ON THE OPTIMAL RATE OF GEOTHERMAL ENERGY EXTRACTION

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A geothermal reservoir is, among other things, a stock of heat energy of a given "quality," stored in an aquifer system. In this study, the stock is considered finite and exhaustible over the relevant economic horizon. An important consideration in xploiting this resource is the "optimal" rate at which the energy stock should be extracted from a particular geothermal anomaly. This is primari y an "economic" quest ion, although any meaningful conclusions on an optimal extraction policy must surely be based on a specific model of the physica hydrothermal processes that occur In the geothermal aquifer. Accordingly, the purpose of this paper is to outline some economic models for optimal extraction, in the context of a particular hydrothermal model.

The discussion focuses on one anomaly. The rate of hydraulic pumping is the major decision variable, and the analysis trades off discounted "value" of energy from the anomaly against the rate of deterioration of the quality (temperature) of the heat stock. All extracted fluid is recycled, and no divergence between private and social benefits and costs is assumed. Secondary or indirect regional benefits and costs associated with development are not considered.

Two economic models are developed. The first is "quasi-steady-state," in that the flow rate, Q, of fluid extraction is constant over all relevant time, although the temperature, T_o, of the extracted fluid varies with time. The second is completely "non-steady-state," assuming both Q and T_o vary with time. A major objective of these notes is to state, as clearly as possible, *the* major assumption of these economic models, so the appropriateness of hydrothermal model selection can be evaluated by physical model researchers.

The Hydrothermal Model

The hydrothermal model adopted for this discussion was developed by Gringarten and Sauty. It assumes a pumped production well for a single phase (hot water) geothermal anomaly with a recharge well as shown in Fig. 1 (actually each well could represent a cluster of wells).

Fluid is withdrawn at the rate Q (cfs) and recharged at the same rate. The temperature of extracted fluid at time **t** is T_o^t, and recharged fluid enters the ground at temperature T_i in period **t**. T_i is the temperature of condensed exhausted steam (on the cool side of the turbine). It is determined by turbine design and does not vary with time, When the temperature of the aquifer matrix has dropped to T_i, no more energy may be extracted.

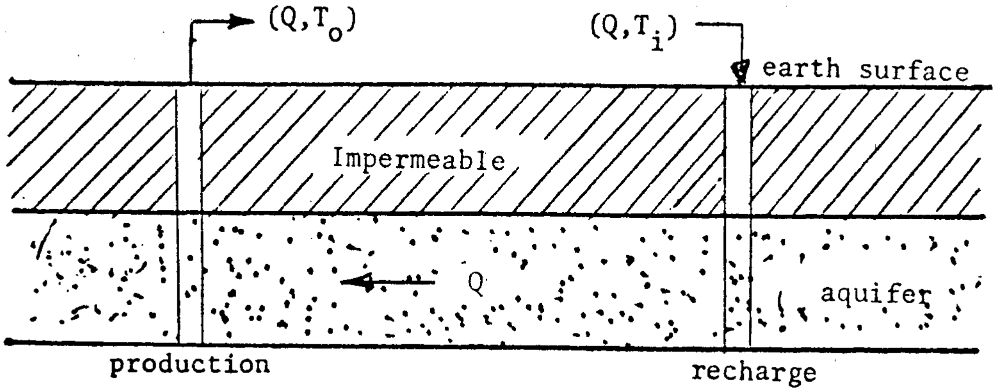

Figure 1

The recirculated fluid is heated by the aquifer matrix from T_i to T_o^t. For the first τ years, $(0 \leq t \leq \tau)$, $T_o^t = T_o^o$, where T_o^o is the initial equilibrium temperature of the unexploited anomaly. The magnitude of τ is inversely proportional to Q:

$$\tau = f(1/Q).$$

The symbol τ denotes time until reduced fluid temperature "breaks through" to the production well.

After the τ^{th} year, T_o^t drops exponentially toward T_i at a rate $g(Q)$, as shown in Fig. 2. In general, T_o^t can be written:

$$T_o^t = \begin{Bmatrix} T_o^o & 0 \leq t \leq \tau \\ T_i + (T_o^o - T_i)e^{-g(Q)(t-\tau)} & \tau < t \end{Bmatrix}$$

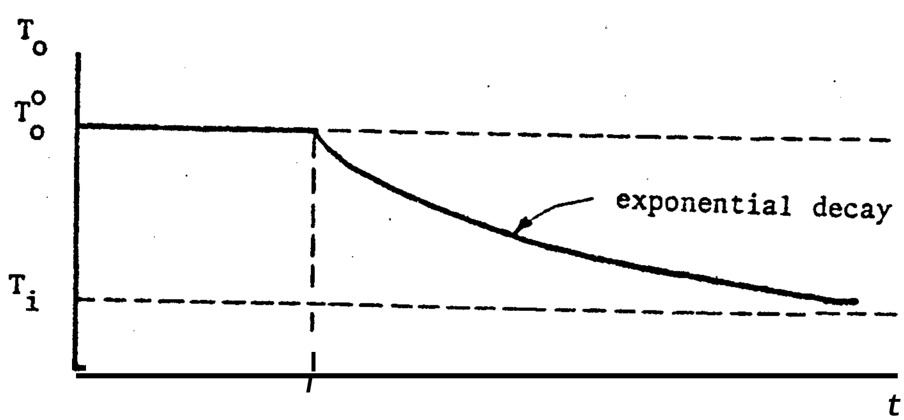

Figure 2

The functions f and g are derived using results of the hydrothermal model.

