NUMERICAL AND ANALYTICAL STUDIES ON HEAT AND MASS TRANSFER IN VOLCANIC ISLAND GEOTHERMAL RESERVOIRS

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The Hawaii Geothermal Project is a multidisciplinary research effort with the major objectives of locating a suitable geothermal resource on the island of Hawaii and utilizing the heat to produce electricity by means of a research-oriented power plant. As a primary first step of this project, the potential geothermal resources on the island of Hawaii must be studied and sufficient information obtained to permit a reasonable prediction to be made of the behavior of the geothermal field, as well as the environmental impact of the utilization of the resources.

The Hawaiian Islands were formed by volcanic action. Because of the high porosity and permeability of the basaltic formation, aquifers at shallow depth are likely to be unconfined from the top. It has been postulated that a magma chamber at shallow depth and the numerous hot intrusives that form a dike complex can provide the heat source for the heating of the groundwater in the aquifer (Furumoto 1975). As the high permeability of the formation permits a continuous recharge from the ocean and rainfall, it has been speculated that most of the geothermal resource at shallow depth in the island is probably warm water at low or moderate temperatures (Macdonald 1973). Because of a self-sealing effect it is possible that impermeable layers are formed at great depth where hot water may be found (Furumoto 1975). As the basaltic rock hardly deforms under pressure, a large scale land surface subsidence resulting from the withdrawal of geothermal fluids is not anticipated. However, the utilization of geothermal resources in the island is not free from other adverse environmental impacts. The most serious potential hazard is the possibility of the contamination of the freshwater lens during the reinjection of the toxic fluids into the formation. The prediction of the fate of the injected fluids under different operating conditions thus merits careful consideration.

The following is a summary of the progress we have made in the past two years on the theoretical study of heat and mass transfer in a volcanic island geothermal reservoir. Many of the results are of a fundamental nature and are, therefore, applicable to other liquid-dominated reservoirs.

Numerical Studies on Heat and Mass Transfer in Liquid-Dominated Geothermal Reservoirs

Free Convection in Island Geothermal Reservoirs. The problem of steady free convection in an island aquifer, confined by caprock at the top and heated by bedrock from below, is considered by Cheng, Yeung & Lau (1975). The effects of thermal conditions at the caprock, the geometry of the reservoir, the variation *of* Rayleigh number, the size of the heating surface, and the dike intrusion on fluid flow and heat transfer characteristics in the reservoir are examined. The numerical results show: (1) As a result of geothermal heating, cold seawater will move inland from the ocean in the lower portion of the aquifer; rise up as a thermal plume after sufficient heat is absorbed; spread around under the caprock and finally discharge to the ocean in the upper portion of the aquifer. This open streamline convective pattern always exists in an island aquifer. (2) If the heating surface is sufficiently large, multiple closed-streamline convective cells will also be generated in the interior portion of the aquifer. These recycling convective cells prevent the complete mixing of cold water from the coast and the warmer water in the interior. (3) The number of these closed-streamline convective cells depends not only on the size of the heating surface but also the manner in which it is heated, i.e., whether it is heated by bedrock from below or the dike complex on the side. (4) Away from the thermal plumes, vertical temperature profiles exhibit a temperature reversal similar to that measured by Keller (1974). (5) The heat transfer rate on the bottom heating surface is independent of the thermal boundary condition at the caprock. (6) The temperature gradient *is* large in the fluid adjacent to the heating surface and in the thermal plume; this boundary layer behavior is very pronounced at large Rayleigh number.

A perturbation analysis was made for steady free convection in an island geothermal reservoir unconfined from the top (Cheng *E* Lau 1974; Lau *E* Cheng 1975). The analysis is applicable for aquifers with low Rayleigh numbers (i.e., aquifers at low temperature or with low permeability) where heat conduction is predominant. The effect of geothermal heating on the upwelling of water table is shown. To investigate the upwelling of water table at high Rayleigh numbers, numerical methods must be used for the solutions of the governing non-linear partial differential equations with non-linear boundary conditions. This phase of the work is currently underway.

Withdrawal and Reinjection of Fluids in Island Geothermal Reservoirs. The problem of withdrawal of fluids *is* investigated by Cheng *E* Lau (1975) and Cheng *E* Teckchandani (1975). It is found that (1) the withdrawal rate is linearly proportional to the withdrawal pressure; (2) a symmetric location of withdrawal sites with respect to heat source will enhance the convective heat transfer from the bottom heating surface and thus prolong the lifespan of a geothermal well; (3) as a result of withdrawal of fluid, the temperature above the sink will decrease while the temperature distribution below the sink is less affected; and (4) the temperature of withdrawal fluid is relatively unaffected by the withdrawal rate. The reasons for (3) and (4) can be explained as follows: As a result of withdrawal of fluid, the flow field below the sink will experience a favorable pressure gradient, thus aiding the upward movement of the convecting flow, which in turn enhances the convective transfer on the bottom hot surface. On the other hand, the flow field above the sink will experience an adverse pressure gradient which will retard the upward movement of the hot fluid or, in some cases, reverse the direction of the convecting flow if the withdrawal pressure is large.

The problem of reinjection of fluid is studied by Cheng & Yeung (1975) and Cheng & Teckchandani (1975). If the injected fluid is colder than the surrounding fluid, a cold region is created above the injection point. For a fixed reinjection rate, the injection pressure decreases as Rayleigh number is increased. In other words, less injection pressure is needed for an aquifer at high temperature or with high permeability. For a fixed Rayleigh number, the injection rate is linearly proportional to the injection pressure.

