HEAT EXTRACTION FROM A HYDRAULICALLY FRACTURED PENNY-SHAPED CRACK IN HOT DRY ROCK

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Heat extraction from a penny-shaped crack having both inlet and outlet holes is investigated analytically by considering the hydraulic and thermal growth of the crack when fluid is injected at a constant flow rate. The rock mass is assumed to be infinitely extended, homogeneous, and isotropic. The equations for fluid flow are derived and solved to determine the flow pattern in the crack. Temperature distributions in both rock and fluid are also determined. The crack width change due to thermal contraction and the corresponding flow rate increase are discussed. Some numerical calculations of outlet temperature, thermal power extraction, and crack opening displacement due to thermal contraction of rocks are presented for cracks after they attain stationary states for given inlet flow rate and outlet suction pressure.

The present paper is a further development of the previous works of Bodvarsson (1969), Gringarten et al. (1975), Lowell (1976), Harlow and Pracht (1972), McFarland (1975), among others, and considers the two dimensional rather than the one-dimensional crack. Furthermore, the crack radius and width are quantities to be determined rather than given a priori.

FLUID FLOW IN A PENNY-SHAPED CRACK

Consider a large penny-shaped crack having a radius R and width w (in the z-direction) as shown in Fig. 1. Fluid is injected from the inlet at the center of the crack and removed in part at the outlet, $x = a$, where x is the distance measured in the vertical direction from the center. The radii of the inlet and outlet holes are denoted by R_0 and R_a, respectively.

The total mass flow rate at the inlet wellbore can be written in the form:

$$q_0 = q_a + q_E + q_L + q_T \tag{1}$$

where q_a is the effective flow rate equal to the outlet flow rate, q_E is the total mass change in the crack, q_L corresponds to the total fluid loss in the crack per unit time, and q_T is the increase of the crack volume due to the thermal contraction of the rock and can be neglected.

If the crack is subjected to a constant inlet flow rate and the crack radius is sufficiently large, the fluid viscosity can be neglected from the equation of linear momentum as shown in a previous paper (Abé, Mura and Keer, 1976):

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$$\frac{\partial p}{\partial r} = -\rho_f \ g \cos\theta, \quad \frac{\partial p}{\partial \theta} = \rho_f \ g r \sin\theta \tag{2}$$

where p is the fluid pressure in the crack, g is the acceleration due to gravity, and ρ_f is the fluid density. Equation (2) is integrated as

$$p(r,\theta,t) = p_0(t) - \rho_f \ gr \cos\theta \tag{3}$$

where p_0 is the fluid pressure at r = 0 and t is time. The density ρ_f has been assumed to be constant.

The fracture mechanics is introduced here by considering a crack opening stress $(\sigma_z)_{z=0} = -p + (S_0 - K_a \rho_\gamma g x)$ where S_0 is the tectonic stress at r = 0, K_a is the coefficient of active rock pressure, and ρ_γ is the density of the rock.

The stress intensity factor at the crack tip and the opening displacement are easily obtained from the results derived by Keer (1964):

$$K = \lim_{r \to R} (r - R)^{1/2} \sigma_z = \frac{\sqrt{2R}}{\pi}[p_0 - S_0 + \tfrac{2}{3} g R (K_a \rho_\gamma - \rho_f) \cos\theta] \tag{4}$$

and

$$w(r,\theta) = \frac{3}{\rho_f D}[p_0 - S_0 + \tfrac{2}{3}(K_a \rho_\gamma - \rho_f) g r \cos\theta]\sqrt{R^2 - r^2} \tag{5}$$

where

$$D = \frac{3\pi E}{8(1-\nu^2)\rho_f} \tag{6}$$

with E and ν being Young's modulus and Poisson's ratio respectively.

The flow rate q_E, defined in (1), is

$$q_E = \frac{d}{dt}\int_0^R \int_{-\pi}^{\pi} \rho_f w \, r \, d\theta dr = \frac{d}{dt}\{\frac{2\pi}{D} R^3 (p_0 - S_0)\}. \tag{7}$$

Now the average stress intensity factor is introduced by the definition

$$\bar{K} = \frac{1}{2\pi}\int_{-\pi}^{\pi} K \, d\theta = \frac{\sqrt{2R}}{\pi}(p_0 - S_0). \tag{8}$$

