A HELE-SHAW MODEL OF HEAT CONVECTION IN POROUS MEDIA UNDER GEOTHERMAL CONDITIONS

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Evidence from New Zealand indicates that geothermal fields occur at reasonably regular intervals of about 15 kilometers apart. Investigators have speculated that these regular intervals may be indications of the scale of the heat convection cells.

Wooding (1975) has described the four well-known approaches to solve the equations of continuity, motion and heat transport for heat convection in porous media. Unfortunately, to date, none of these methods has provided solutions for ail geothermal conditions with a Rayleigh number up to the order of 1000 and also allowed the viscosity to vary. In New Zealand, the permeability is in the order of 3×10^{-11} cm^2, the change of density $\Delta\rho$ is 0.2 gm/cm^3, the length scale L is about 5×10^5 cm, the dynamic viscosity μ is 0.001 to 0.01 poise (depending on the temperature of the fluid) and the thermal diffusivity k is 0.003 cm^2/sec. The range of Rayleigh number would be from 100- 1000.

The Stuart-Watson method has been applied to small viscosity change and low Rayleigh number ($R \sim R_c$ the critical value for neutral stability). For high Rayleigh number the method is not applicable. The Galerkin method has been applied to two-dimensional constant viscosity cases. The variational method has been applied to three-dimensional constant viscosity cases with $R \sim 500$. The methods of numerical solution have been applied to both constant and variable viscosity cases. Due to the great cost involved in computer solution, a relatively large mesh was used at $R = 500$. The accuracy of these results is questionable.

Since none of these above-mentioned equations can provide solutions for variable viscosity in high Rayleigh number regions, physical models are particularly attractive at this stage. If properly designed, physical models should provide (1) approximate final solutions; (2) verification of numerical and analytical solutions; (3) a solution obtained from keeping variable-viscosity terms in the equation versus a solution obtained from assuming constant viscosity in the equation; and (4) change of solution forms with increased Rayleigh number.

Analysis

Two-dimensional experiments cannot fully represent the three-dimensional situation of a geothermal region, but they are useful to demonstrate the influence of large variations of viscosity on the form of convecting plumes of hot water.

For two-dimensional convection in the (x,z) plane, the usual stream-function representation

$$\underset{\sim}{u} = \left(\frac{\partial\psi}{\partial z}\, ,\, -\frac{\partial\psi}{\partial x}\right) \qquad (1)$$

gives, after eliminating pressure from Darcy's law,

$$\frac{\partial}{\partial x}\left(a\,\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial z}\left(a\,\frac{\partial\psi}{\partial z}\right) = \frac{\partial\theta}{\partial x} \qquad (2)$$

where θ is a dimensionless fluid density, and $a = (\mu/k)/(\mu/k)_o$ is a dimensionless viscosity parameter. The suffix o refers to the lowest temperature in the system. Typically, a will vary from 1 at the cold end of the range to about 0.1 at the hot end--an order-of-magnitude change.

A further equation describes heat or mass transport

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\tau}{\partial x}\frac{\partial\psi}{\partial z} - \frac{\partial\theta}{\partial z}\frac{\partial\psi}{\partial x} = \frac{1}{R}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2}\right) \qquad (3)$$

where

$$R = \frac{k_o g\ \Delta\rho\ L}{\mu_o \kappa} \qquad (4)$$

is the Rayleigh number. Also, in (3) τ is a dimensionless time defined by

$$\tau = \frac{kg\Delta\rho \cdot t}{\mu_o E L} \qquad (5)$$

The parameters in (4) and (5) are defined in terms of the problem under study. k_o is the permeability of the medium, g is gravity, $\Delta\rho$ is the density difference between hot and cold, μ_o is the viscosity of the cold fluid, κ is the diffusivity of the density-controlling property, and L is a length scale--typically the depth in the case of a saturated porous layer in which convection is taking place, $E = \{(1-n)\, c_s\rho_s + nc\rho\}/c\rho$ is the ratio of the heat capacity of the medium to that of the fluid. The suffix s refers to the medium.

Two-dimensional systems governed by the above equations may be solved, for example, by physical modelling or by computer methods.