The system is assumed base loaded into a local power grid. The base load energy, E(kwhr), available from the process is proportional to the product of Q and $(T_o^t - T_i)^{\frac{1}{2}}$: $E^t = h(Q(T_o^t - T_i)^{\frac{1}{2}})$

The value of E is determined by the "long-run" value to the grid of a kwhr of base load energy ("long-run" implies grid power system capacity as well as operating costs). This value is the price the power company is just willing to pay for marginal units of baseload power. If we denote this value as a function of time, p(t), we have:

$$p(t) \sim p^o e^{rt} ,$$

where

p^o = price at t = 0.

r = rate of increase of baseload power system costs relative to general price level (in other words, we are dealing in "real" dollars throughout all time).

The costs associated with extraction depend on Q, T_o^o and T_i. Capacity (investment) costs will be incurred for drilling, lining of both holes, piping, pumps, and turbine-generator equipment. After L years, salvage costs are zero, where L equals life of equipment. Operating costs will depend on Q and an downhole pressure, which is related to T_i^t. Rate Q will determine pump capacity and turbine costs, while pressure will affect pumping energy requirements. Let $C(Q, T_o^o, T_i)$ be present worth of all capacity costs (which are incurred at t = 0) ~~and~~ operating costs. The discount rate will be i, and we say a (real) dollar at time t = j has present worth of e^{-ij} at time t = 0.

The "Quasi-Steady-State" Model

The "Quasi-steady-state" model assumes Q is constant over a finite horizon, N, where N is an integer multiple, k, of L, the life of turbine-generator equipment. For the case where k=1, $\pi_1(Q)$, the total discounted net revenue from the system over the first L years, when pumping occurs at rate Q, may be written as:

$$\pi(Q) = \int_0^{\tau=f(\frac{1}{Q})} p(t)h\left(Q \cdot (T_o^o - T_i)^{\frac{1}{2}}\right)e^{-it}\,dt$$

$$+ \int_{\tau=f(\frac{1}{Q})}^{L} p(t)h\left(Q \cdot (T_o^o - T_i)^{\frac{1}{2}} e^{-g(Q)(t-\tau)}\right)e^{-it}\,dt$$

$$- C(Q,T_o^o, T_i).$$

When the integrals are evaluated, the resulting function can be optimized over the pertinent range of Q:

$$\pi_1^*(Q) = \max_Q \pi_1(Q)$$

$$\text{s.t.} \quad Q \geq 0.$$

In general, we can repeat this optimization for $k=2,3,\ldots,$ obtaining $\pi_2^*(Q_2^*)$, $\pi_3^*(Q_3^*)$, etc. Then for some k^*,

$$\pi_{k^*}^*(Q_{k^*}^*) \geq \pi_\ell^*(Q_\ell^*) \ , \qquad \ell = 1,2,3,\ldots,$$

and $N^* + (k^*)L$ is the optimal horizon.

Although this "quasi-steady-state" approach considers horizons of indefinite length, it is somewhat restricted, in that Q is assumed constant for all time. A more flexible approach would allow Q to vary from year to year.

The Non-Steady-State Model

If the restriction on constant Q is relaxed, an investment timing dimension is added to the economic model. An extraction policy is then defined in terms of a vector of pumping rates:

$$\bar{Q} = \{Q_1, Q_2, \ldots, Q_N\} \ ,$$

where Q_t is the pumping rate in the t^{th} year. An optimal policy, $\bar{Q}^*$, is a policy that maximizes the pertinent objective function, namely, discounted net revenues. We are now considering the optimal "staged" development of an anomaly.

The same hydrothermal model is assumed. However, the big difference is that the fluid pumping rate, Q, can be increased in any year (at some incremental investment and operating cost). The goal now is to find not one Q, but a set of Q's, an investment-pumping policy that maximizes discounted net revenues.

To do this we define system state variables, Q_N and T_o^N.

Let:

Q_N = fluid pumping rate from extraction well just before beginning of period N.

T_o^N = temperature of extracted fluid just before beginning of period N.

$V_N^*(Q_N, T_o^N)$ = The optimal "value" of being in state (Q_N, T_o^N) at beginning of period N. This is the present worth (as of beginning of period N) of sum of net revenues in period N, N+1, N+2, ..., assuming an optimum policy is followed; that is, the sume of these net revenues discounted to beginning of period N is equal to $V_N^*(Q_N, T_o^N)$.