It is assumed that when the crack is expanding

$$\bar{K} = \text{constant } K_c. \tag{9}$$

The flow loss is defined by

$$q_L = 2\rho_f \int_0^{2\pi}\int_0^R u_L \, r \, dr d\theta, \tag{10}$$

where u_L is the fluid loss rate per unit area of the crack surface and is assumed here to be a linear function of p;

$$2\rho_f u_L = C_{L0} + C_{L1}(p_0 - \rho_f \ gr \cos\theta) \tag{11}$$

where C_{L0} and C_{L1} are constant. Then, (10) becomes

$$q_L = \pi R^2 (C_{L0} + C_{L1} p_0). \tag{12}$$

Finally, the flow rate q_a in (1) is evaluated from the Bernoulli equation applied to the flow in the neighborhood of the throat of the outlet. Then

$$p_0 - g\rho_f a = -p_a^* + g\rho_f (h_0 - a) + \frac{1}{2\rho_f}\left(\frac{C_v q_a}{\pi R_a^2}\right)^2 \tag{13}$$

where p_a^* is the suction pressure by the outlet pump and the constant C_v (> 1) is an outlet head loss.

Equations (1), (7), (12) and (13) provide a functional form of R with respect to t for given values of q_0, p_a^* and other physical constants and geometrical values of h_0, a, R_a; p_0 is expressed in terms of K_c and R through the relations (8) and (9). The crack radius R increases with time from the initial value R_s which is the value of R before the outlet is introduced. R reaches a stationary value after some time when

$$q_E > 0 \quad \text{and} \quad p_a^*/S_0 < 1/\Delta - 1 \tag{14}$$

where $\Delta = S_0/\rho_f g h_0$. We call this case Case (I). On the other hand, the crack can remain at the initial size R_s when

$$q_E \le 0 \quad \text{and} \quad \bar{K} < K_c. \tag{15}$$

We call this case Case (II). Here, $R = R_s$ and p_0 is obtained as a function of t from (1), (7), (12) and (13) for given values of q_0, p_a^* and other physical and geometrical constants. In the next section we shall calculate the quantity of heat extracted from the outlet in each case (I) and (II). Several numerical examples for R = R(t) and $p_0 = p_0(t)$ were shown in a previous paper (Abé, Keer, Mura 1976).

HEAT EXTRACTION FROM OUTLET

In this section a stationary penny-shaped crack (after R and p_0 have attained their stationary values) is treated as a starting point for the analytical study of two-dimensional heat transmission problems.

We have to determine first the velocity field of the fluid inside the crack. Assumptions of incompressibility and irrotationality of fluid lead to

$$\frac{1}{r}\frac{\partial}{\partial r}(r\, q_r) + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + 2\rho_f\, u_L = 0 \tag{16}$$

$$\frac{\partial}{\partial r}(r\, q_\theta) - \frac{\partial q_r}{\partial \theta} = 0 \tag{17}$$

where

$$q_r = \rho_f w \bar{u}_r, \quad q_\theta = \rho_f w \bar{u}_\theta \tag{18}$$

and $\bar{u}_r$ and $\bar{u}_\theta$ are the components of velocity averaged through the width w. The boundary condition is $q_r = 0$ at r = R. The inlet and outlet are treated as a point source and sink, respectively, since R_0 and R_a are sufficiently small when compared with R. The solution is obtained as

$$q_r = \frac{q_a}{2\pi}\left[\sum_{n=1}^{\infty} \frac{a^n}{R^{2n}} r^{n-1} \cos n\theta - \frac{a}{r} \frac{r \cos \theta - a}{(r \cos \theta - a)^2 + r^2 \sin^2\theta}\right]$$

$$+ \frac{1}{2}(R^2 - r^2)\left[\frac{1}{r}(C_{L0} + C_{L1}p_0) - \frac{3}{4} \rho_f g C_{L1} \cos \theta\right]$$

$$q_\theta = -\frac{q_a}{2\pi}\left[\sum_{n=1}^{\infty} \frac{a^n}{R^{2n}} r^{n-1} \sin n\theta + \frac{a \sin \theta}{(r \cos \theta - a)^2 + r^2 \sin^2\theta}\right]$$

$$+ \frac{1}{8} \rho_f g C_{L1} (3R^2 - r^2) \sin \theta. \tag{19}$$

It should be noticed that (19) is valid even for a non-stationary crack.

