DERIVATION, BY AVERAGING, OF THE EQUATIONS OF HEAT, MASS AND MOMENTUM TRANSFER IN A GEOTHERMAL RESERVOIR

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The following paper is an abstract of a report under completion at the Lawrence Berkeley Laboratory (Assens, 1977); the main purpose of this report is to (1) provide a mathematical derivation of the equations describing the transfer of heat, mass and momentum in a geothermal brine reservoir (especially when heat or mass sources are present), and (2) help in the choice of the dependent variables best suited for solving these equations numerically.

The basic tool is an averaging procedure that allows the derivation of the transport equations in a porous medium from the level of the pores, where each of the solid, liquid and fluid constituents is considered as a separate continuum, to a grosser level where the medium in which transport takes place is itself considered as a continuum without reference to its three components.

Two variants of this averaging have been proposed by chemical engineers: Anderson and Jackson (1967) on one hand, Whitaker (1969, 1973) and Slattery (1972) on the other. The former variant has been recently applied to the derivation of the transport of solute (Blake and Garge, 1976) whereas the later was used in relation to the transport of solute (Gray, 1975), a derivation of Darcy's law (Gray and O'Neill, 1976; Neuman, 1976) and the transport of heat in terms of fluid internal energy (Witherspoon et al. 1975) or enthalpy (Faust, 1976).

In the following pages we restrict the scope of our investigation to a one-component fluid and follow the latter of the aforementioned variants, hoping that we may avoid some of the shortcomings noticed in the literature surveyed while obtaining a more complete form of the transport equations.

THE BALANCE EQUATIONS FOR A CONTINUUM

Hypotheses:

(G1) Continuum approximation

(G2) Negligible thermodynamic fluctuations

(G3) Laminar flow regime

(G4) One component fluid

Mass balance:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0 \qquad (1)$$

Momentum balance (i-direction)

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}(\rho v_i v_j + p\delta_{ij} - \tau_{ij}) - \rho g_i = 0 \qquad (2)$$

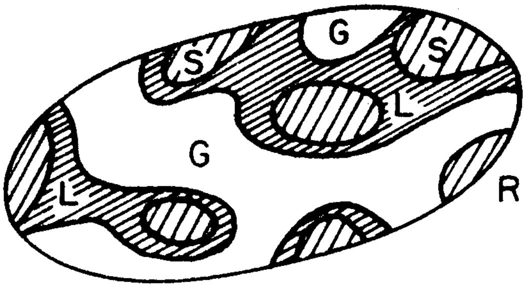
Heat balance:

$$\left\{\begin{array}{c} \frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x_j}(\rho e v_j) \\ \frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_j}(\rho h v_j) \\ \rho c_v \frac{DT}{Dt} \end{array}\right\} + \frac{\partial}{\partial x_j}\left(-\lambda \frac{\partial T}{\partial x_j}\right) \left\{\begin{array}{c} + p\frac{\partial v_j}{\partial x_j} \\ - \frac{\partial p}{\partial t} - v_j \frac{\partial p}{\partial x_j} \\ - \frac{T\alpha}{\rho\beta}\frac{D\rho}{Dt} \end{array}\right\} = \dot{E} + \mu\Phi \qquad \begin{array}{c} (3e) \\ (3h) \\ (3t) \end{array}$$

THE AVERAGING PROCEDURE

Definitions:

Consider, within the porous medium, a fixed representative elementary volume (Bear, 1972) R_α (α = S, L, G).



Let ψ_α be any property of the α-phase, defined physically in R_α and mathematically set equal to zero in $R - R_\alpha$.

Phase average:
$$<\psi_\alpha> \equiv \frac{1}{V}\int_R \psi_\alpha \, dV \tag{4}$$

Intrinsic phase average:
$$<\psi_\alpha>^\alpha \equiv \frac{1}{V_\alpha}\int_{R_\alpha} \psi_\alpha dV \tag{5}$$

Deviation:
$$\psi_\alpha \equiv \begin{cases} \psi_\alpha - <\psi_\alpha>^\alpha & \text{in } R_\alpha \\ 0 & \text{in } R - R_\alpha \end{cases} \tag{6}$$

Any quantity, e.g. $<\psi_\alpha>$, that has no meaning at a finer level than that of the R.E.V. will be subsequently referred to as "locally" defined, whereas a quantity that exists at the level of the pore, e.g. ψ_α, will be referred to as "pointwise" defined.

Hypotheses:

(G5) characteristic lengths: $d \ll \ell \ll L$

(pore) (R.E.V.) (reservoir)

(G6) good behavior of ψ_α:
$$<<\psi_\alpha>> = <<\psi_\alpha>>^\alpha = <\psi_\alpha> \tag{8a}$$
$$<<\psi_\alpha>^\alpha> = <<\psi_\alpha>^\alpha>^\alpha = <\psi_\alpha>^\alpha \tag{8b}$$

Theorems:

General transport theorem (Whitaker, 1969):
$$< \frac{\partial \psi_\alpha}{\partial t} > = \frac{\partial}{\partial t} <\psi_\alpha> - \frac{1}{V}\int_{S_\alpha} \psi_\alpha w_{\alpha j} n_{\alpha j} \, dA \tag{9}$$

Averaging theorem (Whitaker, 1969; Slattery, 1972):
$$< \frac{\partial \psi_\alpha}{\partial x_j} > = \frac{\partial}{\partial x_j} <\psi_\alpha> + \frac{1}{V}\int_{S_\alpha} \psi_\alpha n_{\alpha j} \, dA \tag{10}$$