Analytical Studies on Heat and Mass Transfer in Liquid-Dominated Geothermal Reservoirs

From the numerical solutions for free convection in 'geothermal reservoirs by Cheng, Yeung & Lau (1975), it is observed that boundary layer behavior is pronounced for flow near the heating surface and in the thermal plume at large Rayleigh numbers. Thus the boundary layer approximations analogous to the classical viscous flow theory can be applied. With these approximations, analytical solutions have been obtained for the following problems.

Steady Free Convection About Vertical Intruded Bodies. Within the framework of boundary layer approximations, similarity solutions are obtained for free convection about a vertical hot dike with surface temperature being a power function of distance from the origin; i.e., $T_w = T_\infty + Ax^\lambda$ where T_w and T_∞ are the wall temperature and the temperature of the surrounding fluid away from the dike. For the special case of an isothermal dike ($\lambda = 0$) with a height L and a width S, the local boundary layer thickness $\delta(x)$, and the total surface heat transfer rate Q are given by (Cheng & Minkowycz 1975)

$$\delta(x) = 6.3 \left[\frac{\mu\alpha x}{\rho_\infty g\beta K(T_w - T_\infty)} \right]^{1/2} , \qquad (1)$$

$$Q = 0.88 S k (T_w - T_\infty)^{3/2} \left[\frac{\rho_\infty K\beta g L}{\mu\alpha} \right]^{1/2} , \qquad (2)$$

where p μ, and β are the density, viscosity, and the thermal expansion coefficient of the fluid, K is the permeability of the saturated porous medium, $\alpha = k/(\rho C)_f$ is the equivalent thermal diffusivity where k denotes the thermal conductivity of the saturated rock and $(\rho C)_f$ the product of the density and specific heat of the fluid.

Approximate analytical solutions for free convection about a vertical cylindrical intrusive are also obtained (Minkowycz & Cheng 1975). It is found that the ratio of total heat transfer rate for a cylinder to that for a flat plate depends only on the dimensionless parameter

ξ_L where $\xi_L \equiv \frac{2}{r_0}\left[\frac{\mu\alpha L}{K\rho_\infty \beta g(T_W-T_\infty)}\right]^{1/2}$ with r_0 denoting the radius of the cylinder. The ratio varies between 1 to 3 when ξ_L varies between 0 and 10.

Buoyancy induced Flows Adjacent to Heated impermeable Horizontal Surfaces. Similarity solutions for free convection above a heated horizontal impermeable surface with a power law variation of wall temperature are also obtained by Cheng E Chang (1975). **The** local thermal boundary layer thickness $\delta_T(x)$ and the total surface heat transfer rate for a horizontal heating surface with a length L and a width **S** are:

$$\delta_T(x) = C_1\left[\frac{\mu\alpha x}{\rho_\infty g\beta K(T_W-T_\infty)}\right]^{2/3} , \tag{3}$$

and

$$Q = C_2 Sk(T_W-T_\infty)4/3 \left[\frac{\rho_\infty K\beta L^3}{\mu\alpha}\right]^{1/3} , \tag{4}$$

where the values of C_1 and C_2 depend on the values of A and are tabulated in Table 1.

To gain some feeling of the order of magnitude of various physical quantities given **by** Eqs. (1-4), computations are carried out for a heating surface of 1 km by 1 **km** at a temperature of 300°C embedded in an aquifer at 15°C. The physical properties used for the computations are β = 3.2 x 10^{-4}/C, ρ_∞ = 0.92 x 10^6 g/m^3, C = 1 cal/g-°C, μ = 0.18 g/sec-m, **k** = 0.58 cal/sec-°C-m, and **K** = 10^{-12}m^2. With these values, the boundary layer thickness along a dike increases from zero at the origin to 70 m at 1 km; the total heat transfer rate is 75 MW. For a horizontal heating surface of the same size, the boundary layer thickness increases from zero at the origin to 200 m at 1 km with a total heat transfer rate equal to 20 MW.

Table 1 VALUES OF C_1 AND C_2 **FOR** EQS. (3) & (4)

λ	C_1	C_2
0.5	4.9	1.23
1.0	4.4	1.32
1.5	4.0	1.45
2.0	3.6	1.57

Buoyancy Plumes Above a Horizontal Line Source. Similarity solutions for plume rise above a horizontal line source in a saline aquifer have been obtained by Cheng (1975). The **problem** is an extension of the work by Wooding (1963). The spreading of the buoyancy plume or the boundary layer thickness of the plume is given by

$$\delta = \frac{7.2\ x^{2/3}}{\left[\frac{K\beta g Q}{\alpha^2 \mu C}\right]^{1/3}} , \qquad (5)$$

where Q is the rate of heat generated per unit length of the line source. **Eq.** (5) shows that the spreading of the plume increases as $x^{2/3}$, and decreases as K or Q are increased. Using the previous physical properties and with Q = 20 **kw**, the plume spreading is about 120 m at 1 km above the point source.

Future Work

Continuous effort will be devoted to the numerical solutions of free convection in unconfined geothermal island aquifers with recharge from the ocean and rainfall. The interaction of production and withdrawal wells will also be examined. The effects of non-local thermodynamic equilibrium, the layer structure of the rock formation, and the irregular boundary of the reservoir will also be studied. Boundary layer analysis is being pursued for other problems of free and forced convection in geothermal reservoirs.

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