ANALYTICAL STUDY OF CRACK GROWTH AND SHAPE BY HYDRAULIC FRACTURING OF ROCKS

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Crack shape, orientation, size and growth due to hydraulic fracturing will be investigated as a problem in three dimensional elasticity theory. Since the opening of the crack by hydraulic fracture, the pressurizing of the treatment fluid, the leaking off of the fluid, and the thermal cracking are simultaneous events, the theory of elasticity will be coupled with fluid mechanics and the theory of heat conduction. The results, which include the coupling of elasticity and fluid inclusion, will be obtained by analytical techniques so that they can be presented with analytical formulae when possible.

Stress intensity factors for an elliptic crack. By using the continuous dislocation method developed by Mura (1963) and Willis (1968), the stress component σ_{33}, which is the most important component, along an elliptical crack (Figure 1) under a linearly changing applied stress $\sigma^{o}_{33} = A + Bx_1 + Cx_2$ has been obtained as follows:

$$\sigma_{33} = \frac{a_2 \left(x_1^2/a_1^4 + x_2^2/a_2^4\right)^{\frac{1}{2}}}{\left(x_1^2/a_1^2 + x_2^2/a_2^2 - 1\right)^{\frac{1}{2}}} \left(\frac{A}{E} + \frac{Bx_1}{3E_1} + \frac{Cx_2}{3E_2}\right) \qquad (1)$$

where

$$E = \int_0^{\pi/2} (1 - k^2 \sin^2\varphi)^{\frac{1}{2}}\, d\varphi$$

$$k^2 = \left(a_1^2 - a_2^2\right)/a_1^2 > 0$$

$$E_1 = \int^{\pi/2} \sin^2\varphi (1 - k^2 \sin^2\varphi)^{\frac{1}{2}}\, d\varphi$$

$$E_2 = \int_0^{\pi/2} \cos^2\varphi (1 - k^2 \sin^2\varphi)^{\frac{1}{2}}\, d\varphi \qquad (2)$$

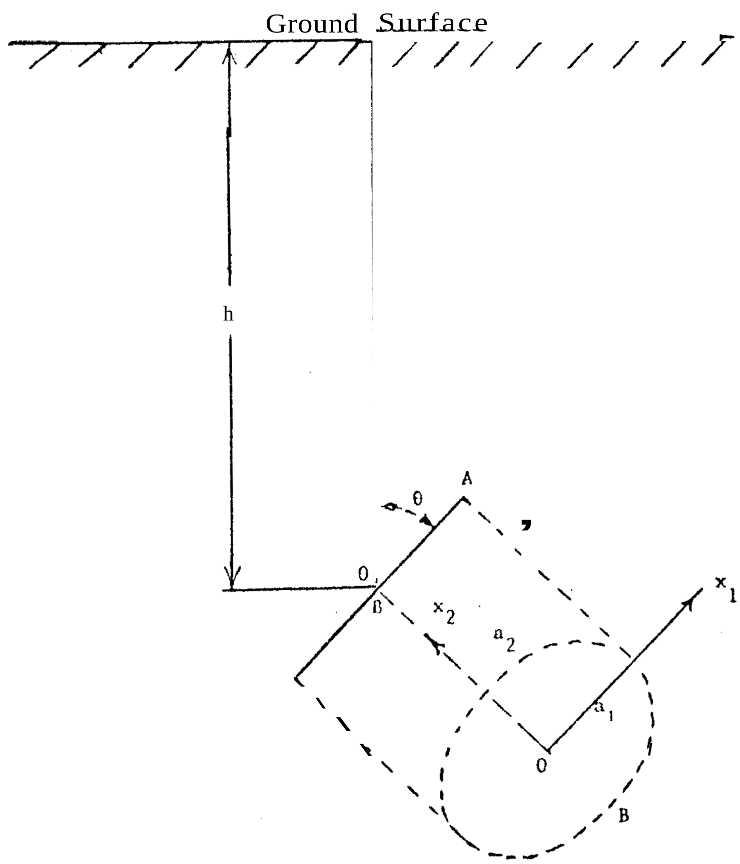
On the crack surface the boundary condition

