NUMERICAL CALCULATION OF MULTIPHASE FLUID AND HEAT FLOW IN HYDROTHERMAL RESERVOIRS

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In this paper one aspect of an ongoing research program will be described. The overall objective is to develop reliable computer simulators whereby field information for a specific liquid- or vapor-dominated geothermal system can be used to predict reservoir performance and, in addition, subsurface environmental effects. These latter effects include land surface subsidence, induced seismic activity, and pollution of fresh water aquifers by geothermal brines. The approach is to develop large-scale computer programs and to validate them using both laboratory measurements and field data.

So far, separate codes have been developed for describing the multidimensional multiphase unsteady flow of steam and water and of heat in a heterogeneous geologic setting in the absence of rock deformation, and for calculating the response of a multidimensional rock matrix to prescribed pore pressure changes without specific consideration of fluid flow. These codes are presently being combined to produce a single fully-interactive fluid flow/rock deformation simulator. The separate codes in themselves may be of some interest, however; S. K. Garg discussed the finite-element rock-deformation simulator in another presentation at this workshop. Here, the fluid-flow simulator will be discussed.

Mathematical Formulation

Brownell ~~et al.~~ (1975) have presented elsewhere the equations governing the flow of water and steam in a non-deforming rock matrix. These may be summarized as follows:

Fluid Mass Conservation:

$$\phi \frac{\partial \rho}{\partial t} = \dot{m} + \nabla \cdot \left| k \left([\alpha_L \rho] \, [\nabla P - \rho_L \vec{g}] + [\alpha_V \rho] \, [\nabla P - \rho_V \vec{g}] \right) \right|$$

Energy Conservation:

$$\frac{\partial}{\partial t} [E_s + \phi E] = \dot{e} + \nabla \cdot \left| k \left([\beta_L E] \, [\nabla P - \rho_L \vec{g}] + [\beta_V E] \, [\nabla P - \rho_V \vec{g}] \right) + \bar{\kappa} \, \nabla T \right|$$

Local Thermal Equilibrium:

$$T_s = T$$

where

	S = 0 (all liquid)	0 < S < 1 (multiphase)	S = 1 (all vapor)
α_L	$1/\mu$	$\frac{1-Q}{1-S}\frac{R_L}{\mu_L}$	0
α_V	0	$\frac{Q}{S}\frac{R_v}{\mu_v}$	$1/\mu$
β_L	$1/\mu$	$\alpha_L[1-QE_{vap}/E]$	0
β_v	0	$\alpha_v[1+(1-Q)E_{vap}/E]$	$1/\mu$

and

E = Bulk fluid internal energy per unit fluid volume.
$\mathcal{E}_{vap}$ = Latent heat of vaporization per unit fluid volume.
E_S = Solid internal energy per unit total volume.
ρ = Bulk fluid density.
ρ_v = Vapor phase density = $\rho Q/S$.
ρ_L = Liquid phase density = $\rho(1-Q)/(1-S)$.
Q = Steam quality.
S = Steam saturation.
$\mu(\mu_L;\mu_v)$ = Bulk fluid (liquid; vapor phase) viscosity.
$R_v(R_L)$ = Relative vapor (liquid) permeability.
k = Absolute solid permeability.
$\bar{\kappa}$ = Mixture (rock-liquid-vapor) heat conductivity.
ϕ = Porosity.
P = Pressure.
T = Fluid temperature.
T_S = Solid temperature.
$\vec{g}$ = Acceleration of gravity.
$\dot{m}$ = Local fluid mass source/sink rate.
$\dot{e}$ = Local heat source/sink rate.

These balance laws are to be solved subject to appropriate initial and boundary conditions. Furthermore, constitutive relations must be prescribed both for the rock matrix and for the institial fluid. For the rock, the density, porosity, directional absolute peremeabilities, relative permeability functions, heat capacity and thermal conductivity must be supplied at each point in the system. For the fluid, a large number of properties must be known as functions of water density (ρ) and internal energy (E). These include pressure (P), temperature (T), steam quality (Q), vapor saturation (S), latent heat of vaporization (E_{vap}), and separate viscosities (μ_ℓ, μ_v) and thermal conductivities (κ_ℓ, κ_v) for liquid and vapor. For this purpose, a rather elaborate system of subroutines was developed which uses large data tables and various interpolation schemes valid up to ultra-high pressures (several megabars) and temperatures to 3000°C.

