GEOTHERMAL ENERGY FROM A BOREHOLE IN HOT DRY ROCK - A PRELIMINARY STUDY

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A simple procedure for calculating heat transfer between circulated water and hot dry rock in a single, concentric-annulus geothermal well is presented. Also presented are the results of an application of the procedure to a proposed well in Sweden.

The concept examined consists essentially of pumping water, at naturally-occurring temperatures, down a cylindrical well drilled several kilometers into the earth, and extracting it in a heated state through an annular cylinder (see for example, Smith et al., 1973). Two different schemes embodying the same concept are examined in the present investigation. These are illustrated in Figs. 1 and 2. In the first, cold water is pumped into the inner cylinder and extracted through the annular space. In the second, the situation is exactly reversed.

The problem, in both schemes, is to predict the outlet temperature of water under given conditions on the one hand; and on the other, to predict the temperature drop in the rock mass surrounding the well, otherwise known as energy depletion of the geothermal reservoir. (See Gringarten et al., (1975) for analytical solutions to a plane-crack situation embodying a similar concept.) Such prediction was undertaken in two stages.

Methodology Adopted

In order to render the problem tractable to simple mathematical analysis in Stage 1, certain simplifying assumptions are made. By virtue of these assumptions, the process of thermal energy gain by the flowing water was decoupled from that of energy loss by the surrounding rock mass. Then, an ordinary differential equation was formulated to represent the energy-gain process, and solved analytically with specified rock temperatures as boundary condition. A series of such solutions was obtained and the influence of significant parameters was investigated. During this stage, only a rough estimate of rock-mass energy depletion was made. This estimate indicated that such a depletion would be minimal.

In Stage 2, the coupled, and essentially unsteady-state, processes were modelled by partial-differential equations, which were solved by an integrated, finite-difference technique. A computer program embodying this technique was used to investigate the influence of significant parameters. Such investigations are continuing and the results presented here are preliminary.

Mathematical Details

1. Simplifying Assumptions

The following assumptions are made, in both Stage 1 and Stage 2 calculations, concerning the process of thermal energy gain by circulating water:

a) The flow is fully-developed, i.e., essentially one-dimensional in nature, both within the inner tube and in the annular space.

b) The flow is turbulent.

c) Within the temperature range encountered, the circulating-water properties remain essentially constant.

d) The rock-mass temperature varies linearly with depth below the earth's surface.

2. Stage 1 Calculations

The ordinary differential equation governing the steady-state energy gain by circulating water in Scheme I is

$$\frac{dT_W}{dx} + \left\{\frac{hP}{\dot{m}C_W}\right\} T_W = \left\{\frac{hP}{\dot{m}C_W}\right\}(T_{R,L} - ax) \tag{1}$$

and, in Scheme II is

$$\frac{dT_W}{d\xi} + \left\{\frac{hP}{\dot{m}C_W}\right\} T_W = \left\{\frac{hP}{\dot{m}C_W}\right\}(T_{R,O} + a\xi) \tag{2}$$

The symbols in the above equations are defined in Fig. 3, and in the nomenclature'. The solution to Equation (1) is (Sharma, 1975)

$$T_W = T_{W,O}\, e^{-\left\{\frac{hP}{\dot{m}C_W}\right\}x} - ax + \left[T_{R,L} + a\left\{\frac{\dot{m}C_W}{hP}\right\}\right]\left(1 - e^{-\left\{\frac{hP}{\dot{m}C_W}\right\}x}\right) \tag{3}$$

and to Equation (2) is

$$T_W = T_{W,O}\, e^{-\left\{\frac{hP}{\dot{m}C_W}\right\}\xi} + a\xi + \left[T_{R,O} + a\left\{\frac{\dot{m}C_W}{hP}\right\}\right]\left(1 - e^{-\left\{\frac{hP}{\dot{m}C_W}\right\}\xi}\right) \tag{4}$$

These solutions, obtained w th known values of rock temperature, are represented in Figs. 4 and 5. n order to make these representations, the values of heat-transfer coeffic ent h were extracted from the work of Kays and Leung (1963). Furthermore a borehole of 4 in. was presumed and a thermal gradient "α" of 0.034.

It can be observed from Fig. 5 that Scheme II is preferable to Scheme I. However, the influence of circulating water flow in cooling the adjacent rock and hence depleting the geothermal energy source cannot be observed by the decoupled technique. In order to calculate this depletion, it is necessary to perform calculation of the coupled processes.