Next, the energy equation for the fluid is derived. For heat transfer problems at small fluid velocity, the mechanical energy terms are small in the energy equation. The effect of heat conduction in fluid (water) may also be small compared with those of heat convection and transfer terms. Furthermore, the time derivative term of the fluid temperature T can be neglected because of smallness (Bodvarsson, 1969, Lowell, 1976). It is assumed that the rock temperature T_r is approximately equal to T on the crack surface and T is constant through the crack width (Bodvarsson, 1969, Gringarten et al., 1975, Lowell, 1976). In this way the energy equation for the fluid, after averaging through the crack width, can be written in the form:

$$q_r \frac{\partial T}{\partial r} + q_\theta \frac{\partial T}{r \partial \theta} = \frac{2\lambda}{C_f} \frac{\partial T_r}{\partial z}\bigg|_{z=0} \tag{20}$$

where C_f and λ are the specific heat of the fluid and the heat conductivity of the rock respectively. The position of the boundary $z = w/2$ has been replaced by $z = 0$, since w is very small compared with the radius R and the distance a.

When the energy system operates effectively, the thermal penetration depth in the rock is very small compared with R and a so that the heat flux is almost perpendicular to the fracture surface. Thus the rock temperature T_r may simply be governed by the following equation:

$$\frac{\partial^2 T_r}{\partial z^2} = \frac{C_\gamma \rho_\gamma}{\lambda} \frac{\partial T_r}{\partial t} \tag{21}$$

where C_γ is the specific heat of the rock. It is noted that this simplification does not mean that T_r is independent of r and θ. Harlow and Pracht (1972) have used the same equation as (21).

The temperatures $T(r,\theta,t)$ and $T_r(r,\theta,z,t)$ which are the solutions of (20) and (21) must satisfy the following conditions:

$$T_r(r,\theta,z,t_s) = T_\infty \tag{22a}$$

$$T(0,\theta,t) = T_0 \tag{22b}$$

$$T(r,\theta,t) = T_r(r,\theta,0,t) \tag{22c}$$

where T_∞ is the initial temperature (or the far-field tectonic temperature), T_0 is the temperature of the inlet fluid, and t_s is the time at which the outlet is provided.

The solution of (20) and (21) is written as

$$T_r = T_0 + (T_\infty - T_0)\,\mathrm{erf}\left[\frac{1}{\sqrt{t - t_s}}\left\{\sum_{n=0}^{\infty} f_n(r)\cos n\theta + \frac{1}{2}\left(\frac{C_\gamma\rho_\gamma}{\lambda}\right)^{1/2} z\right\}\right] \tag{23}$$

with

$$f_n(0) = 0, \tag{24}$$

where f_n are solutions of

$$a_0 \frac{df_0}{dr} + \frac{1}{2}\sum_{n=1} a_n \frac{df_n}{dr} - \frac{1}{2r}\sum_{n=1} nb_n \frac{df_n}{dr} = kr\left(1 + \frac{r^2}{a^2}\right)$$

$$a_0 \frac{df_1}{dr} + a_1 \frac{df_0}{dr} + \frac{1}{2}\sum_{n=1}\left(a_{n+1}\frac{df_n}{dr} + a_n\frac{df_{n+1}}{dr}\right) - \frac{1}{2r}\sum_{n=1}\left\{nb_{n+1}f_n + (n+1)b_n f_{n+1}\right\}$$

$$= -2k\,\frac{r}{a} \tag{25}$$

$$a_0 \frac{df_p}{dr} + a_p \frac{df_0}{dr} + \frac{1}{2}\sum_{n=1}\left(a_{n+p}\frac{df_n}{dr} + a_n\frac{df_{n+p}}{dr}\right) - \frac{1}{2r}\sum_{n=1}\left\{nb_{n+p}f_n + (n+p)b_n f_{n+p}\right\}$$

$$+ \frac{1}{2}\sum_{n=1}^{p-1} a_n \frac{df_{p-n}}{dr} + \frac{1}{2r}\sum_{n=1}^{p-1}(p-n)b_n f_{p-n} = 0 \qquad (p \geq 2)$$

where

$$k = \frac{2\pi}{C_f q_0}(C_\gamma \rho_\gamma \lambda)^{1/2},$$

$$a_0 = \left(1 - \frac{r^2}{R^2}\right)\left[1 + \frac{\pi R^2}{q_a}\left\{\left(1 + \frac{r^2}{a^2}\right)\left(C_{L0} + C_{L1}p_0\right) + \frac{3}{4}\rho_f\, g\, C_{L1}\frac{r^2}{a}\right\}\right]$$

$$a_1 = r\left(1 - \frac{r^2}{R^2}\right)\left[\frac{a}{R^2}\left(1 - \frac{R^2}{a^2}\right) - \frac{\pi R^2}{aq_a}\left\{2\left(C_{L0} + C_{L1}p_0\right) + \frac{3}{4}\rho_f\, g\, C_{L1} a\left(1 + \frac{r^2}{a^2}\right)\right\}\right]$$

