OPTIMAL MANAGEMENT OF A GEOTHERMAL RESERVOIR

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In a presentation to the First Stanford Geothermal Workshop last year, we outlined the basic philosophy, assumptions and general approach to finding an optimal rate of energy extraction from a hot water geothermal field. In this paper, we present the explicit relationships governing the physical processes and economic factors of our model, as well as the modifications to the model that have been necessary to accommodate the more specific articulation of these relationships. The conceptual modifications of the earlier model are subtle, but of great importance in making our work more useful for geothermal resource management.

This study is concerned with the optimal management, and in particular the optimal timing of energy extraction from a geothermal reservoir. For the conclusions of this optimization problem to be meaningful, the analysis must be carried out in the context of a particular hydro-thermal model. Furthermore, some assumption regarding the future value of geothermal energy must be made. Accordingly, we adopt the hydro-thermal model developed by Gringarten and Sauty (1), and assume that the value of geothermal energy is known as a function of time and the quantity of the extracted energy. We note however that our optimization model can be modified to accommodate other hydrothermal models such as that of Kasameyer and Schroeder, which combines fractured and porous media flow (2). In view of the increase in the attractiveness of geothermal energy for space heating (3,6), we also assume that the extracted energy is used for generation of steam to be used for this purpose. However, we are well aware that the hot brine, depending on the parameters of a particular field, may be more economically utilized for some other purpose (e.g., electric power generation, direct utilization of hot water for domestic and industrial use, mineral extraction and desalination). In this paper, we restrict our attention to the case where the decision has already been made to use the geothermal energy for space heating.

The quantity of the extracted energy is a function of both the rate at which hot brine is extracted and the degree to which it is cooled before reinjection in the reservoir. Hence, we seek an extraction rate, a reinjection temperature and an economic life that maximize the net discounted value of the extracted energy.

The Hydro-Thermal Model

The hydro-thermal model adopted for this study was developed by Gringarten and Sauty (1). It assumes a pumped production well for a single phase saturated confined hot water aquifer with a recharge well as shown in Fig. 1 (actually each well can represent a cluster of wells). Although the aquifer is confined vertically, it is assumed to extend horizontally to infinity.

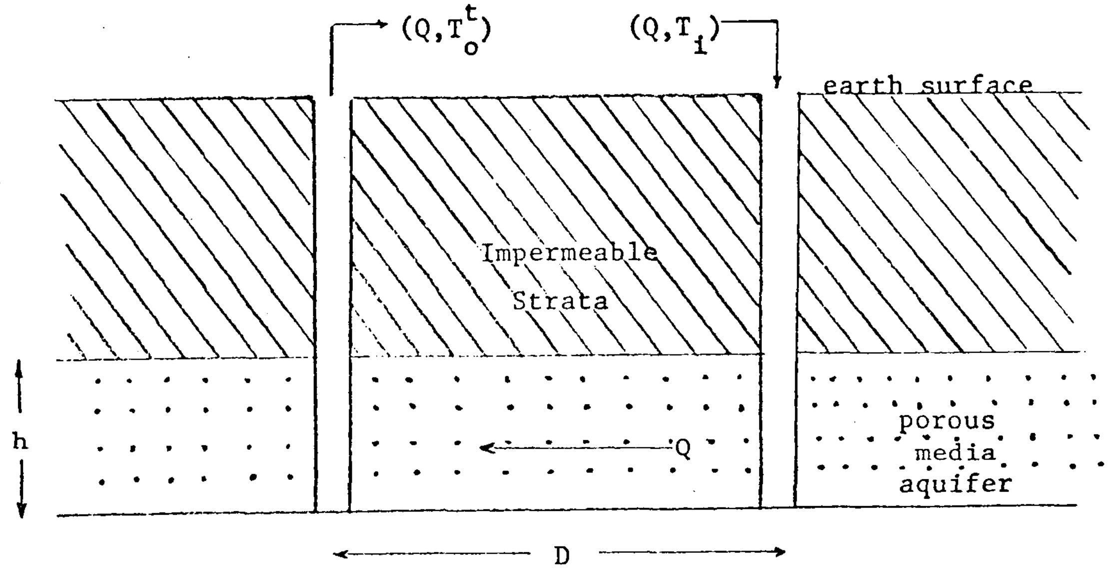

Figure 1

Fluid is withdrawn at the rate Q and recharged at the same rate. The temperature of extracted fluid at time t is T_o^t. Recharged fluid enters the ground at temperature T_i^t at time t.