Modified averaging theorem (Gray, 1975):

$$< \frac{\partial \psi_\alpha}{\partial x_j} > = \varepsilon_\alpha \frac{\partial}{\partial x_j} <\psi_\alpha>^\alpha + \frac{1}{V}\int_{S_\alpha} \tilde{\psi}_\alpha \, n_{\alpha j} \, dA \qquad (11)$$

Modified transport theorem (Assens, 1977):

$$< \frac{\partial \psi_\alpha}{\partial t} > = \varepsilon_\alpha \frac{\partial}{\partial t} <\psi_\alpha>^\alpha - \frac{1}{V}\int_{S_\alpha} \tilde{\psi}_\alpha \, w_{\alpha j} n_{\alpha j} \, dA \qquad (12)$$

Nice relations:

$$\frac{1}{V}\int_{S_\alpha} n_{\alpha j} \, dA = - \frac{\partial \varepsilon_\alpha}{\partial x_j} \qquad (13a)$$

$$\frac{1}{V}\int_{S_\alpha} w_{\alpha j} \, n_{\alpha j} \, dA = \frac{\partial \varepsilon_\alpha}{\partial t} \qquad (13b)$$

THE AVERAGED BALANCE EQUATIONS

Assume that (H1) the average of the product of two or more deviations of variables not strongly dependent on velocity is negligible compared to the product of the averages of these variables.

Mass balances:

Solid - Assume (H2) incompressible, non reacting solid.

$$\frac{\partial \rho_S}{\partial t} = 0 \qquad (14)$$

Fluid phase $\alpha(\alpha = L, G)$:

$$\underbrace{\frac{\partial}{\partial t} (\varepsilon_\alpha <\rho_\alpha>^\alpha)}_{\text{(storage)}} + \underbrace{\frac{\partial}{\partial x_j} (<\rho_\alpha>^\alpha <v_{\alpha j}>)}_{\text{(convection)}} + \underbrace{\frac{1}{V}\int_{S_{\alpha\beta}} \rho_\alpha (v_{\alpha j} - w_{\alpha j}) \, n_{\alpha j} \, dA}_{\text{(phase change)}} + \underbrace{\frac{\partial}{\partial x_j} <\tilde{\rho}_\alpha \tilde{v}_{\alpha j}>}_{\text{(dispersion)}} = 0 \qquad (15)$$

where the dispersion term may be omitted subject to (H3) low correlation between density and velocity.

Momentum balances:

Solid - Assume (H4) indeformable solid matrix

$$v_{Sij} = 0 \tag{16}$$

Fluid - Assume (H5) negligible inertia

$$\langle v_{\alpha i}\rangle^{\alpha} = - \sum_{\beta=L,G} \frac{1}{\varepsilon_{\beta}\langle\mu_{\beta}\rangle^{\beta}} k_{\alpha\beta ij} \left(\frac{\partial}{\partial x_j} \langle p_{\beta}\rangle^{\beta} - \langle\rho_{\beta}\rangle^{\beta} g_j\right) \tag{17}$$

yields Darcy's formula in the case of a (H6) rigid gas-liquid interface with no slip.

Heat Balances:

Solid - Define the "stagnant" thermal conductivity λ'_S by:

$$- \varepsilon_S \lambda'_S \frac{\partial}{\partial x_j} \langle T_S\rangle^S \equiv - \langle\lambda_S\rangle^S \left(\varepsilon_S \frac{\partial}{\partial x_j} \langle T_S\rangle^S + \frac{1}{V} \int_{S_S} \tilde{T}_S n_{Sj} dA\right) \tag{18}$$

Then, by H1:

$$\left.\begin{array}{c} \frac{\partial}{\partial t} (\varepsilon_S \langle\rho_S\rangle^S \langle e_S\rangle^S) \\ \text{(storage)} \\ \varepsilon_S \langle\rho_S\rangle^S \langle c_{vS}\rangle^S \frac{\partial}{\partial t} \langle T_S\rangle^S \end{array}\right\} + \underbrace{\frac{\partial}{\partial x_j} \left(-\varepsilon_S \lambda'_S \frac{\partial}{\partial x_j} \langle T_S\rangle^S\right)}_{\text{(conduction within solid)}} + \underbrace{\frac{1}{V} \int_{S_S} \left(-\lambda_S \frac{\partial T_S}{\partial x_j}\right) n_{Sj} dA}_{\text{(conduction across fluid-solid interface)}} = \langle \dot{E}_S\rangle \tag{19}$$

Fluid - Define the tensor of thermal dispersion $\lambda''_{\alpha ij}$ as follows:

$$\frac{\partial}{\partial x_j} \left(-\varepsilon_{\alpha} \lambda''_{\alpha ij} \frac{\partial}{\partial x_i} \langle T_{\alpha}\rangle^{\alpha}\right) \equiv \frac{\partial}{\partial x_j} \langle \rho_{\alpha} \tilde{e}_{\alpha} \tilde{v}_{\alpha j}\rangle + \langle \tilde{p}_{\alpha} \frac{\partial \tilde{v}_{\alpha j}}{\partial x_j} \rangle \tag{20}$$

Then define the "effective" thermal conductivity tensor:

$$\lambda^{*}_{\alpha ij} \equiv \lambda'_{\alpha} + \lambda''_{\alpha ij} \tag{21}$$

