FINITE ELEMENT SOLUTION OF GEOTHERMAL ENERGY EXTRACTION

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The objective of this theoretical work is to simulate numerically the following two basic problems in the area of geothermal energy extraction: (1) initiation and extension of fracture in hot dry rocks by hydraulic fracture; and (2) circulation of water through the fractured zone and back up to the ground surface. In addition, it has become evident that the following third problem area also requires careful consideration: (3) the study of thermally induced secondary cracks and their effects on power production. The basic method of approach involves a finite-element numerical simulation coupled with some analytical computations.

Basic Equations

The basic field equations for the water flow and heat transfer in a crack have been formulated for one-and two-dimensional cases. These equations include (a) crack width varying arbitrarily in time and space; (b) heat convection due to flow of water, heat conduction in water, and heat supply by conduction from the rock; (c) an accurate mathematical model for the thermodynamic properties of water (according to 1968 ASME Steam Tables), i.e., the pressure-density-temperature relationship (with the dependence of compressibility, thermal expansion coefficient and heat capacity on pressure and temperature). The basic differential equations have been obtained by applying the conditions of conservation of mass, of linear momentum, and of energy to the cross-section of crack, using an assumed velocity profile. In the case of unidirectional flow, for example, these equations are:

$$\text{mass;}\quad \frac{\partial}{\partial t}(\rho w) + \frac{\partial q}{\partial x} = 0 \;; \tag{1}$$

$$\text{momentum;}\quad \frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\alpha \frac{q^2}{\rho w}\right) = - w \frac{\partial p}{\partial x} - 2\tau + \rho w g_x + \frac{4}{3}\mu \frac{\partial}{\partial x^2}\left(\frac{q}{\rho}\right) \tag{2}$$

$$\text{energy;}\quad \rho w C_w \frac{\partial T}{\partial t} + C_w q \frac{\partial T}{\partial x} + \left\{ \frac{3}{2}(\beta - \alpha)\left(\frac{q}{\rho w}\right)^2 \frac{\partial q}{\partial x} + (\alpha - 1)\frac{q}{\rho w}\frac{\partial q}{\partial t} - (\beta - \alpha)\left(\frac{q}{\rho w}\right)^3 \frac{\partial(\rho w)}{\partial x} \right\} = \left\{ \frac{2\mu}{3}\left[\frac{\partial^2}{\partial x^2}\left(\alpha \frac{q^2}{\rho^2 w}\right) - \frac{2q}{\rho w}\frac{\partial^2}{\partial x^2}\left(\frac{q}{\rho}\right)\right] \right\} + 2\tau \frac{q}{\rho} + \frac{\partial}{\partial x}\left(kw \frac{\partial T}{\partial x}\right) + 2h(T_r - T) \;; \tag{3}$$

equation of state for water; $p = f(\rho, T)$; (4)

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where ρ is the mass density; q is the mass flux across the width of the crack; p is the pressure; τ is the shear force at the interface between fluid and rock; g_x is the average body force on the fluid,; C_w, k, and h define the heat capacity, heat conductivity within the fluid, and heat conductivity between fluid and rock, respectively; T and T_r are the fluid's and the rock's temperature, respectively. The equation of state for water in the range of pressure and temperature relevant to the geothermal problem is given in the 1968 ASME steam tables. This equation is used in the numerical calculations.

The parameters α and β, as well as the shear force τ, in Eqs. *(2)* and (3) depend on the geometry of the velocity profile across the crack width. For a parabolic profile, for example, one has $\alpha = 1.2$, $\beta \simeq 1.54$ and $\tau = \frac{6\mu q}{\rho w^2}$, where μ is the fluid viscosity. However, in the operation stage, the fluid velocity can be high and a turbulent flow with an almost uniform profile of mean velocity may be a more appropriate assumption.

Finite Element Formulation for the Fluid

To obtain the corresponding finite-element equations, linear spatial variation for pressure, temperature, and mass flow within each element is assumed. Then a systematic application of Galerkin's method gives the basic finite-element equations for the fluid flow. For the unidirectional flow, for example, these equations are

$$\underset{\sim}{K}_1 \dot{\underset{\sim}{p}} + \underset{\sim}{K}_2 \dot{\underset{\sim}{T}} + \underset{\sim}{K}_3 \underset{\sim}{p} + \underset{\sim}{f} = \underset{\sim}{0} , \tag{5}$$

$$\underset{\sim}{K}_4 \dot{\underset{\sim}{T}} + \underset{\sim}{K}_5 \underset{\sim}{p} + \underset{\sim}{K}_6 \underset{\sim}{T} + \underset{\sim}{g} = \underset{\sim}{0} , \tag{6}$$

$$\underset{\sim}{G} \underset{\sim}{q} + \underset{\sim}{K}_1 \dot{\underset{\sim}{p}} + \underset{\sim}{K}_2 \dot{\underset{\sim}{T}} + \underset{\sim}{f}^{(0)} = \underset{\sim}{0} . \tag{7}$$