3. Stage 2 Calculations

The Stage 2 calcu ations involved the solution of the coup ed problem. This problem is expressed mathematically thus:

Circulating-water energy gain

$$\rho_W C_W \frac{\partial T_W}{\partial t} + \rho_W C_W U_W \frac{\partial T_W}{\partial \xi} = \frac{2h}{R_I}(T_R - T_W) \quad (5)$$

with the initial and boundary conditions:

$$\begin{aligned} T_W &= T_{W,0} \quad , \quad 0 < \xi < L \; ; \quad t < 0 \\ &= T_{W,0} \quad , \quad \xi = 0 \quad , \quad t \geq 0 \end{aligned} \quad (6)$$

Rock-rnass energy loss

$$\rho_R C_R \frac{\partial T_R}{\partial t} = \frac{\partial}{\partial \xi}\left(k_R \frac{\partial T_R}{\partial \xi}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(k_R r \frac{\partial T_R}{\partial r}\right) \quad (7)$$

with the initial and boundary conditions:

$$T_R \; (\xi) \qquad T_{R,0} + a\xi, \quad \left. \begin{array}{l} 0 < \xi < L \\ R_I < r < R_0 \end{array} \right\} ; \; t < 0$$

$$\left. \begin{array}{l} T_R = T_{R,0} \\ \\ {}_R(\xi) \qquad r = R_0 \end{array} \right\} \quad \geq 0$$

$$ {}_R \frac{\partial T_R}{\partial r} \qquad h(T_W - T_R), \quad r = R_I, \quad t \geq 0 \tag{8}$$

The coupled solutions to Equations *(5)* and (7) with boundary conditions (6) and (8) were obtained with an integrated finite-difference procedure described by Sharma (1975). Investigations in this connection are still continuing. However, a result of the application of the procedure to Scheme II (Fig. *2)* is illustrated in Fig. 6. This result appears to indicate that the rock-face temperature drops more rapidly due to the circulating-water energy gain, than the replenishment possible due to heat conduction from the surrounding rock mass. It is clear that a simple borehole type of approach is not sufficient for extracting geothermal energy in meaningful quantities. It is concluded from the present study that either explosives or hydraulic fracturing will have to be used in order to provide sufficient contact area for effective energy extraction. In countries such as Sweden where abnormally high horizontal stresses exist, it is likely that horizontal fractures can be created, thereby providing a vast surface area of contact at constant high temperature (this is clearly an advantage over other parts of the world where vertical fractures are more common). We are currently studying the problem of geothermal energy gain from circulating water flow through arbitrarily-shaped fractures. This study is based upon a mathematical model involving the numerical solution of partial differential equations governing convective heat and mass transfer. In a continuing study, account will be taken of phase change, dissolution, turbulence and other influences on thermal energy transfer.

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Nomenclature

a	-	geothermal gradient
c_R , c_W	-	specific heat capacity of rock mass and circulating water respectively
h	-	water-rock surface heat-exchange coefficient
L	-	depth of borehole
$\dot{m}$	-	circulating water mass flow rate
P	-	perimeter of borehole
r	-	radius coordinate
R_I , R_O	-	radii of borehole and rock mass considered respectively
t	-	time coordinate
U_W	-	velocity of circulating water
ξ	-	depth coordinate
ρ	-	density
X	-	inverse depth coordinate

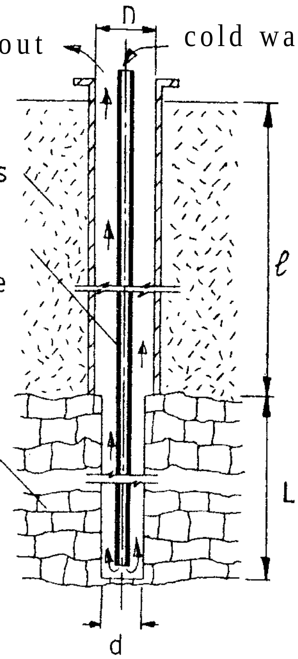


Fig. 1 Illustration of geothermal well; Scheme I.

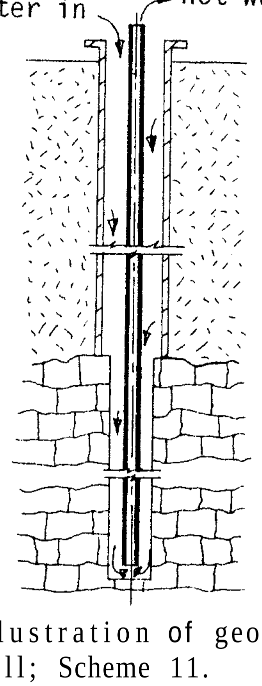
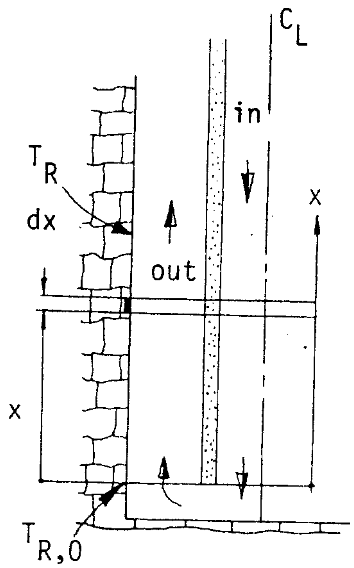


Fig. 2 Illustration of geothermal well; Scheme 11.