Finally assume (H7) negligible viscous dissipation

$$\frac{\partial}{\partial t}\left(\varepsilon_\alpha \langle\rho_\alpha\rangle^\alpha \langle e_\alpha\rangle^\alpha\right) + \frac{\partial}{\partial x_j}\left(\langle\rho_\alpha\rangle^\alpha \langle e_\alpha\rangle^\alpha \langle v_{\alpha j}\rangle\right) + \frac{\partial}{\partial x_j}\left(-\varepsilon_\alpha \lambda^*_{\alpha ij} \frac{\partial}{\partial x_i} \langle T_\alpha\rangle^\alpha\right)$$

(storage) (convection) (conduction within phase α)

$$+ \frac{1}{V}\int_{S_\alpha} \left[(\rho_\alpha e_\alpha + p_\alpha)(v_{\alpha j} - w_{\alpha j}) - \lambda_\alpha \frac{\partial T_\alpha}{\partial x_j}\right] n_{\alpha j}\, dA + \langle p_\alpha\rangle^\alpha \left(\frac{\partial \varepsilon_\alpha}{\partial t} + \frac{\partial}{\partial x_j}\langle v_{\alpha j}\rangle\right)$$

(convection + conduction across interfaces separating phase α from phase β and solid) (pressure work)

$$- \frac{1}{V}\int_{S_{\alpha\beta}} \tilde{p}_\alpha (v_{\alpha j} - w_{\alpha j})\, n_{\alpha j}\, dA = \langle \dot{E}_\alpha\rangle \tag{22}$$

(residual pressure work) (internal generation)

COMBINING THE AVERAGED EQUATIONS

Fluid momentum balance: Define (locally) the fluid mass flux:

$$\dot{m}_j \equiv \langle\rho_L\rangle^L \langle v_{Lj}\rangle + \langle\rho_G\rangle^G \langle v_{Gj}\rangle \tag{23}$$

Assume (H8) negligible capillarity, then define (pointwise) the reservoir pressure p:

$$p \equiv \begin{cases} p_S \text{ in } R_S \\ p_L \text{ in } R_L \\ p_G \text{ in } R_G \end{cases} \quad \text{with} \quad \begin{cases} \langle p\rangle = \langle p_S\rangle^S = \langle p_L\rangle^L = \langle p_G\rangle^G \\ p_L = p_G \ (= p) \text{ along } S_{LG} \end{cases} \tag{24}$$

Then:

$$\dot{m}_j = -\left(\frac{k_L\langle\rho_L\rangle^L}{\varepsilon_L\langle\mu_L\rangle^L} + \frac{k_G\langle\rho_G\rangle^G}{\varepsilon_G\langle\mu_G\rangle^G}\right) k_{ij}\frac{\partial}{\partial x_i}\langle p\rangle + \left(\frac{k_L(\langle\rho_L\rangle^L)^2}{\varepsilon_L\langle\mu_L\rangle^L} + \frac{k_G(\langle\rho_G\rangle^G)^2}{\varepsilon_G\langle\mu_G\rangle^G}\right) g_j \tag{25}$$

<u>Fluid mass balance</u>: Define (locally) the fluid density $<\rho_F>^F$ as follows:

$$\varepsilon_F <\rho_F>^F \equiv \varepsilon_L <\rho_L>^L + \varepsilon_G <\rho_G>^G \qquad (26)$$

then:

$$\frac{\partial}{\partial t} (\varepsilon_F <\rho_F>^F) + \frac{\partial \dot{m}_j}{\partial x_j} = 0 \qquad (27)$$

<u>Fluid heat balance</u>: Define (locally) the fluid internal energy $<e_F>^F$ and internal heat generation rate $<\dot{E}_F>$ as follows:

$$\varepsilon_F <\rho_F>^F <e_F>^F \equiv \varepsilon_L <\rho_L>^L <e_L>^L + \varepsilon_G <\rho_G>^G <e_G>^G \qquad (28)$$

$$<\dot{E}_F> \equiv <\dot{E}_L> + <\dot{E}_G> \qquad (29)$$

Then:

$$\frac{\partial}{\partial t} (\varepsilon_F <\rho_F>^F <e_F>) + \frac{\partial}{\partial x_j} (\dot{m}_j <e_F>^F) + \frac{\partial}{\partial x_j} (-\varepsilon_L \lambda^*_{Lij} \frac{\partial}{\partial x_i} <T_L>^L - \varepsilon_G \lambda^*_{Gij} \frac{\partial}{\partial x_i} <T_G>^G)$$

$$- \frac{1}{V} \int_{S_s} (-\lambda_S \frac{\partial T_S}{\partial x_j}) n_{Sj} \, dA + <p_L>^L (\frac{\partial \varepsilon_L}{\partial t} + \frac{\partial}{\partial x_j} <v_{Lj}>) + <p_G>^G (\frac{\partial \varepsilon_G}{\partial t} + \frac{\partial}{\partial x_j} <v_{Gj}>)$$

$$+ \frac{1}{V} \int_{S_{LG}} (\tilde{p}_G v_{Gj} - \tilde{p}_L v_{Lj}) n_{Lj} dA + \frac{1}{V} \int_{S_{LG}} (\tilde{p}_L - \tilde{p}_G) w_{Lj} n_{Lj} dA = <\dot{E}_F> \qquad (30)$$