Description of Hele-Shaw Model

A typical physical model of two-dimensional convection is shown in Fig. 1 below. A Hele-Shaw cell is formed from two strips of 1/4-inch thick polished plate glass, separated by strips of waterproof adhesive tape. In the example shown, a cavity 20 cm x 1 cm is formed in the (x,z) plane. The thickness of the cavity is very small so that a slow viscous flow occurs between the plates. The flow averaged between the plates is analogous to two-dimensional flow in a porous medium of permeability $k = d^2/12\ cm^2$, where d is the plate spacing.

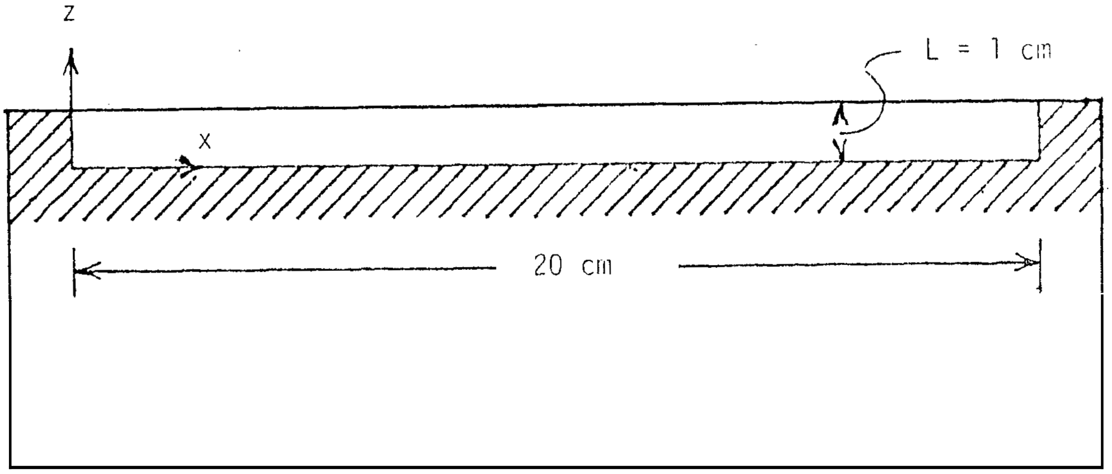

Figure 1. Plan View of Hele-Shaw Cell

The cell is mounted with the x-axis horizontal, and with the z-axis at an angle α to the horizontal so that the effective gravity component is $g \sin \alpha$. This is useful for adjusting the Rayleigh number R to a desired value.

In the case shown, the cell is filled with a fluid of density ρ_1 and viscosity μ_1 (say) which represents a geothermally heated fluid. The cell is immersed in a transparent tank filled with fluid of density ρ_o and viscosity μ_o, representing cold groundwater.

Since the width of the cell is very large compared with L, the sidewall boundary conditions will be ignored. The following initial and boundary conditions are taken to apply:

$$w = 0, \ \theta = 0 \quad (t = 0, \ 0 \leq z \leq 1) \tag{6a}$$

$$w = \varepsilon(x), \ \theta = 1 \quad (t = 0, \ z = 1) \tag{6b}$$

where w is vertica velocity, $\varepsilon(x)$ is a small noise signa ,

$$w = 0, \ \frac{\partial \theta}{\partial z} = 0 \quad (t > 0, \ z = 0) \tag{7a}$$

(insulating impermeable boundary condition)

$$\frac{\partial w}{\partial z} = 0, \ \theta = 1 \quad (t > 0, \ z = 1) \tag{7b}$$

(constant pressure, constant density, boundary condition).

These conditions could correspond to a geothermal field situation where a period of volcanicity has injected a large amount of heat, in the form of magmatic steam, into a deep groundwater aquifer, so that the whole aquifer is initially very hot. After the volcanicity has quieted down, cold surface water intrudes from above, displacing the hot water which appears at geothermal areas.

The experiment is aimed

(1) To model the deep groundwater motion (at least in two dimensions) which is not accessible to geophysical observation at the present time,

(2) To provide an experimental relationship between the heat flux out of the region (expressed as the Nusselt number) and the Rayleigh number.