In these equations superimposed dot denotes the partial time derivative; $\underset{\sim}{p}$, $\underset{\sim}{T}$, and $\underset{\sim}{q}$ denote pressure, temperature, and mass flow at the two nodes of each element: and the coefficient matrices, as well as the corresponding forcing functions, are either constant or nonlinear functions of p, w, and $\underset{\sim}{q}$, as well as linear function of $\dot{p}$, $\dot{w}$, and $\dot{\underset{\sim}{q}}$.

Equations (5) - *(7)* together with the equation of state for water, Eq. (4) are sufficient for the calculation of the fluid flow, provided that the crack width is known at all nodal points. For an assumed crack width (obtained in a previous time step) these equations are solved for ρ, q, p, and T is time steps with iteration at each step, until the maximum change of each quantity with respect to the previous value is less than a prescribed limit; see, however, the following discussion.

Combination of Finite Element Model with Analytical Solutions

Test runs of the finite element program for the solid in combination with the finite element program for the flow in the crack have indicated that the requirements for computer time are extremely high and convergence very slow. The extension jumps of the crack in the finite element grid must be very small, or else enormous spurious changes of pressure in the fluid are obtained. Dense spacing of the nodes in turn requires very short time steps, in order to avoid numerical instabilities. However, the response of the elastic rock due to pressure in the crack as well as cooling from the crack can be quite accurately described by analytical formulas, and this allows reduction of computer time as well as higher accuracy of numerical calculations. It is therefore concluded that the following numerical approach is most effective:

(a) In case of hydraulic fracturing, the pressure in wate in excess of hydrostatic remains essentially uniformly distributed at a 1 times and water temperature is equal to that of the adjacent rock. n this case, the finite-element program for the solid alone may be used treating the fluid pressure as the input;

(b) In case of operation stage, the finite-element program for the fluid flow and the heat transfer in the fluid may be used in conjunction with analytical solutions for the elastic solid (rock) and the heat conduction in the rock (using the concept of cooling penetration depth and the heat transfer coefficient)..

Finite Element Formulation for the Solid (Rock)

For analyzing the fracture of the solid (rock), a two-dimensional finite element program with a water-filled crack has been written. The criterion for the propagation of the crack can be formuldted in this program either by means of a stress-intensity factor, or by means of a strength (limiting stress value in the finite element). The former type of strength criterion is usually more appropriate, provided the rock is relatively homogeneous and flawless and the crack is sufficiently large. Among the various methods of evaluating the stress intensity factor in the finite element analysis, the method of calibrated crack-tip element *of* ordinary type has been chosen as the most efficient one. This program must be subjected to more testing, and the method by which the boundary conditions representing the surrounding infinite solid can be best simulated, must be identified.

Examples and Estimates

In order to develop an understanding for the various physical processes which are involved in this general area, some simple analytical results have been developed. In the following, these results are briefly discussed.

Crack Extension: The extension of a crack in a rock mass as a function of the total mass flow can be estimated in the following manner. If the maximum crack opening is Δ and the crack radius is R, then for an elliptical opening the total fluid volume in the crack is given by $\frac{4}{3}\pi\Delta R^2$.

On the other hand, according to the Griffith criterion, one has $p - S = AR^{-\frac{1}{2}}$ and $\Delta = B(p-S)R$ where $A = \frac{\pi E\gamma}{2(1-\nu^2)}$, S is the tectonic stress normal to the face of the crack, $B = \frac{4(1-\nu^2)}{\pi E}$, γ is the surface energy, E is the elastic modulus of the rock, and ν is the corresponding Poisson's ratio. If $M = \rho V$ is the total mass of the fluid in the crack, one then obtains

$$R = R_o \left(\frac{M}{M_o}\right)^{2/5}, \tag{8}$$

$$p - S = (p_o - S)\left(\frac{M}{M_o}\right)^{-1/5}, \tag{9}$$

where subscript o refers to the initial values. For example, if the fluid is pumped in at a constant rate q, one has $M = M_o + qt$, and Eqs. (8) and (9) give the crack radius and the corresponding pressure as functions of time; the latter is illustrated in Fig. 1. Except for the transient effects, it is seen that (p-S) remains relatively constant as the crack grows.