$$a_2 = r^2\left(1 - \frac{r^2}{R^2}\right)\left[\frac{a^2}{R^4}\left(1 - \frac{R^2}{a^2}\right) + \frac{3\pi R^2}{4aq_a}\rho_f\, g\, C_{L1}\right]$$

$$a_m = \frac{r^m a^m}{R^{2m}}\left(1 - \frac{r^2}{R^2}\right)\left(1 - \frac{R^2}{a^2}\right) \qquad (m \geq 3); \tag{26}$$

$$b_0 = 0$$

$$b_1 = -\frac{2r}{a} - \frac{ra}{R^2}\left(1 - \frac{r^2}{R^2}\right)\left(1 - \frac{R^2}{a^2}\right) + \frac{3\pi R^2}{4q_a}\rho_f g C_{L1} r\left(1 - \frac{r^2}{3R^2}\right)\left(1 + \frac{r^2}{a^2}\right)$$

$$b_2 = -\frac{r^2a^2}{R^4}\left(1 - \frac{r^2}{R^2}\right)\left(1 - \frac{R^2}{a^2}\right) - \frac{3\pi R^2}{4aq_a}\rho_f g C_{L1} r^2\left(1 - \frac{r^2}{3R^2}\right)$$

$$b_m = -\frac{r^m a^m}{R^{2m}}\left(1 - \frac{r^2}{R^2}\right)\left(1 - \frac{R^2}{a^2}\right) \qquad (m \geq 3).$$

The heat extraction rate or thermal power output at the outlet is

$$Q_f = q_a C_f (T_a - T_0) \tag{27}$$

where T_a is the fluid temperature at the outlet, or by (23) and (22c)

$$Q_f = q_a C_f (T_\infty - T_0)\operatorname{erf}\left[\frac{1}{\sqrt{t - t_s}} \sum_{n=0}^{\infty} f_n(a)\right]. \tag{28}$$

ILLUSTRATIVE EXAMPLES

The data employed here and in the following are given below:

R_a/R_0	$= 0.5$	C_f	$= 1.0$ cal/gr °C
C_v	$= 1.25$	C_γ	$= 0.25$ cal/gr °C
ρ_f	$= 1.0$ gr/cm^3	λ	$= 6.2 \times 10^{-3}$ cal/cm sec °C
ρ_γ	$= 2.65$ gr/cm^3	T_∞	$= 250$°C
K_a	$= 0.49$	T_0	$= 65$°C
ν	$= 0.25$	α_T	$= 8.0 \times 10^{-6}$/°C
$S_0/\rho_f g h_0$	$= 1.3$	$\pi K_c/\sqrt{2R_0} \cdot S_0$	$= 1.118$.

Furthermore, B_1 is taken as zero since the effect of the pressure on the fluid loss should not be large as discussed by Hall and Dollarhide (1964).

The outlet fluid temperature T_a and the thermal power output Q_f in Case (I) are graphed as functions of time in Figs. 2a and 2b. The corresponding relations in Case (II) are graphed in Figs. 3a and 3b. The effect of the position of the outlet hole is also shown in Figs. 2a and 2b. The outlet flow rate q_a considered here is not necessarily large (Table 1).

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Table 1. Stationary Cracks

Case		h_0 m	q_0 gr/sec	p_a^*/S_0	q_a gr/sec	$\Delta p_0/S_0$	R/R_0
I	1	3000	1.451×10^5	-0.23916	5.305×10^4	0.24009	11500
	2	2000	1.409	-0.24264	5.151	0.24396	5750
II	A	3000	1.289	-0.23852	4.647	0.23923	10000
	B		1.232	-0.23882	4.649	0.23954	
	C		1.162	-0.23916	4.653	0.23988	
	D	2000	1.183	-0.24172	3.819	0.24245	5000
	E		1.127	-0.24215	3.823	0.24288	
	F		1.059	-0.24264	3.827	0.24337	