Assume (H9) <u>temperature equilibrium between both fluid phases</u> then define (pointwise) the fluid temperature T_F, (locally) the fluid effective thermal conductivity tensor λ^*_{Fij} and a coefficient of solid-fluid heat transfer h_{SF} (Combarnous and Bories, 1974):

$$T_F \equiv \begin{cases} 0 & \text{in } R_S \\ T_L & \text{in } R_L \\ T_G & \text{in } R_G \end{cases} \qquad \text{with} \qquad \langle T_F\rangle^F = \langle T_L\rangle^L = \langle T_G\rangle^G \tag{31}$$

$$\varepsilon_F \lambda^*_{Fij} \equiv \varepsilon_L \lambda^*_{Lij} + \varepsilon_G \lambda^*_{Gij} \tag{32}$$

$$a h_{SF}(\langle T_S\rangle^S - \langle T_F\rangle^F) \equiv \frac{1}{V} \int_{S_S} (-\lambda_S \frac{\partial T_S}{\partial x_j})\, n_{Sj}\, dA \tag{33}$$

with $a \equiv \frac{A_S}{V}$, solid-fluid interfacial area per unit volume of porous medium. Then

$$\underbrace{\frac{\partial}{\partial t}(\varepsilon_F \langle\rho_F\rangle^F \langle e_F\rangle^F)}_{\text{(storage)}} + \underbrace{\frac{\partial}{\partial x_j}(\dot{m}_j \langle e_F\rangle^F)}_{\text{(convection)}} + \underbrace{\frac{\partial}{\partial x_j}(-\varepsilon_F \lambda^*_{Fij} \frac{\partial}{\partial x_i} \langle T_F\rangle^F)}_{\text{(conduction within fluid)}}$$

$$+ \underbrace{a h_{SF}(\langle T_F\rangle^F - \langle T_S\rangle^S)}_{\text{(conduction across solid-fluid interface)}} + \underbrace{\langle p\rangle \frac{\partial}{\partial x_j}(\langle v_{Lj}\rangle + \langle v_{Gj}\rangle)}_{\text{(pressure work)}} + \underbrace{\frac{1}{V}\int_{S_{LG}} \tilde{p}(v_{Gj} - v_{Lj}) n_{Lj}\, dA}_{\text{(residual pressure work)}}$$

$$= \underbrace{\langle \dot{E}_F\rangle}_{} \text{ (internal generation)} \tag{34}$$

<u>Solid heat balance</u>: By H8 and H9,

$$\left.\begin{array}{l} \underbrace{\frac{\partial}{\partial t}(\varepsilon_S \langle\rho_S\rangle^S \langle e_S\rangle^S)}_{\text{(storage)}} \\ \varepsilon_S \langle\rho_S\rangle^S \langle c_{vS}\rangle^S \frac{\partial}{\partial t} \langle T_S\rangle^S \end{array}\right\} + \underbrace{\frac{\partial}{\partial x_j}(-\varepsilon_S \lambda^*_{Sij} \frac{\partial}{\partial x_i} \langle T_S\rangle^S)}_{\text{(conduction within solid)}} + \underbrace{a h_{SF}(\langle T_S\rangle^S - \langle T_F\rangle^F)}_{\text{(conduction across solid-fluid interface)}}$$

$$= \langle \dot{E}_S\rangle \text{ (internal generation)} \tag{35}$$

<u>Solid-fluid heat balance</u>:

Assume (H10) <u>solid-fluid temperature equilibrium</u> then define (pointwise) the reservoir temperature T and (locally) the reservoir effective thermal conductivity tensor λ^*_{ij}:

$$T \equiv \begin{cases} T_F \text{ in } R_L \cup R_G \\ T_S \text{ in } R_S \end{cases} \quad \text{with } \langle T\rangle = \langle T_S\rangle^S = \langle T_F\rangle^F \tag{36}$$

$$\lambda^*_{ij} \equiv \varepsilon_S \lambda^*_{Sij} + \varepsilon_F \lambda^*_{Fij} \quad (\text{with } \varepsilon_S + \varepsilon_F = 1) \tag{37}$$

Then:

$$\underbrace{\frac{\partial}{\partial t} \left(\varepsilon_S \langle\rho_S\rangle^S \langle e_S\rangle^S + \varepsilon_F \langle\rho_F\rangle^F \langle e_F\rangle^F\right)}_{\text{(storage)}} + \underbrace{\frac{\partial}{\partial x_j} \left(\dot{m}_j \langle e_F\rangle^F\right)}_{\text{(convection)}} + \underbrace{\frac{\partial}{\partial x_j} \left(-\lambda^*_{ij} \frac{\partial}{\partial x_i} \langle T\rangle\right)}_{\text{(conduction)}}$$

$$+ \underbrace{\langle p\rangle \frac{\partial}{\partial x_j} \left(\langle v_{Lj}\rangle + \langle v_{Gj}\rangle\right)}_{\text{(pressure work)}} = \underbrace{\langle \dot{E}_S\rangle + \langle \dot{E}_F\rangle}_{\text{(internal generation)}} \tag{38e}$$