Heat Extraction: For the heat extraction in a steady-state operation the following equation estimates the temperature of the fluid along a "stream tube" (see Figs. 2 and 3):

$$T_w = T_r^{in} - \left\{ \left[T_r^{in} - T_w^{in} + \left(T_r^{in} - T_r^{o}\right)/\alpha\ell_o\right] e^{-\alpha x} + \left(T_r^{in} - T_r^{o}\right)\left(\frac{x}{\ell_o} - \frac{1}{\alpha\ell_o}\right) \right\}, \tag{10}$$

where

T_w = water temperature

T_r = rock temperature

= length of the stream tube

a = $2\bar{h}/C_w q$

q = mass flow per unit length measured normal to the stream tube

C_w = water heat capacity,

and where "in" denotes the "inlet" and "o" denotes the outlet values. Here $\bar{h}$ is given by

$$\bar{h} \simeq \sqrt{\frac{\rho_r k_r C_r}{12t}} , \qquad (11)$$

where

ρ_r = rock's mass-density

k_r = rock's heat conductivity

C_r = rock's heat capacity

t = time

Fig. 2 shows the result obtained from (10) for a case in which ℓ_o = 1200m, T_r^{in}= 300°C, T_r^o = 240°C, T_w^{in} = 65°C, and q = 0.2 kg/m sec. These results check very accurately with the numerical results. This is shown in Fig. 3 where the results of the finite-element solution of the complete set of equations are shown by solid lines.

It should be noted that the thermal boundary layer in the rock, in which a thermal gradient exists, is very small when compared with the length of the crack. This requ res that the numerica calculations be coupled with some analytical est mates.

The example of F gs. 2 and 3 does not nclude the secondary cracking due to the very large thermal stresses which may develop as the rock is cooled. Both analytical and numerical calculations have shown that these secondary cracks are very likely to develop and change the nature of the heat flow as well as that of the fluid flow. These and other related problems are now being studied.