The recirculated fluid is heated by the aquifer matrix from T_i^t to T_o^t (and this tends to cool the matrix). For the first τ years ($0^o \leq t \leq \tau$), $T_o^t = T_o^o$ where T_o^o is the initial equilibrium temperature of the unexploited anomaly and τ denotes the time until reduced fluid temperature "breaks through" to the production well. The breakthrough time is function of Q and is described by:

$$\tau(Q) = \frac{t_u}{6} ,$$

where t_u is a dimensionless expression for time,

$$t_u = \frac{2\pi h D^2 \rho_a c_a}{Q \rho_f c_f}$$

h is the thickness of the aquifer, D the well separation, Q the pumping rate and $\rho_a c_a$ and $\rho_f c_f$ the heat capacities of the aquifer matrix and the fluid, respectively. Thus $\tau(Q)$ is inversely proportional to Q.

The temperature after breakthrough is given by a function $\overline{g}(T_i^t, t/t_u)$ which gives the ratio of the temperature drop through the heat exchanger experienced by the brine at the time t, to the analogous drop at time zero:

$$\frac{T_o^t - T_i^t}{T_o^o - T_i^t} = \overline{g}\,(T_i^t,\ t/t_u)$$

It can be easily shown that the variation in T_i^t is small. Hence, we have used the results of the hydro-thermal model to approximate $\overline{g}$ by a function g which assumes T_i does not vary with time. However, althouth T_i may be assumed constant with time, its value does affect heat removed per unit of time (for a given Q), and hence discounted net revenues. That is, for a given steam temperature, lower values of T_i yield greater heat flows per time but require more expensive heat exchangers and also cause the aquifer to cool more rapidly.

The expression for g has been developed (5) and is given by

$$g\,(t/t_u) = \begin{cases} 1 & \text{if } 0 \leq t \leq \tau \\ 0.338e^{-0.0138t/t_u} + 0.337e^{-0.656t/t_u} + 1.368e^{-8.006t/t_u} & \text{if } t \geq \tau. \end{cases}$$

Therefore, after the τ^{th} year, T_o^t drops exponentially toward T_i at a rate $g(t/t_u)$ as shown in Figure 2.

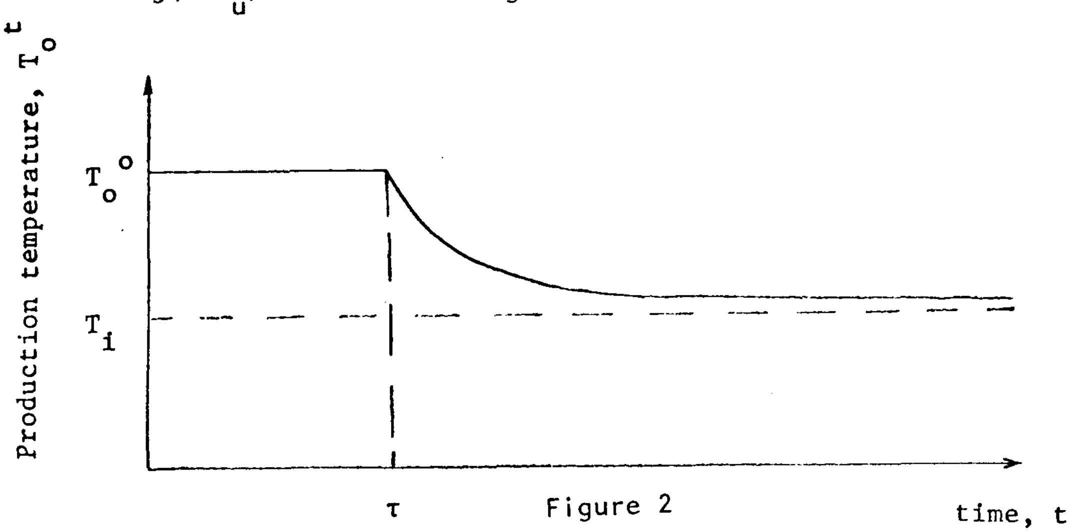

Figure 2

The Economic Model

We have now described the fundamental relationship between temperature and time for a given Q. Since extractable energy is proportional to $T_o^t - T_i$, it is possible to take T_o^t as the quality of the resource. As $T_o^t \to T_i$ the cost of heat extraction (per BTU) increases and there is a time when it is no longer economical to extract more heat. Since a certain amount of heat is lost in transfer and transmission, we need the difference between the production and injection temperatures to remain greater than a prespecified number δ. Thus we will need L^*, the optimal lifetime of the project, to be no greater than L_δ, where L_δ is such that $T_o^{L_\delta} - T_i = \delta$ and is a decreasing function of δ.