$\Delta p_0/S_0 \equiv p_0/S_0 - 1/\Delta$

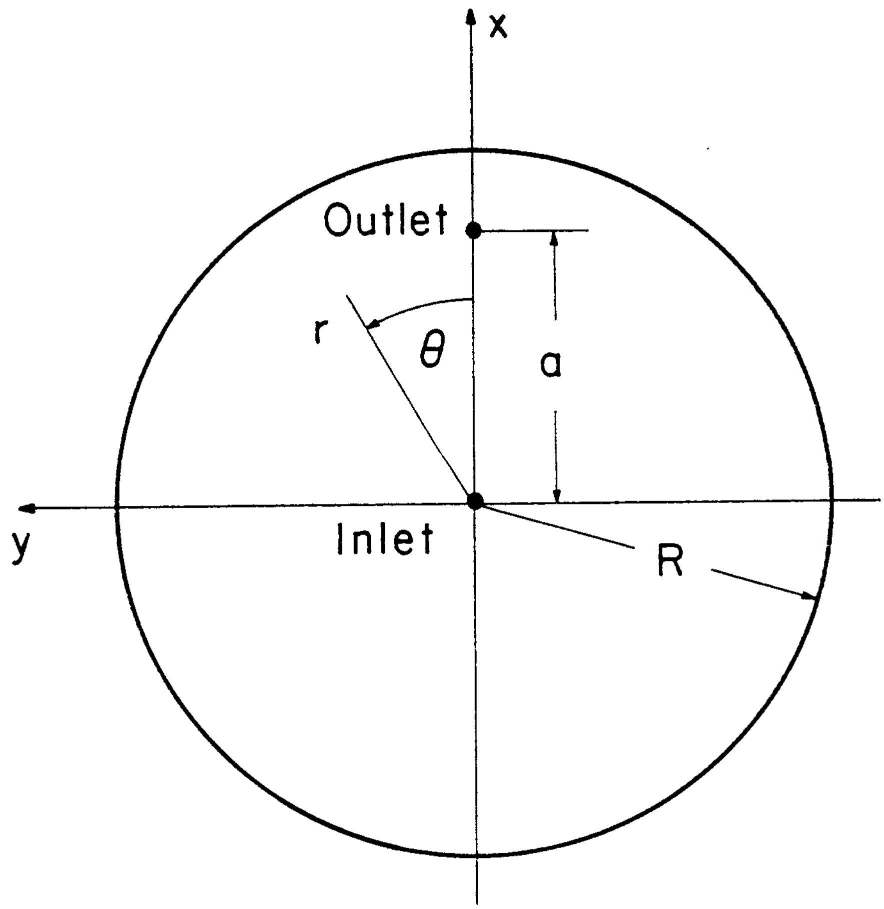

Fig. 1

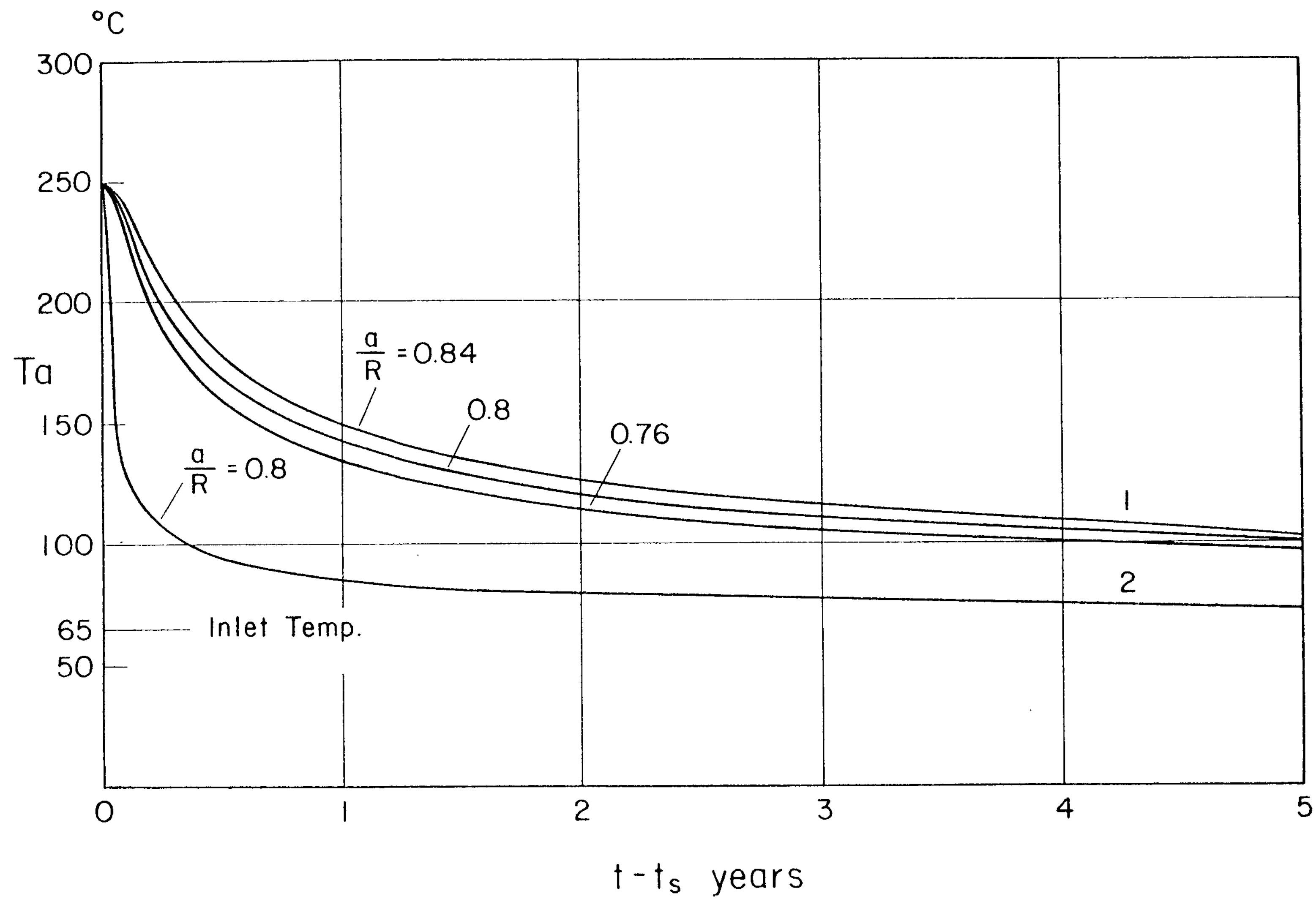
°C
300
250
200
Ta
150
100
65
50
Inlet Temp.
a/R = 0.84
0.8
0.76
a/R = 0.8
1
2
0
1
2
3
4
5
t - ts years