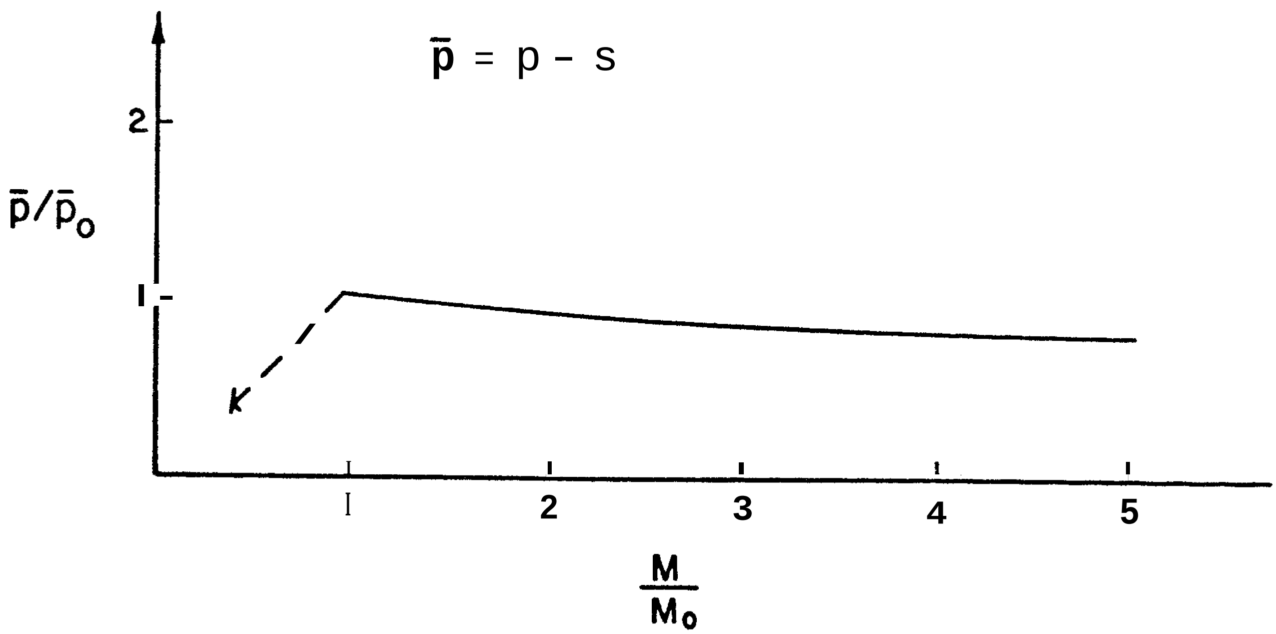

Figure 1

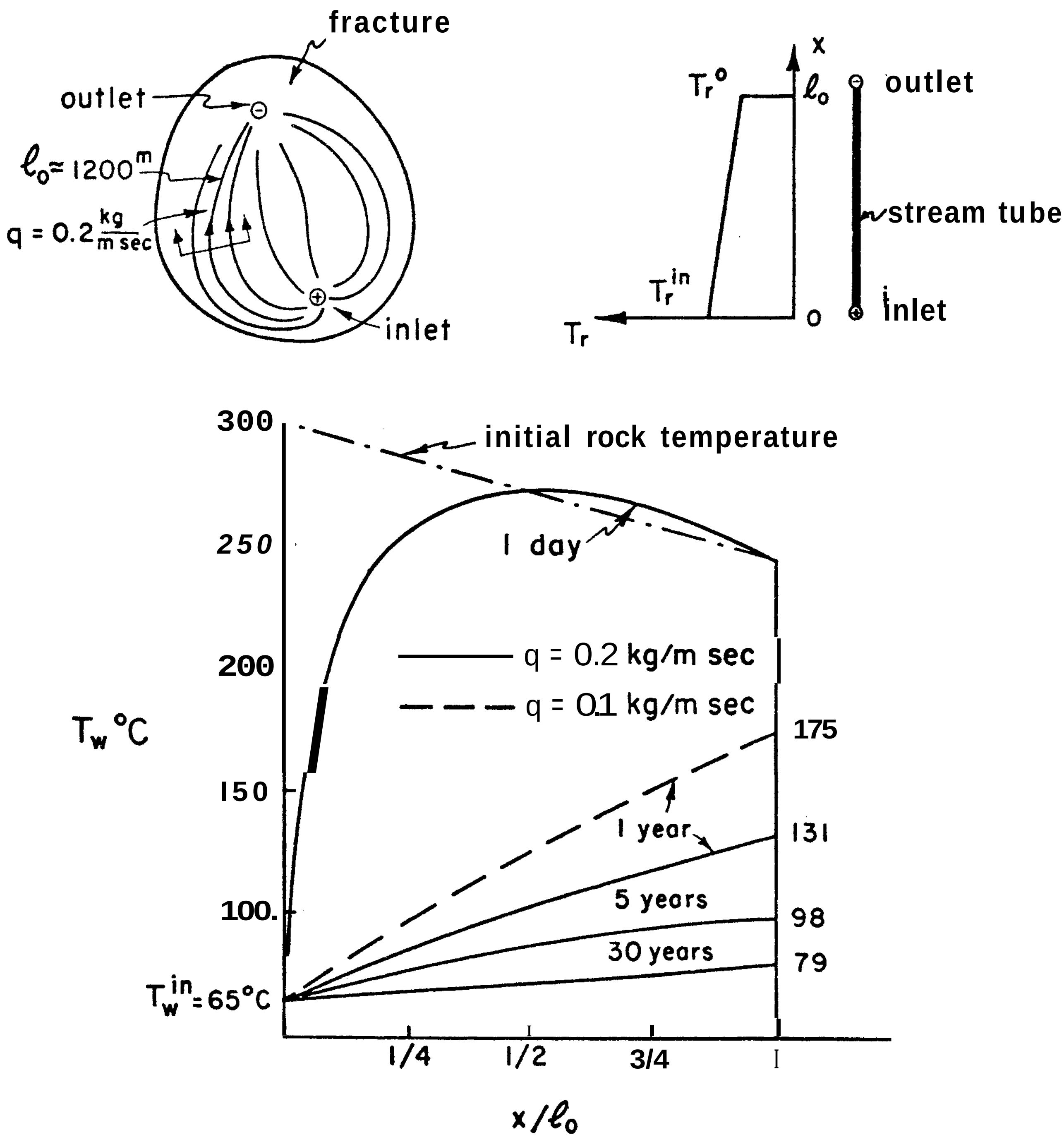
fracture
outlet
$\ell_0 \approx 1200^m$
q = 0.2 kg/m sec
inlet
x
T_r^o
ℓ_o
outlet
stream tube
T_r^{in}
inlet
T_r
o
300
250
200
150
100.
initial rock temperature
1 day
q = 0.2 kg/m sec
q = 0.1 kg/m sec
T_w °C
175
1 year
131
5 years
98
30 years
79
T_w^{in}=65°C
1/4
1/2
3/4
1
x/ℓ_o