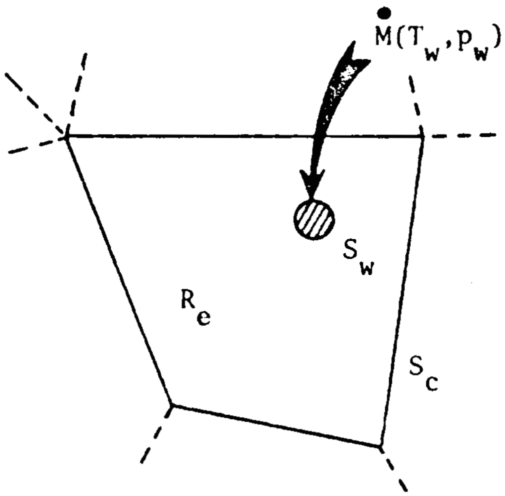
Similarly, in terms of (fluid) enthalpy:

$$\frac{\partial}{\partial t} \left(\varepsilon_S \langle\rho_S\rangle^S \langle e_S\rangle^S + \varepsilon_F \langle\rho_F\rangle^F \langle h_F\rangle^F\right) + \frac{\partial}{\partial x_j} \left(\dot{m}_j \langle h_F\rangle^F\right) + \frac{\partial}{\partial x_j} \left(-\lambda^*_{ij} \frac{\partial}{\partial x_i} \langle T\rangle\right)$$

$$- \frac{\partial}{\partial t} \langle p\rangle - \left(\langle v_{Lj}\rangle + \langle v_{Gj}\rangle\right) \frac{\partial}{\partial x_j} \langle p\rangle = \langle \dot{E}_S\rangle + \langle \dot{E}_F\rangle \tag{38h}$$

ELEMENTAL AVERAGING

(Or, how to fit external heat or mass sources, such as wells or leaking boundaries, into the balance equations.)



The Procedure

Partition the reservoir into a set of "elements". Any element R_e is bounded by

i) an interface S_c connecting R_e to neighboring elements.

ii) a surface S_w separating R_e from the "outside".

Elemental average:

$$\bar{\psi} \equiv \frac{1}{V_e} \int_{R_e} \langle\psi\rangle \, dV \tag{39}$$

Hypothesis G5 modified:

$$\begin{array}{ccccccc} (d<<) & \ell & << & \ell_e & << & L \\ \text{(pore)} & \text{(R.E.V.)} & & \text{(element)} & & \text{(reservoir)} \end{array} \qquad (40)$$

Divergence theorem:

$$\int_{R_e} \frac{\partial}{\partial x_j} \langle\psi_j\rangle dV = \int_{S_c U S_w} \langle\psi_j\rangle n_j dA \qquad (41)$$

Elemental mass balance:

$$\bar{\varepsilon}\, V_e \frac{\partial \bar{\rho}_F}{\partial t} + \int_{S_c} \dot{m}_j n_j dA = \dot{M} \qquad (42)$$

Elemental momentum balance: Immaterial

Elemental heat balance: By approximation of the pressure term in (38e):

$$V_e \frac{\partial}{\partial t} [(1-\bar{\varepsilon})\bar{\rho}_S \bar{e}_S + \bar{\varepsilon}\bar{\rho}_F \bar{e}_F] + \int_{S_c} \dot{m}_j \langle e_F\rangle^F n_j dA + \int_{S_c} (-\lambda^*_{ij} \frac{\partial}{\partial x_i} \langle T\rangle)\, n_j dA$$

$$+ \bar{p} \int_{S_c} (\langle v_{Lj}\rangle + \langle v_{Gj}\rangle) n_j dA = V_e(\dot{\bar{E}}_S + \dot{\bar{E}}_F) + \dot{M}\, (\bar{e}_w + \frac{\bar{p}}{\bar{\rho}_w}) + A_w\, (\frac{\lambda^*}{\ell_e})_w\, (\bar{T}_w - \bar{T})$$

(43e)

Alternatively:

$$V_e \frac{\partial}{\partial t} [(1-\bar{\varepsilon})\bar{\rho}_S \bar{e}_S + \bar{\varepsilon}\bar{\rho}_F \bar{h}_F] + \int_{S_c} \dot{m}_j \langle h_F\rangle^F n_j dA + \int_{S_c} (-\lambda^*_{ij} \frac{\partial}{\partial x_i} \langle T\rangle) n_j dA$$

$$- V_e \frac{\partial p}{\partial t} - \int_{R_e} (\langle v_{Lj}\rangle + \langle v_{Gj}\rangle) \frac{\partial\langle p\rangle}{\partial x_j} dV = V_e(\dot{\bar{E}}_S + \dot{\bar{E}}_F) + \dot{M}\bar{h}_w + A_w (\frac{\lambda^*}{\ell_e})_w (\bar{T}_w - \bar{T})$$

(43h)

TOWARD NUMERICAL SOLUTION

The Pressure-Work Term

The main difference between equations (43e) and (43h) lies in the pressure terms in the left side. These terms may be viewed as coupling the heat balance to the mass balance.

An order of magnitude analysis (Appendix B) shows that, under steady state conditions, the contribution of the pressure-work term to the heat balance is systematically lower with equation (43h) than with equation (43e); that term cannot however be neglected, even in equation (43h), since it is of the same order of magnitude as the convection term when the element considered is devoid of any external mass source.

Equation (43h) exhibits an additional pressure term involving the time derivative of the elemental pressure: we expect that term to yield a significant contribution to the heat balance of those elements only that include wells and only for a "short" period of time following every drastic change in the rate of mass generation.

Provided such drastic changes cover a relatively small part of the simulated life span of the geothermal reservoir, we think that, all in all, equation (43h) may be easier to solve than equation (43e).

The Integrated Finite Difference Method

Let us now compare equations (43e) and 43h) from the standpoint of ease in programming by the Integrated Finite Difference Method (Lasseter et al., 1975; Assens, 1976).

The relevant characteristics of that method are that

1) every element may be connected to any number of surrounding elements and

2) the elemental balances are obtained by adding to the storage and generation terms the contribution (fluxes) of every connection.