Fig. 2a

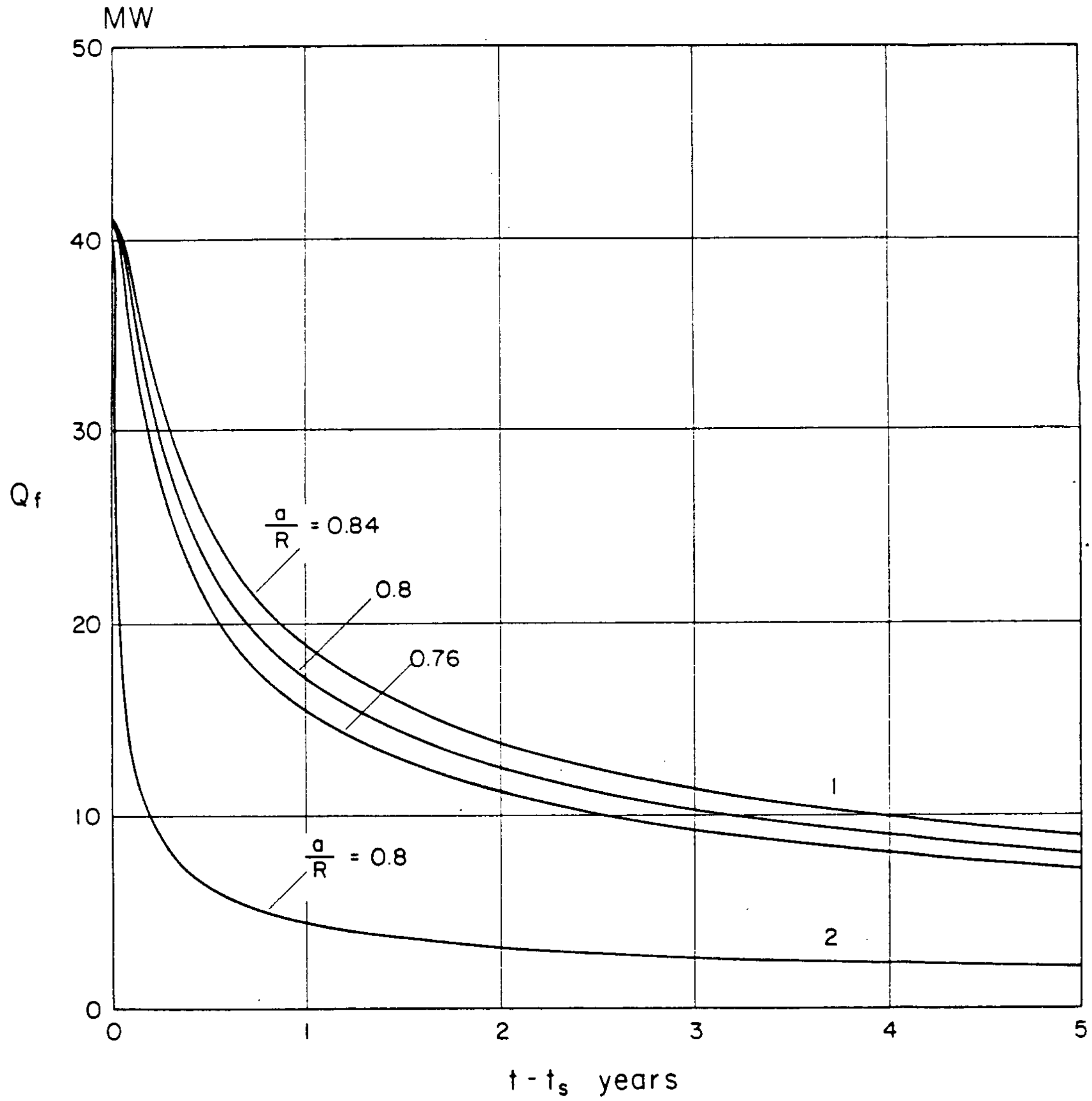
MW
50
40
30
20
10
0
Q_f
$\frac{a}{R}$ = 0.84
0.8
0.76
1
$\frac{a}{R}$ = 0.8
2
0
1
2
3
4
5
$t - t_s$ years