(a)

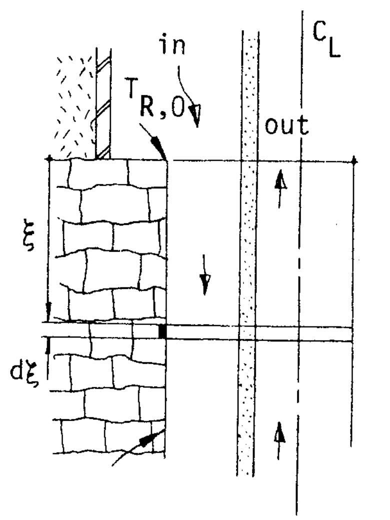


Fig. 3 Illustration *of* coordinate system.

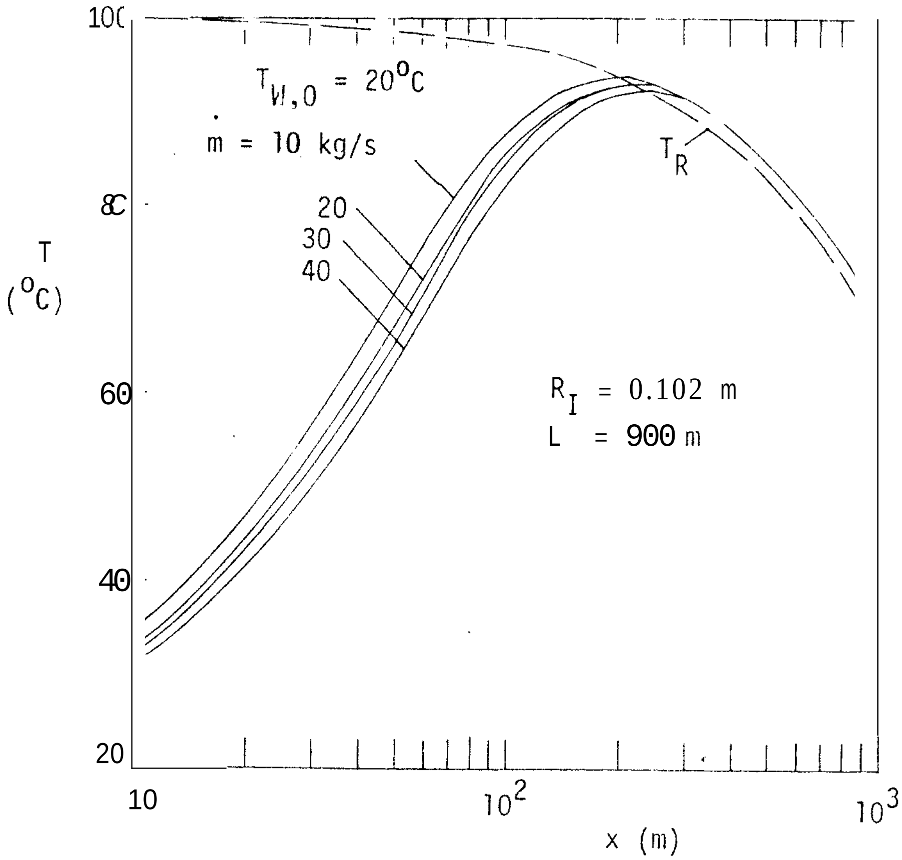


Fig. 4 Effect of flowrate on temperature rise of circulating water, Scheme I.