Now suppose that $N = 100$ and the discount rate is large enough so that the present worth in year zero of $V^*_{N+1}(Q_{N+1}, T_o^{N+1})$ is zero. Then this is tantamount to saying $V^*_{N+1}(Q_{N+1}, T_o^N) \equiv 0$. This implies that the present worth at time $t = 0$ of value of energy from this anomaly after N years, is zero regardless of the value of Q_{N+1} and T_o^N. This effectively defines a horizon of "economic" relevance.

Let:

$R_N(Q_N + \Delta Q_N, T_o^N$ = revenues in year N from pumping (and selling power) at rate $Q_N + \Delta Q_N$ and temperature T_o^N.

$I_N(\Delta Q_N, T_o^N)$ = capital investment in year N to increase pumping rate by ΔQ_N, assuming temperature during that period is T_o^N.. Of course it is not likely that an optimal policy would include a capacity investment in the last year. Nevertheless, this option is available in this year, as in all the other N-1 years. This investment cost would also cover incremental power transmission costs.

$C_N(Q_N + \Delta Q_N, T_o^N)$ = operating cost during year N associated with producing at rate Q + ΔQ_N and temperature T_o^N.

T_o^{N+1} = $\phi(Q_N + \Delta Q_N, T_o^N)$, where $\phi(\cdot,\cdot)$ is a functional expression relating Q_N, ΔQ_N and T_o^N to T_o^{N+1}. This "transfer" function reflects the parameters of a non-steady-state hydrothermal model. Perhaps Professor Witherspoon's hydraulically steady state hydrothermal model could be used to estimate pertinent values of $Q(\cdot,\cdot)$.

Then we have:

$$V^*_N(Q_N, T_o^N) = \max_{\Delta Q_N} \Big\{ R_N(Q_N + \Delta Q_N, T_o^N) - I_N(\Delta Q_N, T$$

$$- C_N(Q_N + \Delta Q_N, T_o^N) + \alpha V_{N+1}(Q_N + \Delta Q_N, T_o^{N+1}) \Big\} ,$$

$$\text{s.t.} \quad T_o^{N+1} = \phi(Q_N + \Delta Q_N, T_o^N) ,$$

where:

$$a = 1/(1+i)$$

However, since $V_{N+1}(Q_{N+1}, T_o^{N+1}) \simeq 0$,

$$V^*_N(Q_N, T_o^N) = \max_{\Delta Q_N} \Big\{ R_N(Q_N + \Delta Q_N, T_o^N)$$

$$_N(\Delta Q_N, T_o^{N}) \qquad _N(Q_N + \Delta Q_N, T_o^{N}) \Big|$$

Basically, this says the best value of the system in state (Q_N, T_o^N) at the beginning of period N, can be found by maximizing the expression in braces over all values of ΔQ_N. The value of ΔQ_N that maximizes will be ΔQ_N^*. Most likely ΔQ_N^* will be zero for this last period.

We find ΔQ_N^* for each pertinent value of Q_N, T_o^N, and then move back to the beginning of period N-1, writing:

$$V_{N-1}^*(Q_{N-1}, T_o^{N-1}) = \max_{\Delta Q_{N-1}} \Big\{ R_N(Q_{N-1} + \Delta Q_{N-1}, T_o^{N-1}) - I_{N-1}(\Delta Q_{N-1}, T_o^{N-1}) - C_{N-1}(Q_{N-1} + \Delta Q_{N-1}, T_o^{N-1}) + \alpha V_N^*(Q_{N-1}+\Delta Q_{N-1}, \phi(Q_{N-1}+\Delta Q_{N-1}, T_o^{N-1}) \Big\} .$$

This is the typical two-stage optimization problem. Assuming $V_N^*(\cdot,\cdot)$ has some positive value, we trade-off value of energy extracted in period N with that removed in period N-1, as various values of ΔQ_{N-1} are considered. This is done for all pertinent values of Q_{N-1}, T_o^{N-1} and then we move back to period N-2 and repeat the two-stage optimization again. If energy extraction from this anomaly is at all economically feasible, at least one of the ΔQ_t will be positive.

This backward stepping recursive algorithm is then used iteratively until we compute $V_o^*(Q_o, T_o^o)$, where

$Q_o = 0$

T_o^o = initial, equilibrium temperature of the aquifer.

We can then move forward through this set of equations and find $\bar{Q}^*$, the optimal pumping policy vector.

Proposed Work

The next step in developing these economic models is to quantify the functions L, g, h and ϕ, and obtain solutions to the models outlined above. Perhaps the most important part of these results would be a sensitivity analysis indicating the relative importance of the above functions and such parameters as the discount rate.

A logical extension would be to investigate various geometries and spacing (in plan view) of production and recharge wells with these economic models. This extension would consider multiple well clusters for a single anomaly. A more comprehensive extension would include development of multiple, hydraulically-independent anomalies.