$$\sigma_{33} + \sigma^{o}_{33} = 0$$

is satisfied. For the crack as shown in Fig. 1

$$\sigma^{o}_{33} = \rho_o g(h - x_1 \cos\theta) + p - S \tag{3}$$

and therefore, $A = -\rho_o gh + p - S$, $B = \rho_o g \cos\theta$, $C = 0$, where p is the pressure necessary for crack opening, ρ_o the density of the rock, h, the depth of the crack, S, tectonic stress and θ is the orientation of the crack surface relative to the surface of the earth (Fig. 1).



Fig, 1

The maximum width of the crack also has been obtained as

$$w = \left(1 - x_1^2/a_1^2 - x_2^2/a_2^2\right)^{\frac{1}{2}} A \, a_2 \, \frac{2(1 - \nu)}{\mu E} \tag{4}$$

where ν is Poisson's ratio, μ is the shear modulus, and E is defined in Eq. (2).

The stress intensity factor is the coefficient of $(x_1^2/a_1^2 + x_2^2/a_2^2 - 1)^{\frac{1}{2}}$ in Eq. (1). The stress intensity factor is not constant along the crack edge. However, certain angles θ of crack inclination give equal stress intensity factor at the crack tips of the major and minor principal axes of the crack. Thus, θ can be obtained as

$$\theta = \cos^{-1}\left[\frac{3E_1}{E}\frac{A}{a_2\rho_o g}(1 - a_2/a_1)\right] \tag{5}$$

Axisymmetrical crack growth in hydraulic fracturing. It is found that the growth rate of a penny-shaped crack can be predicted as a continuous function of time, w en the crack is fractured by water under hydraulic pressure.

The fundamental equat ons are

$$\frac{\partial(\rho w)}{\partial t} + \frac{1}{r}\frac{\partial(rq)}{\partial r} = 0$$

$$\frac{\partial p}{\partial r} = -\frac{12\mu}{\rho w^2} q - \frac{6}{5wr}\frac{\partial}{\partial r}(rq^2/\rho w) - \frac{1}{w}\frac{\partial q}{\partial t} \tag{6}$$

where p is the fluid pressure in the crack and q is the rate of mass flow defined by

$$q = \rho w \bar{u} \tag{7}$$

where $\bar{u}$ is the average radial fluid velocity. According to Sneddon and Elliott (1946), we have for the width of the crack and the stress intensity factor:

$$w = \frac{8(1 - \nu^2)}{\pi E}\int_r^R \frac{r_1 dr_1}{\sqrt{r_1^2 - r^2}}\int_o^1 \frac{x(p - S)}{\sqrt{1 - x^2}}dx \tag{8}$$

$$K = \frac{1}{\pi}\sqrt{\frac{2}{R}}\int_{R_o}^R \frac{r(p - S)}{\sqrt{R^2 - r^2}}dr \tag{9}$$

where R is the crack radius, R_o, is the wellbore radius, and E is Young's modulus for the crack; S is the tectonic stress.

Since w = 0 at r = R and (6) has the inverse cube singularity for w, we assume that the second equation in (6) holds for the domain, $R_o \leq r \leq R_1$, where $R_1 < R$ and indicates the radius of the wetted domain. It follows that

$$\bar{u}(R_1) = \frac{dR_1}{dt} \tag{10}$$

Investigation of order of magnitude of the terms in (6) leads to the conclusion that the last two terms in the right-hand side can be neglected. The following global equation of the conservation of mass is also employed

$$\int_{R_o}^{R_1} \rho \, rw \, dr = \int_o^t q_o R_o \, dt \qquad (11)$$

where q_o is the flow rate at the wellbore.