Using the IFDM, we evaluate the pressure-work term in (43e) as the product of the pressure within R_e by the sum of all the volumetric fluxes across all the connecting surfaces relative to R_e. On the other hand, we found no simple way to evaluate the (integral) pressure-work term in (43h); we however acknowledge that this might be straightforward when using the Finite Element Method.

We are currently looking for a way of approximating the pressure-work term in (43h) so that we may be able to apply IFDM to the solution of that particular equation.

Choosing a Set of Dependent Variables

The selection of (43h) over (43e) implies the choice of the fluid enthalpy as one of the two dependent variables needed to fully describe the behavior of the geothermal system.

The very form of the mass balance equation (42) leads us to the choice of the fluid density as the other variable. A more material reason for that choice is however that the mass balance for the entire reservoir will thus be kept more accurately than if pressure had been taken as the second dependent variable (Pritchett, 1975: in this later case density, then considered as a parameter, would be evaluated indirectly, based on the values of enthalpy and pressure obtained by solving the transport equations. In the former case only density increments are computed, and the values of the density obtained at the end of the previous time step accordingly updated, thus ensuring better "historical" consistency.

Whenever temperature equilibrium is not fully established, as may be the case in the process of storing hot or cold water underground, equation (43h) needs to be replaced by the elemental averages of both equations (34) and (35): the solid temperature is with little doubt best chosen as the third dependent variable then required.

CONCLUSION

Averaging the mass balance equations from the level of the pore to that of a "R.E.V." of porous medium yields an equation the form of which is analogous to that of the basic equations but for the introduction of the porosity and a dispersion term which we expect to be negligible whenever no correlation exists between density and velocity.

The momentum balance equation yields an explicit expression for the velocity, provided inertia is negligible.

The main change from the basic heat balance equation to the averaged equation is the substitution of a tensor of "effective" thermal conductivity of the porous medium for the individual scalar conductivities of each solid, liquid and gaseous constituents.

Since these equations hold only within the porous reservoir stricto sensu, a further step of averaging is required to include the external heat or mass sources such as wells or leaking boundaries: this classical averaging yields equations that express the balance of heat and mass for any element of a partition of the reservoir, in a form suitable for numerical solution.

Comparing the two forms of the heat balance obtained in terms of either fluid internal energy (43e) or enthalpy (43h), we find that, beside a highly transient pressure term in equation (43h), the basic difference lies in a pressure-work term that couples the heat balance to the mass balance: an elementary order of magnitude analysis indicates that coupling is minimum when heat balance is expressed in terms of enthalpy, thus favoring the selection of the fluid enthalpy as one of the dependent variables that describe the behavior of the reservoir.

This choice however does not allow us to use the Integrated Finite Differences Method in the current state of our art.

Both computational simplicity and mass balance accuracy lead to the selection of the fluid density as second dependent variable, supplemented by the solid temperature whenever solid-fluid temperature equilibrium is not established.

NOMENCLATURE

Whenever relevant, the dimension is listed in the second column in terms of mass (M), length (L), time (t) and temperature (T).

Roman lower case letters

a	(L^{-1})	solid-fluid interfacial area per unit volume of porous medium
c_e, c_h		coupling ratios pressure-work/heat convection
c_v	$(L^2t^{-2}T^{-1})$	specific heat capacity at constant volume
d	(L)	characteristic length of the pore space
e	(L^2t^{-2})	(specific) internal energy
g_i	(Lt^{-2})	i^{th} component of the gravitational acceleration
h	(L^2t^{-2})	(specific) enthalpy
h_{SF}	$(Mt^{-3}T^{-1})$	coefficient of solid-fluid heat transfer
k_{ij}	(L^2)	(second order) tensor of absolute permeability
k_α		relative permeability of the α phase
$k_{\alpha\beta ij}$	(L^2)	second order permeability tensor
ℓ	(L)	characteristic length of the R.E.V.
ℓ_e	(L)	characteristic length of the element R_e
$\dot{m}_j$		j^{th} component of the fluid mass flux
$n_{\alpha j}$		j^{th} component of the unit normal to S_α, directed from R_α outward
p	$(ML^{-1}t^{-2})$	pressure
t	(t)	time
v_j	(Lt^{-1})	j^{th} component of the point velocity
$\bar{v}_j$	(Lt^{-1})	j^{th} component of the elemental velocity
w_j	(Lt^{-1})	j^{th} component of the (point) velocity of S
x_j	(L)	j^{th} coordinate

Roman capital letters

$A_\bullet$	(L^2)	area of $S_\bullet$
$\dot{E}_\alpha$	$(ML^{-1}t^{-3})$	energy generated within the α phase, per unit time, per unit volume
L	(L)	characteristic length of the reservoir
$\dot{M}$	(Mt^{-1})	mass generation rate
$R_\bullet$		any space domain
S_c		boundary between R_e and the surrounding elements
S_w		boundary between R_e and the exterior (with respect to the reservoir).
S_α		boundary between R_α and $R - R_\alpha$
$S_{\alpha\beta}$		boundary between R_α and R_β
T	(T)	temperature
U		mathematical symbol: "union"
$V_\bullet$	(L^3)	volume of $R_\bullet$

Greek lower case letters

α	(T^{-1})	coefficient of (isobaric) thermal expansion
β	$(M^{-1}Lt^2)$	coefficient of isothermal expansion
δ_{ij}		Kronecker symbol
ε		porosity of the porous medium
ε_α		volumetric fraction of the α phase
λ_α	$(MLt^{-3}T^{-1})$	(intrinsic) thermal conductivity of the α phase
λ'_α	$(MLt^{-3}T^{-1})$	stagnant thermal conductivity
$\lambda''_{\alpha ij}$	$(MLt^{-3}T^{-1})$	tensor of thermal dispersion
λ^*_{ij}	$(MLt^{-3}T^{-1})$	tensor of effective thermal conductivity of the porous medium
μ	$(ML^{-1}t^{-1})$	dynamic viscosity
ρ	(ML^{-3})	density
τ_{ij}	$(ML^{-1}t^{-2})$	viscous stress tensor