Fig. 2b

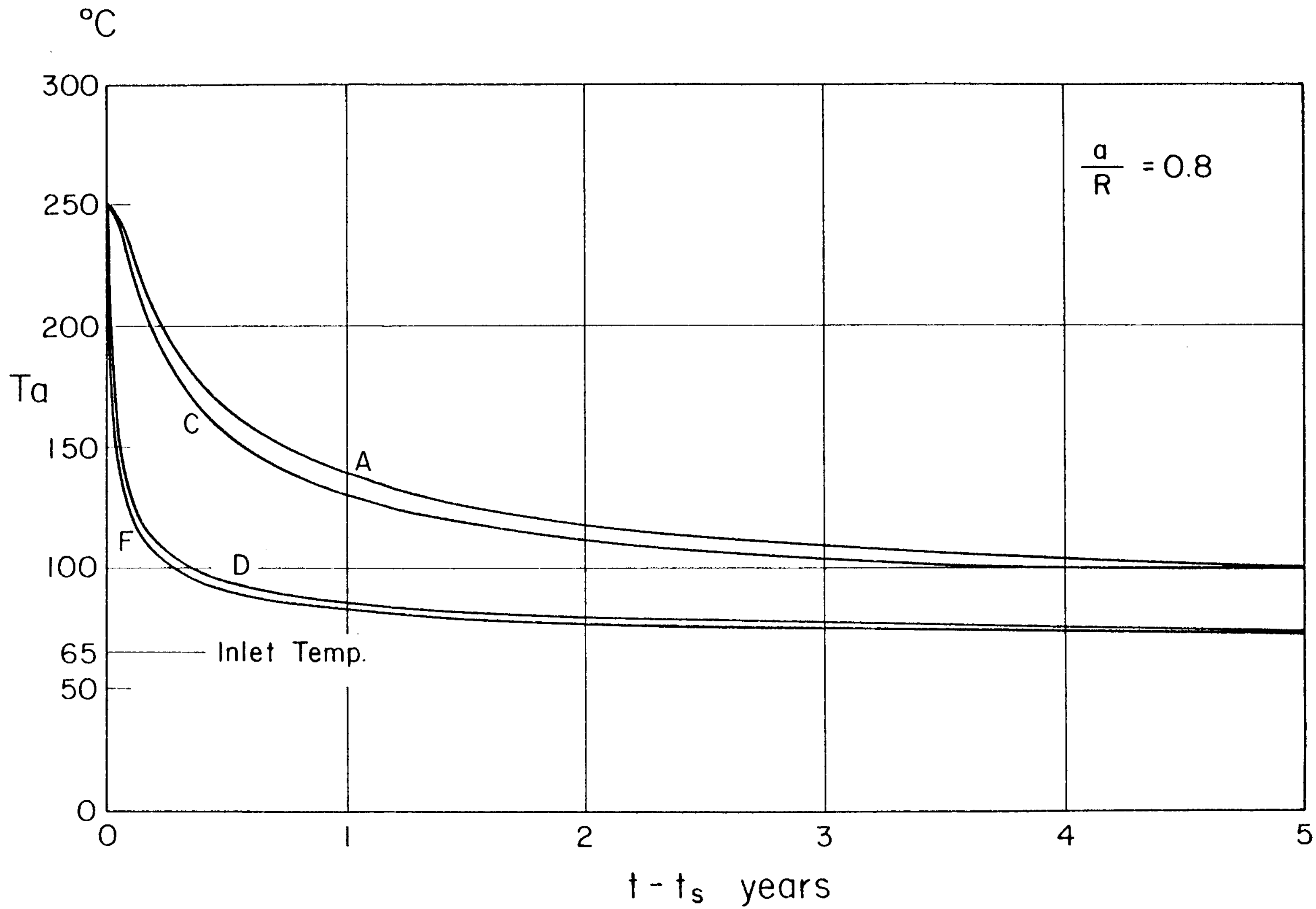
°C
300
250
200
Ta
150
100
65
50
0
a/R = 0.8
A
C
D
F
Inlet Temp.
0
1
2
3
4
5
t - ts years