Results obtained by solving (6) to (9) and (11) are given in Figs. 2 and 3, where

$$t_D = 3\pi E q_0 t/8(1 - \nu^2)\rho S R_o^2, \quad w_D = \pi E w(R_o)/8(1 - \nu^2) R_o S, \quad \Delta p_D = (\bar{p} - S)/S$$

and $\bar{p}$ is the average pressure, The solutions by neglecting the effect of the elliptic integral in w which arises from (8) and the effect of the term $\partial(\rho w)/\partial t$ in (6) are shown in Fig. 2 by putting subscripts (1) and (2), respectively. It is found that the former effect is small for large values of R/R_o while the latter one is significant. Results, which are valid for a wide range of R/R_o, are shown in Fig. 3. The effect of the stress intensity factor of the rock is found to be significant even for large values of R/R_o.

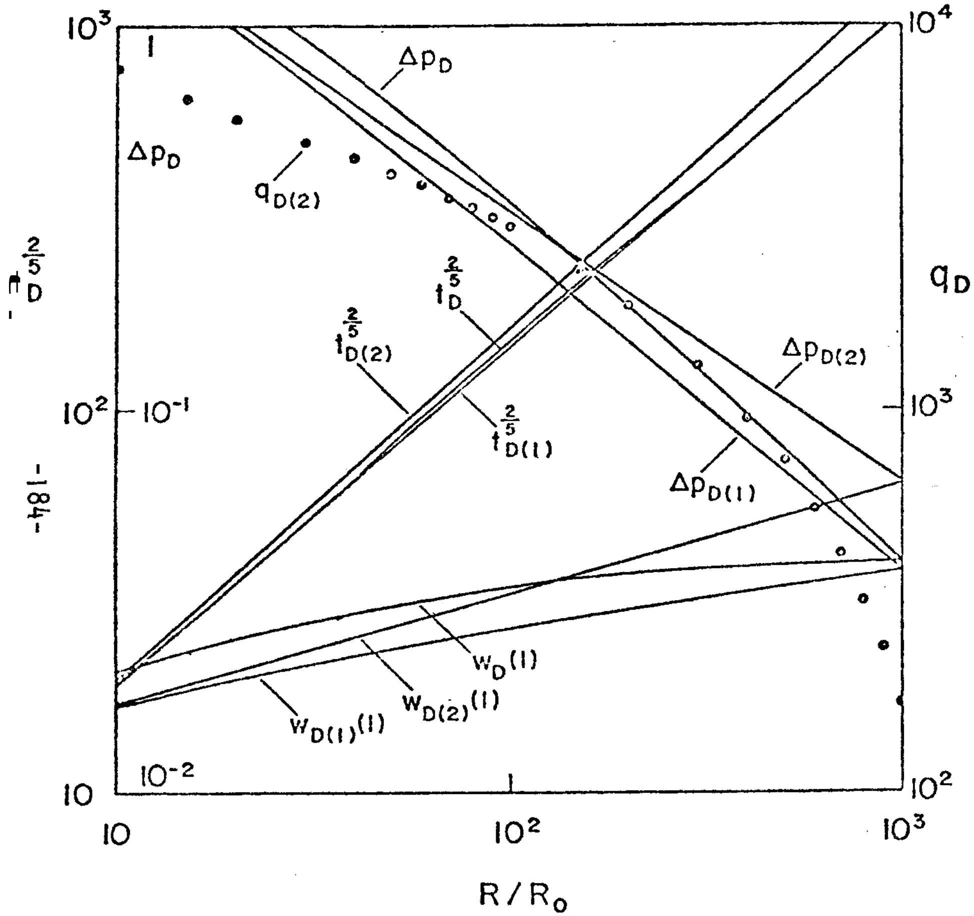


Fig. 2. Effects of terms in the basic equations on crack growth under constant flow rate $q_D = Cq_o/S^4R_o^2 = 7.6961 \times 10^3$.
$C = 3\pi^3E^3\mu/128(1-\nu^2)^3\rho$.