Greek capital letter

Φ	(t^{-2})	viscous dissipation function

Subscripts

c	connection
e	element
F	fluid
G	gas
i,j,k	spatial coordinates
L	liquid
S	solid
w	exterior
α	phase identifier (α = S, L, G)
β	phase identifier (β = S, L, G)

Mathematical symbols and notations

U	union of two sets
$\bar{\bullet}$	elemental average
$\equiv$	is, by definition, equal to
$< >$	phase average over R
$< >^{\alpha}$	intrinsic phase average over R_{α}
$\tilde{\bullet}$	deviation from intrinsic phase average
$\frac{D\bullet}{Dt} = \frac{\partial\bullet}{\partial t} + v_j \frac{\partial\bullet}{\partial x_j}$	substantial derivative
$\nabla\bullet$	gradient

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APPENDIX A

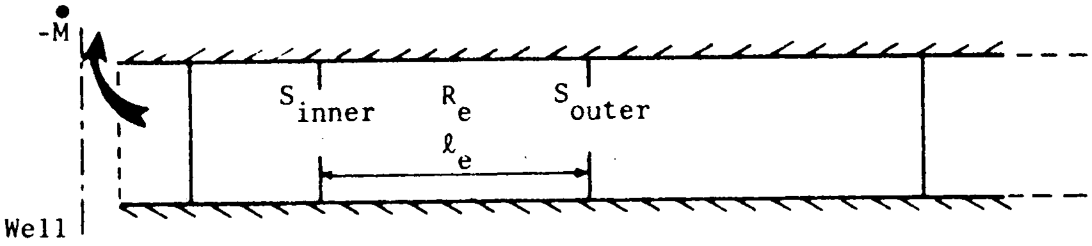
The basic assumptions

G1	Continuum approximation
G2	Negliglbe thermodynamic fluctuations
G3	Laminar flow regime
G4	One component fluid
G5	$d << \ell << \ell_e << L$
G6	Good behavior of the variables
H1	The average of the product of two or more deviations of variables not strongly dependent on velocity is negligible compared to the product of the averages of these variables
H2	incompressible, non reacting solid
H3	low correlation between density and velocity
H4	indeformable solid matrix
H5	negligible inertia
H6	rigid gas-liquid interface with no slip
H7	negligible viscous dissipation
H8	negligible capillarity
H9	temperature equilibrium between both fluid phases
H10	solid-fluid temperature equilibrium (optional)

APPENDIX B

Order of magnitude analysis of the pressure-work term

Consider an homogeneous, isotropic, isothermal reservoir with uniform thickness, partitioned into a set of toric elements centered on a fully penetrating well produced at a constant rate $-\dot{M}$.



Let the heat convection term (CNV) be the reference with which the magnitude of the pressure-work term (PWK) is to be evaluated.

Case 1

First consider the element immediately surrounding the well and assume that the fluid is one-phase throughout:

$$PWK_e \simeq \bar{p} \int_{S_c} \langle v_j \rangle n_j dA = \bar{p} \frac{\dot{M}}{\langle \rho \rangle_c} \sim \bar{p} \frac{\dot{M}}{\bar{\rho}}$$

$$PWK_h \simeq \bar{v} \nabla \bar{p} V_e \sim \left(\frac{\dot{M}}{\bar{\rho} A_c}\right) \left(\frac{\Delta \bar{p}}{\ell_e}\right) (A_c \ell_e) = \frac{\dot{M} \Delta \bar{p}}{\bar{\rho}}$$

$$CNV_e = \dot{M} \langle e \rangle_c \sim \dot{M} \bar{e} \quad \text{Similarly:} \quad CNV_h \sim \dot{M} \bar{h}$$

Thus:

$$c_e \equiv \frac{PWK_e}{CNV_e} \sim \frac{\bar{p}}{\bar{\rho} \bar{e}} \qquad \text{whereas} \qquad c_h \equiv \frac{PWK_h}{CNV_h} \sim \frac{\Delta \bar{p}}{\bar{\rho} \bar{h}}$$

Let us compare both coupling ratios c_e and c_h:

$$\frac{c_h}{c_e} \sim \left(\frac{\Delta\bar{p}}{\bar{p}}\right)\left(\frac{\bar{e}}{\bar{h}}\right) \sim \frac{\Delta\bar{p}}{\bar{p}}$$

In most cases the element closest to the well is sufficiently narrow for the pressure drop across it to be a small fraction of the average pressure; whence c_h may be expected to be at least one order of magnitude smaller than c_e:

$$\frac{c_h}{c_e} \sim 10^{-1}$$

Let us further particularize our analysis and consider the following "typical" saturated conditions (see e.g. Wukalovitch, 1958):

$T = 293.6\ ^\circ C$	$p = 80$ atm $(7.9\ 10^6 N/m^2)$
$\rho_L = 725\ kg/m^3$	$\rho_G = 41.6\ kg/m^3$
$e_L = 1.30\ 10^6$ J/kg	$e_G = 2.57\ 10^6$ J/kg
$h_L = 1.31\ 10^6$ J/kg	$h_G = 2.76\ 10^6$ J/kg

Then:

$$c_e \sim \begin{cases} .8\ 10^{-2} & \text{for steam} \\ 7.\ 10^{-2} & \text{for liquid water} \end{cases}$$

and c_h is at least one order of magnitude smaller.