Computer Code and Applications

The system of balance equations is solved by a finite difference technique which has been described elsewhere (Pritchett et al., 1975). Essentially, an implicit-time, first-order (upstream) space representation of the equations is employed; the iterative Alternating-Direction-Implicit (ADI) technique is used to reduce a single multidimensional problem to an equivalent sequence of one-dimensional problems. These one-dimensiona problems are, of course, nonlinear in themselves--these nonlinearities are removed by iteration within the one-dimensional "module."

The numerical scheme has been incorporated into a simulator wh ch possesses considerable flexibility. Several geometries can be conside ed: (1) 1-D slab, (2) 1-D cylindrical, (3) I-D spherical, (4) 2-D planar or areal, (5) 2-D axisymmetric, or (6) 3-D Cartesian. Each computational zone may contain a different rock type, and any face of any zone may be a boundary. Provision is made for all practical boundary condition options: (1) impermeable, insulated, (2) impermeable, prescribed heat flux, (3) impermeable, prescribed temperature, (4) prescribed mass flux, insulated, (5) prescribed mass and heat flux, (6) prescribed mass flux and temperature, and (7) prescribed pressure and fluid heat content. Boundary condition parameters may be functions of time.

The simulator has been extensively tested, using both simplified analytic problems with known solutions and bench-scale experimental results. Work is currently in progress to simulate the field production history at the Wairakei field in New Zealand. Garg, et al. (1975) presented some of the test results against laboratory data at the United Nations Conference in San Francisco last May. Briefly, one-dimensional simulations were performed of laboratory experiments carried out by Kruger and Ramey (1974) and Arihara (1974) at Stanford. These experiments involved flow in a narrow 60 cm long tube packed with sandstone. In these experiments, non-isothermal and multiphase flow occurred. Results computed by the simulator included pressure and temperature distributions within the tube as functions of time--agreement was generally within experimental scatter for all cases considered.

This numerical reservoir simulator is therefore considered operational and possesses several desirable features. Mass and energy are conserved exactly, since the numerical scheme is based squarely upon density and internal energy rather than other auxiliary quantities. Proper treatment of flow-type (i.e., prescribed-pressure) boundaries eliminates artificial computational "energy sources" at these boundaries, even under conditions of flow reversal. The use of the implicit upstream difference technique suppresses the computational "jitter" produced by many other simulators--artificial oscillations of this sort occasionally cause computational catastrophes in single-phase regions near the saturation line.

A recent paper by Coats et al. (1973) describes a serious computational difficulty they encountered when performing a 2-D areal simulation of a five-spot steamflood of an oilfield. The "five-spot" pattern is a checkerboard-like system with alternating injection and production wells. Coats found that if he treated this problem with a grid oriented such that a line connecting adjacent production and injection wells lies at 45° with respect to the axes the computed water interface expands outward in a roughly circular manner, whereas if the grid is oriented with coordinate lines connecting adjacent production and injection wells, thin "fingers" of injected fluid penetrate outward rapidly. Times of water breakthrough at the production well differed by a factor of three for these calculations. To investigate this problem, our geothermal reservoir simulator was used to calculate a five-spot cold water injection into a producing hot-water field, using both grid orientations. Times of cold water breakthrough computed in these two calculations agreed within a few percent, which is less than the resolution of the finite-difference grid employed. Therefore, it is believed that the present method is not subject to this difficulty, at least for problems of geothermal interest.

Several applied calculations have been performed so far using the simulator. One series of computations reported at the United Nations Symposium by Garg et al. (1975) show that, in a bounded geothermal reservoir with no internal or external heat or mass sources, reinjection of waste water will substantially augment the producing life and the total energy deliverability of the system. At present, under a parallel in-house project, a series of calculations are underway which illustrate the effect of information flashing upon wellhead pressure histories during drawdown and shut-in well testing (Rice, 1975). As mentioned earlier, a simulation of the Wairakei system is now being undertaken.

Coupling of the simulator with the rock-response finite-element code is now in progress. Completion of this task will permit more accurate seismic and subsidence predictions, and will also aid in extending our capability to include cases wherein rock composition produces a significant fraction of the reservoir drive (such as the geopressured systems of the Gulf Coast). Also in progress is the extension of the water equation of state to consider brines, and the addition of a solute-conservation equation to the simulator. These features are desirable when considering very saline systems such as the Salton Sea geothermal field.

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