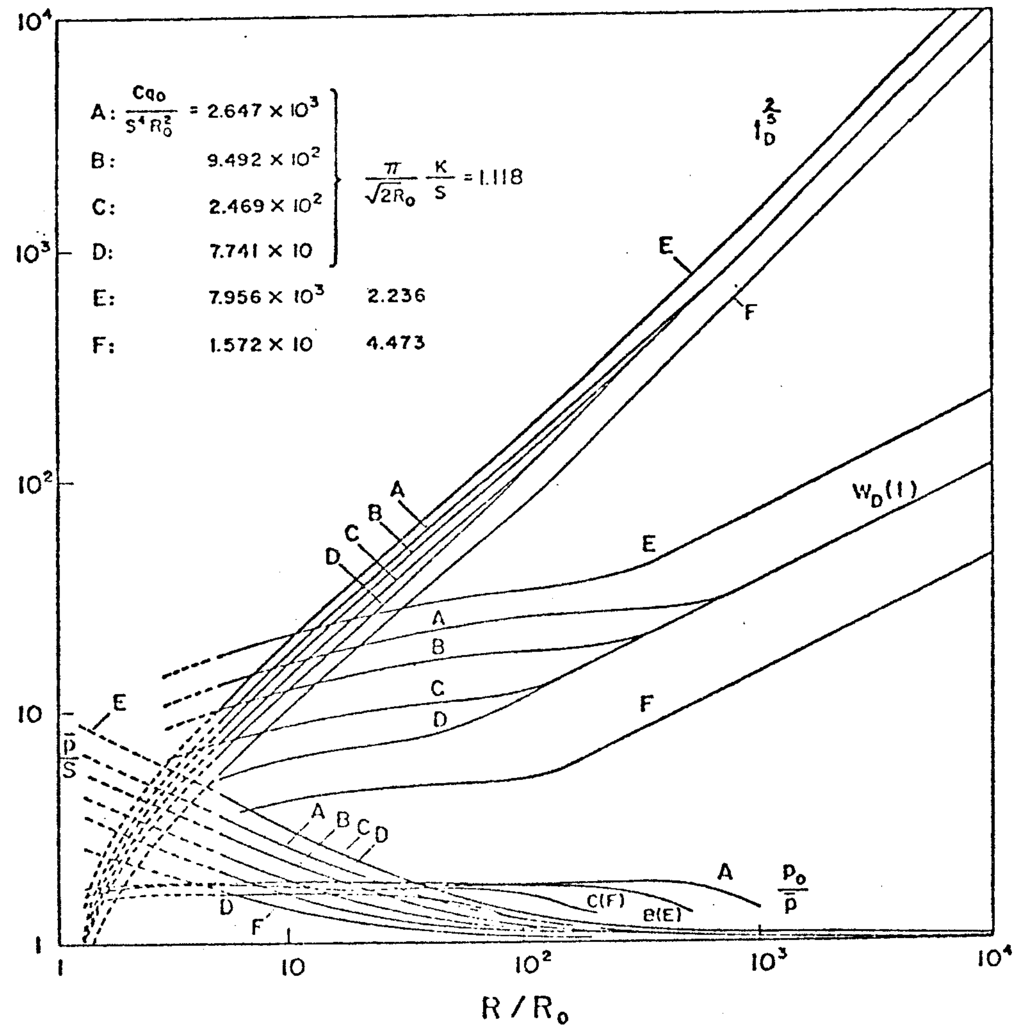


Fig. 3. Expansion of crack by action of fluid flow with constant fl rate; relati between nondimensional width $w_D(1)$, nondimensional time t_D, pressure ratios $p_o/\bar{p}$ and $\bar{p}/S$ with nondimensional fracture radius R/R_o.