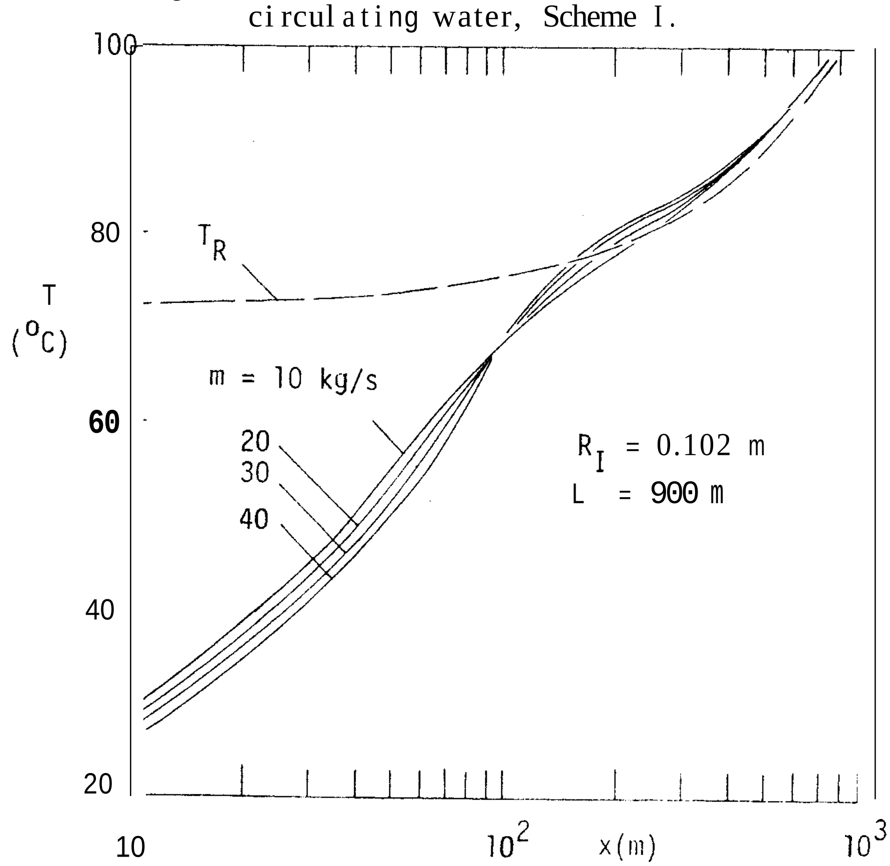


Fig. 5 Effect of flowrate on temperature rise of circulating water, Scheme 11.

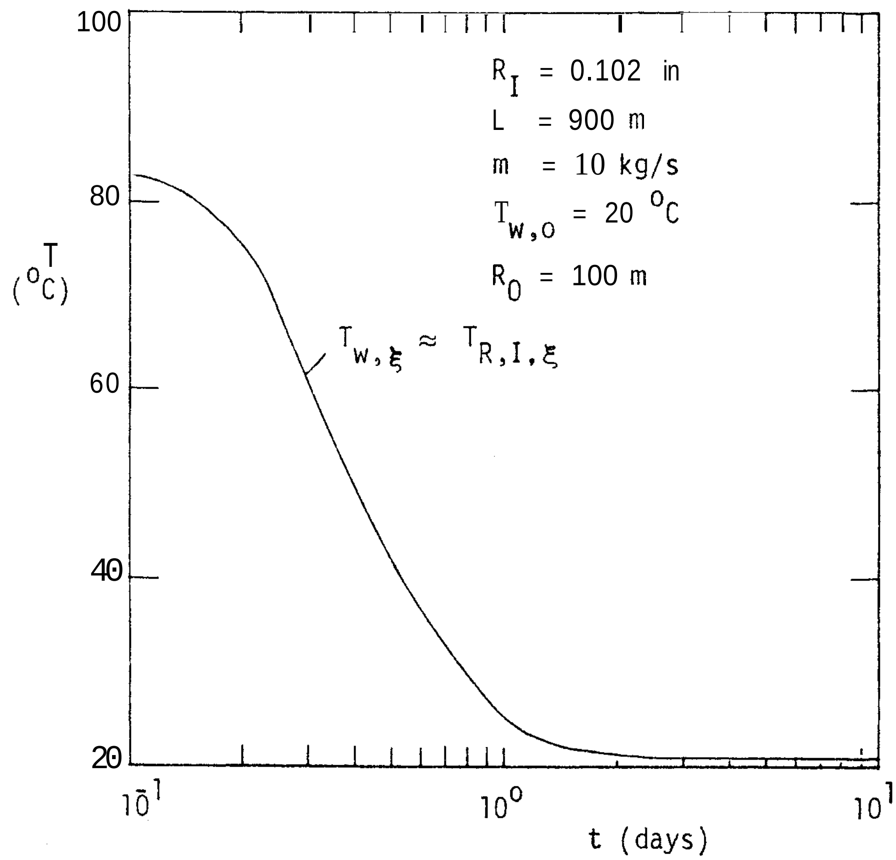


Fig. 6 Outlet temperature of circulating water; coupled problem.