Case 2:

Then consider any element away from the well and assume that the fluid is one-phase throughout:

$$PWK_e \simeq \bar{p}\left[\int_{S_{inner}} \langle v_j\rangle n_j dA + \int_{S_{outer}} \langle v_j\rangle n_j dA\right] = \bar{p}\left(\frac{\dot{M}}{\langle\bar{\rho}\rangle}_{outer} - \frac{\dot{M}}{\langle\bar{\rho}\rangle}_{inner}\right)$$

$$\sim \frac{\dot{M}\bar{p}\Delta\bar{\rho}}{\bar{\rho}^2} \qquad \text{with } \Delta\bar{\rho} \equiv \langle\rho\rangle_{outer} - \langle\rho\rangle_{inner}$$

$$PWK_e \sim \beta\bar{p}\dot{M}\frac{\Delta\bar{p}}{\bar{\rho}}$$

$$PWK_h \sim \frac{\dot{M}\Delta\bar{p}}{\bar{\rho}}$$

$$CNV_e = \dot{M}(e_{outer} - e_{inner}) \equiv \dot{M}\Delta\bar{e}$$

$$CNV_h = \dot{M}\Delta\bar{h}$$

Thus:

$$c_e \sim \beta\bar{p}\,\frac{\Delta\bar{p}}{\bar{\rho}\Delta\bar{e}} \quad , \qquad c_h \sim \frac{\Delta\bar{p}}{\bar{\rho}\Delta\bar{h}}$$

$$\frac{c_h}{c_e} \sim \frac{1}{\beta\bar{p}}\,\frac{\Delta\bar{e}}{\Delta\bar{h}}$$

where $\beta\bar{p} \sim 1$ for steam considered as a perfect gas and (see e.g. Helgeson and Kirkham, 1974):

$$\beta\bar{p} \sim (3.10^{-4} m^2/N)(7.9\ 10^6\ N/m^2) \simeq 2.10^3 \text{ for liquid water}$$

Assuming a piezometric gradient of 10^{-1} and an element width ℓ_e of 100 meters yields the following approximate figures:

$$\Delta\bar{p} \simeq 7.\ 10^4\ N/m^2$$

$$\Delta\bar{e}_L \simeq -\ 180\ J/kg \qquad \Delta e_G \simeq -\ 2800\ J/kg$$

$$\Delta\bar{h}_L \simeq +\ 80\ J/kg \qquad \Delta\bar{h}_G \simeq +\ 3800\ J/kg$$

Then, for liquid water:

$$\frac{c_h}{c_e} \sim 10^{-3} \ , \quad c_e \sim 10^3 \ , \quad c_h \sim 1$$

for steam:

$$\frac{c_h}{c_e} \sim 1 \ , \quad c_e \sim 1 \ , \quad c_h \sim 1$$

Case 3:

Finally, once again consider any element away from the well but assume that complete flashing (or condensation) occurs within it; given the assumption of isothermal process, the vapor zone lies closest to the well:

$$PWK_e \sim \bar{p}\,\left(\frac{\dot{M}}{\bar{\rho}_G} - \frac{\dot{M}}{\bar{\rho}_L}\right) = \dot{M}\bar{p}\,\frac{\bar{\rho}_L - \bar{\rho}_G}{\bar{\rho}_L\bar{\rho}_G} \sim \dot{M}\,\frac{\bar{p}}{\bar{\rho}_G}$$

$$PWK_h \sim \bar{v}_G \nabla\bar{p}_G V_{eG} + \bar{v}_L \nabla\bar{p}_L V_{eL} \sim (\bar{v}_G + \bar{v}_L)\ \nabla\bar{p} V_e \sim \left(\frac{\dot{M}}{\bar{\rho}_G A_c} + \frac{\dot{M}}{\bar{\rho}_L A_c}\right)\left(\frac{\Delta\bar{p}}{\ell_e}\right)(A_c \ell_e) \sim \frac{\dot{M}\Delta\bar{p}}{\bar{\rho}_G}$$

$$CNV_e \sim \dot{M}\ (\bar{e}_G - \bar{e}_L) \qquad , \qquad CNV_h \sim \dot{M}\ (h_G - h_L)$$

Thus:

$$c_e \sim \frac{\bar{p}}{\bar{\rho}_G(\bar{e}_G - \bar{e}_L)} \sim 10^{-1}\ , \quad c_h \sim \frac{\Delta\bar{p}}{\bar{\rho}_G(\bar{h}_G - \bar{h}_L)} \sim 10^{-3}\ , \quad \frac{c_h}{c_e} \sim \frac{\Delta\bar{p}}{\bar{p}}\ \frac{\bar{e}_G - \bar{e}_L}{\bar{h}_G - \bar{h}_L} \sim 10^{-2}$$

Summary

Case number		c_e	c_h	c_e/c_h
1	liquid water	10^{-1}	10^{-2}	10^{-1}
	steam	10^{-2}	10^{-3}	10^{-1}
2	liquid water	10^{3}	1	10^{-3}
	steam	1	1	1
3		10^{-1}	10^{-3}	10^{-2}

Table 1

Under steady-state conditions:

i) equation (43h) involves a lesser amount of coupling than does equation (43e).

ii) the pressure-work term cannot be safely neglected.