It is necessary that the fluid initially in the cell be marked with a dye so that convective motions may be photographed and studied. This colored fluid can also be used for photometric work--to determine how much of the initial fluid has been displaced from the cell and so arrive at estimates for the Nusselt number, at various values of the Rayleigh number.

Field values of parameters based upon measurements in the Taupo Volcanic Zone, particularly Wairakei geothermal field, give the following approximate results:

Permeability k (vertical) = 3×10^{-11} cm^2

Density difference $\Delta\rho$ = 0.2 gm/cm^3

Vertical scale L = 5 Km

Cold water viscosity μ_o = 10^{-2} poise

Thermal diffusivity κ = 3×10^{-3} cm^2/sec

$$R = \frac{kg\Delta\rho L}{\mu_o \kappa} \simeq \frac{3 \times 10^{-11} \times 10^3 \times 0.2 \times 5 \times 10^5}{10^{-2} \times 3 \times 10^{-3}} = 100$$

For the time constant based upon cold water viscosity, $E \approx 1$, and $\frac{1}{R}\frac{EL^2}{\kappa} \approx 10^{12}$ seconds or roughly 30,000 years. However, since the hot water viscosity $\mu_1 \approx 0.1\ \mu_o$, the Rayleigh number based on this value would be roughly 1000, and the time constant would be reduced to the order of 3000 years. It seems likely that the flow pattern would exhibit some properties of both high and low Rayleigh number.

In two recent experimental runs using sucrose solutions in the Hele-Shaw cell, the parameters were approximately as follows:

	Run No. 3	Run No. 4
L	1 cm	1 cm
k	2.63×10^{-5} cm^2	2.63×10^{-5} cm^2
ρ_o	1.28 gm/cm^3	1.28 gm/cm^3
μ_o	0.5 poise	0.5 poise
ρ_1	1.16 gm/cm^3	1.225 gm/cm^3
μ_1	0.05 poise	0.167 poise
μ_o/μ_1	10	3
κ	0.3×10^{-5} cm^2/sec	0.3×10^{-5} cm^2/sec
$\Delta\rho$	0.12 gm/cm^3	0.055 gm/cm^3
α	6°.5	3°.5
R	250	50
E	1	1
$\frac{1}{R}\frac{EL^2}{\kappa}$	0.37 hour	1.5 hours
$\tau(max)$	74	49

The second-last entries in this table are the cold-water time scales, while the final entries, $\tau(max)$, are the maximum values of dimensionless time achieved experimentally.

It is interesting to note that $\tau = 50$ corresponds to an elapsed time of 1.5 million years on the prototype cold-water time scale, or 150,000 years on the hot-water time scale. These figures bracket the estimated life of the geothermal activity in the Taupo Volcan'ic Zone. It follows that the hypothesis of a phase of initial volcanism is not contradicted by experiment, which shows that plumes of hot water art: still present at $\tau = 50$.

As a supplement to Run No. 4, at $R = 50$ and $\mu_o/\mu_1 = 3$, a computer run was made using a program which solves the convection equations by finite-difference methods. This program is due principally to Dr. M. J. O'Sullivan, of the University of Auckland, but has been modified to include a modest variation of viscosity and also to compute transport of dissolved salt. The development *of* convective plumes in the computer run, for convection in a rectangle of width:height ratio 4, is qualitatively similar to that observed in the physical model. However, there has not been time to make detailed comparisons, especially of Nusselt numbers.

The entire study is based on the assumption of homogeneous geological conditions which is normally not valid. By varying the size of the gap between the plates and by placing sources and sinks in the flow, different geological conditions may be simulated. This is planned for future studies.

Acknowledgment

This model was suggested to the author by Dr. Robin Wooding, and we are jointly conducting this research project.

Reference

Wooding, R. A., 1975. Methods of solution of the equations for convection in porous media, with geothermal applications. Presented at the Stanford Workshop on Geothermal Reservoir Engineering and Well Stimulation, Stanford, California, Dec. 15-17.