There are at least two ways to consider the value of the energy. The first is to assume that the value of the energy increases with time at the rate of e^{rt} where r is the (continuous) rate of increase of real (as opposed to inflated) energy price with time, i.e. $P_t = P_o e^{rt}$ where P_t is the value of the energy at time t and P_o is determined by the cost of alternative sources of energy. The second approach is to assume that demand for the energy is price sensitive, using the area under the demand curve as an index of willingness to pay, and hence benefit or value to society. If demand y, is price dependent, then we can write:

$$y=f(p).$$

This can be mathematically inverted, yielding:

$$p=f^{-1}(y).$$

Then willingness to pay for y_o BTU/hr, $w(y_o)$ is:

$$w(y_o) = \int_0^{y_o} p\,dy = \int_0^{y_o} f^{-1}(y)\,dy.$$

We will assume $w_t(y_o)$ increases with time so that

$$w_t(y_o) = w_o(y_o)e^{rt}$$

For the first criterion, the basic optimization problem (θ_1) is then

$$\theta_1: \quad \underset{Q,T_i,L}{\text{Maximize}}\ \Pi = \int_0^{\tau(Q)} P_o e^{rt} Q c_f \rho_f\ (T_o - T_i)\ e^{-it} dt$$

$$+ \int_{\tau(Q)}^{L} P_o e^{rt} Q c_f \rho_f (T_o - T_i) g(t/t_u) e^{-it} dt$$

$$- \int_0^{L} C(Q,T_i) e^{-it} dt$$

subject to

$$T_o^L - T_i \geqq \delta$$

$$Q \geqq 0 \quad ,$$

where Q is the extraction rate, τ the breakthrough time, $c_f \rho_f$ the heat capacity of the brine, i the discount rate, L the economic life of the project and $C(Q,T_i)$ the function describing the annual capital and operting costs.

For the second criterion the problem (Θ_2) is

$$\theta_2: \quad \underset{Q,\ T_i,\ L}{\text{Maximize}}\ \Pi = \int_0^{\tau(Q)} w_o (Q c_f \rho_f\ (T_o - T_i)) e^{rt} e^{-it} dt$$

$$+ \int_{\tau(Q)}^{L} w_o (Q c_f \rho_f (T_o - T_i) g(t/t_u)) e^{rt} e^{-it} dt$$

$$- \int_0^{L} C(Q,T_i) e^{-it} dt$$

subject to $T_o^L - T_i \geqq \delta$

$$Q \geqq 0.$$

A study of the various components of the production and surface equipment has established the relationship between the capacity of each component and the decision variables. By combining these relationships with empirical cost data, we have obtained the function C enabling us to obtain optimal solutions to problems Θ_1 and Θ_2. The components of C are costs for wells and casing, pipes and pipe cleaning, heat exchangers, well assemblies, pumps and pump operating costs. The pump cost is dependent both on the flow rate and the drawdown generated in the production well, which is in turn dependent on the flow rate. An important part of the cost function is the relationship between heat exchanger area A and effectiveness of exchange:

$$\varepsilon = 1-e^{-kA/Q}$$

where k is a constant. We have combined this with the definition of effectiveness.

$$\varepsilon = \frac{T_o^t - T_i}{T_o^t - T_s} \quad ,$$

to incorporate heat exchanger area and T_i into the cost function. A linear demand curve has been assumed to solve Θ_2.

A final note on the optimization model is that the maximum possible flow, $\overline{Q}$, from each production well is determined not only by pump technology, but by the requirement that the flow into the production well be laminar in order to be consistent with the assumption of the Gringarten-Sauty model. Hence, we assume the total flow Q, is achieved by using a number of wells drilled reasonably apart from each other to minimize pressure interference. Each of these wells has an upper bound of $\overline{Q}$ on capacity.

Proposed Work

The next step in our study is a sensitivity analysis indicating the relative importance of the physical, cost and economic parameters of our model in determining the optimal policy.

A logical extension of our work is the development of a dynamic decision process in which the extraction rate Q will be allowed to vary with time. An extraction strategy is then defined in terms of a vector of pumping rates:

$$Q = (Q_1, Q_2, \ldots\ldots\ldots, Q_L)$$

where Q_t is the pumping rate in the t^{th} year. We will seek an optimal strategy that maximizes the total discounted net revenues. Initial consideration of this extension has shown that the dynamic programming approach, suggested in last year's presentation, is not consistent with the Gringarten & Sauty model. This is because the derivative of T_o^t with respect to t ($t>\tau$) is dependent on the history of Q (i.e. $Q_{t-1}, Q_{t-2}, \ldots\ldots Q_1$) in the Gringarten and Sauty model. This dependence is effectively precluded by the dynamic programming approach. The solution to the multiple extraction rate problem is therefore not yet at hand. However, since multiple extraction policies may have advantages for the optimal management of geothermal resources, we intend to consider this problem further. Another extension would be to investigate various geometries and spacing of production and recharge wells. The geometry and well separation not only affect the breakthrough time, but also the hydraulic drawdown and hence pumping costs and production well capacities. Heat losses in surface piping will also be considered in this extension.

Our optimization model can be extended to cases where the hot brine is intended for uses other than space heating, in particular, electric power generation.

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