times 10^{-4}/^{\circ}C$

$\rho_s = 2.5$g/cm^3, $c_s = 0.2$cal/g.$^{\circ}$C, $D_f = 5.0 \times 10^{-4}$ cal/$^{\circ}$C.cm.sec,

$D_{pm} = 3.0 \times 10^{-3}$ cal/$^{\circ}$C cm.sec.

The initial temperature distribution for both fractures and porous medium is displayed on each figure. As expected, a non-zero value of h retards translation of the fracture temperature front, increases translation of the porous medium front, and increases dispersion of both. As the fronts progress, the pressure gradient (not shown) decreases from the initial, essentially isothermal value, due primarily to the decrease in fluid viscosity with rising temperature.

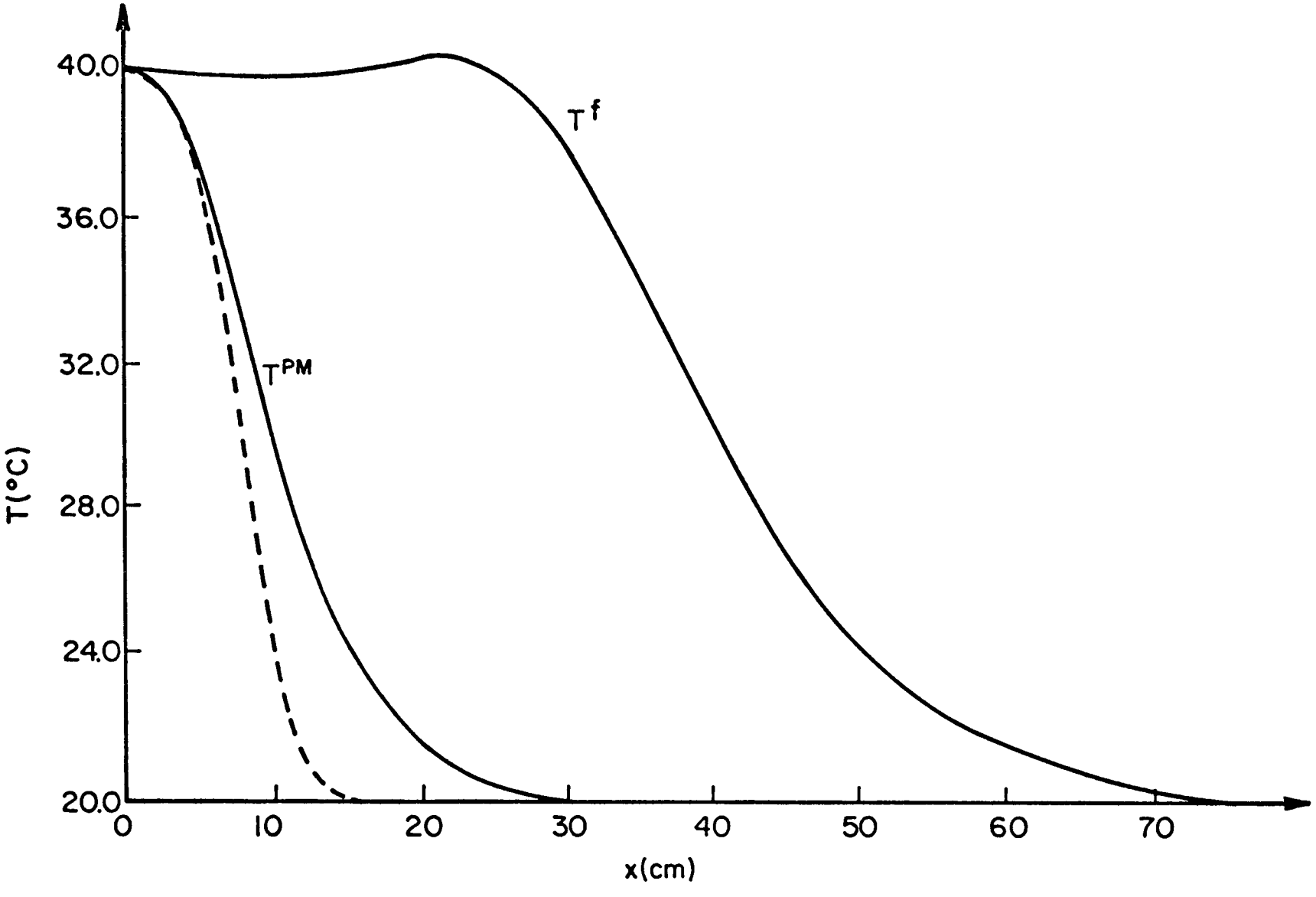
References

Closmann, P. J., "An Aquifer Model for Fissured Reservoirs," Society of Petroleum Engineers Journal, October 1975, 385-398.