Fig. 3a

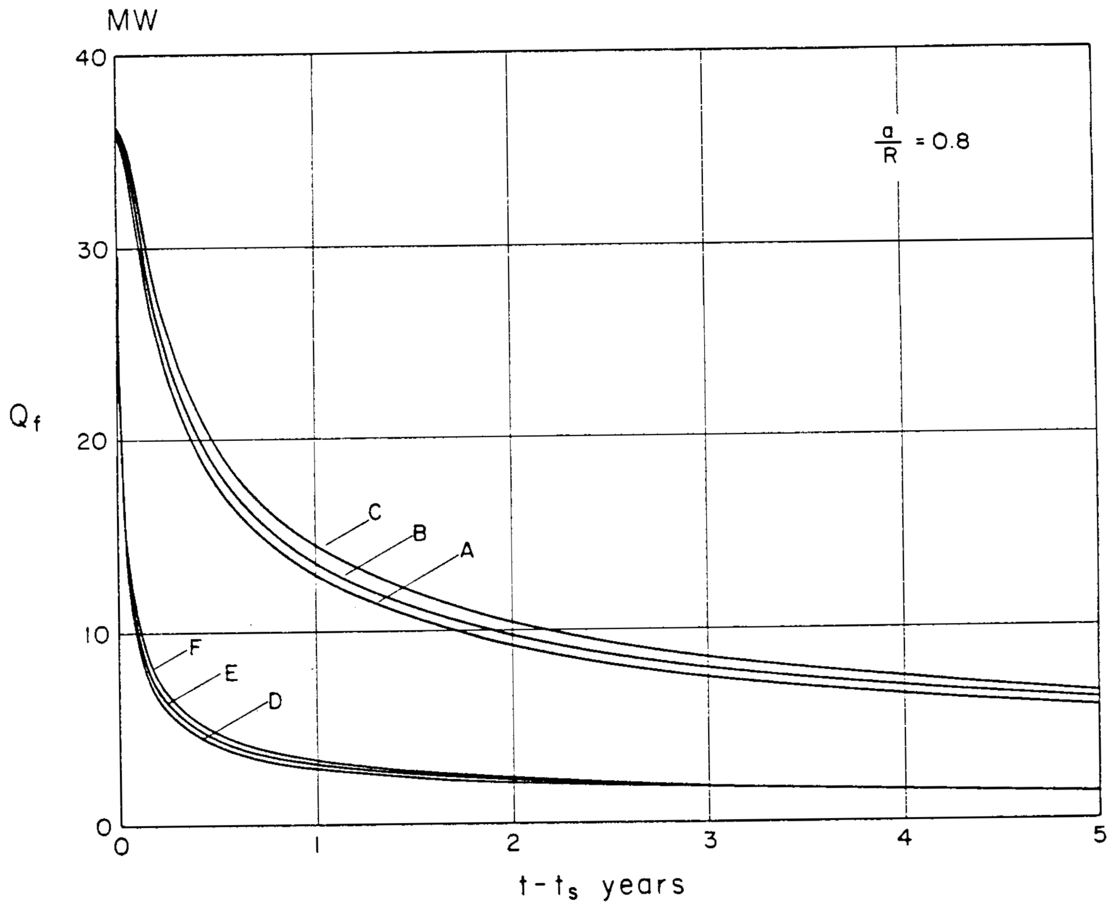

Fig. 3b