Gay, W. G. and P. C. Y. Lee, "On the Theorems for Local Volume Averaging of Multiphase Systems," International Journal of Multiphase Flow, in press 1976.

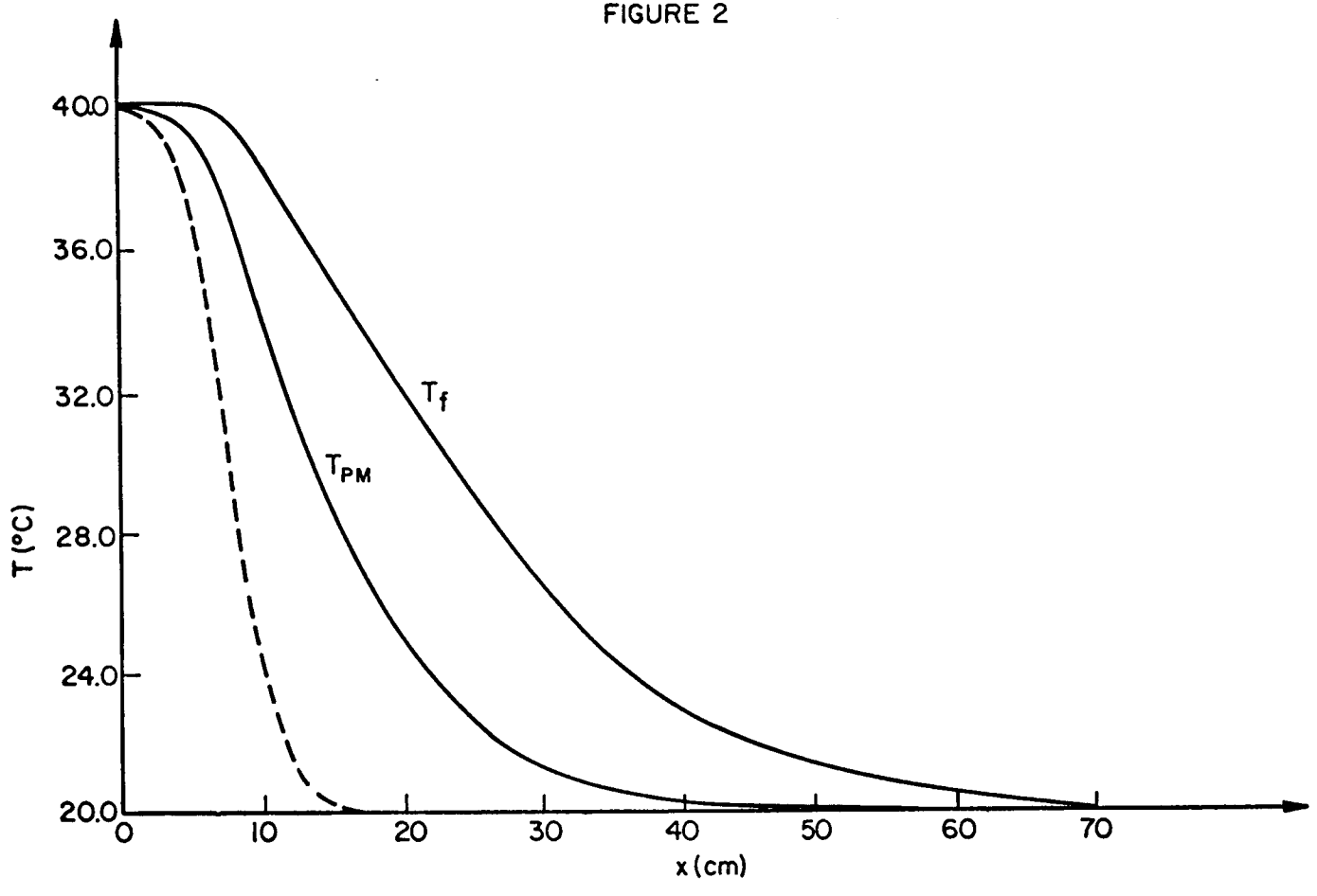
Mercer, W. M., G. F. Pinder, and I. G. Donaldson, "A Galerkin Finite Element Analysis of the Hydrothermal System at Wairakei, New Zealand," Journal of Geophysical Research, June 1975, 2608-2621.

Van Genuchten, M. T., G. F. Pinder, and E. O. Frind, "Simulation of Two-dimensional Contaminant Transport with Isoparametric Hermitean Finite Elements," Water Resources Research, in press 1976.



h=0
$t = 3.0 \times 10^3$ sec
DASHED LINE SHOWS INITIAL TEMPERATURE
FIGURE 1

FIGURE 2



$h = 2.0 \times 10^{-5}$ CAL/cm^2 sec °C
$t = 5.25 \times 10^3$ sec
DASHED LINE SHOWS INITIAL TEMPERATURE