Figure 2

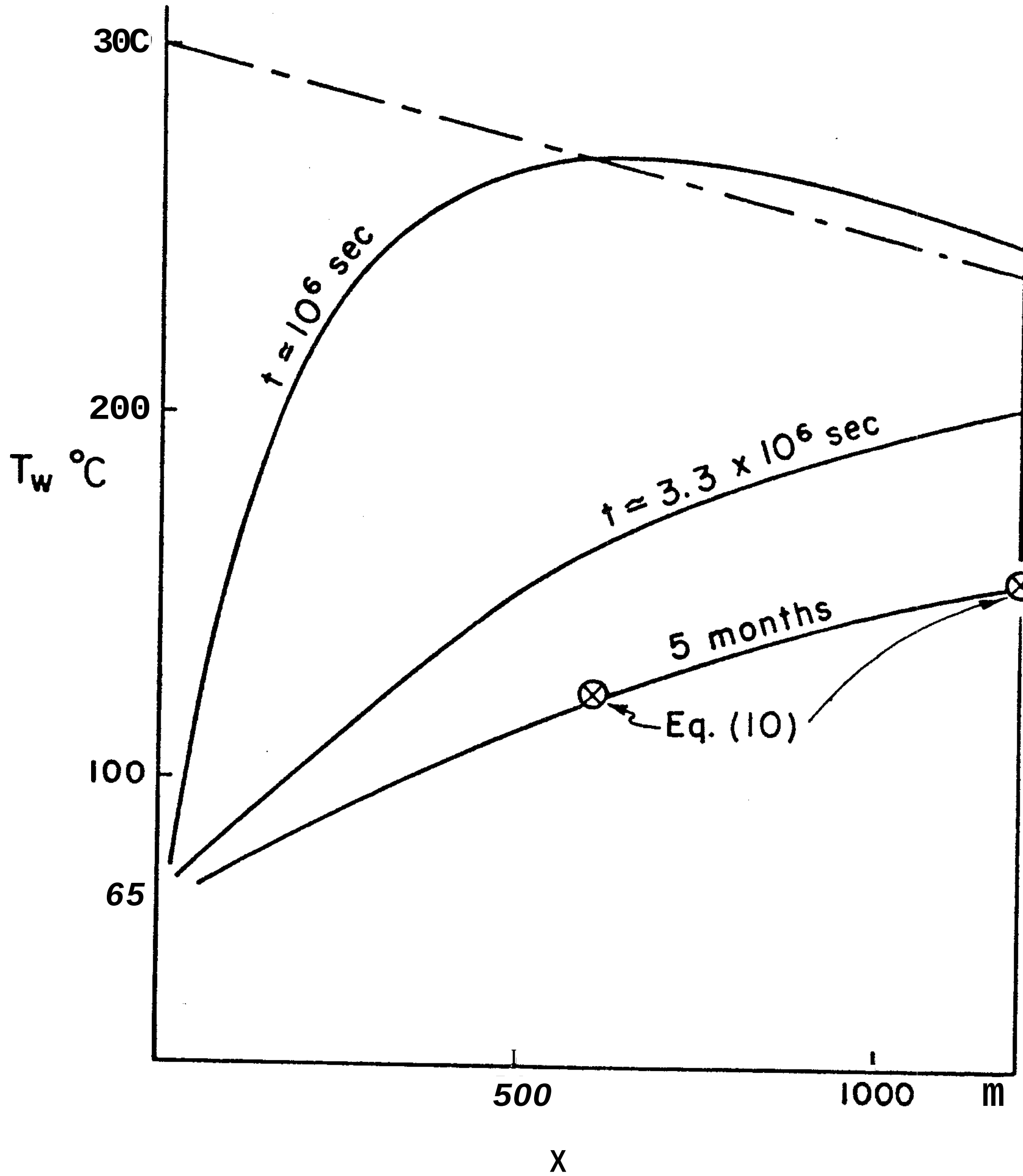
30C
200
100
65
T_w °C
t ≃ 10⁶ sec
t ≃ 3.3 x 10⁶ sec
5 months
Eq. (10)
500
1000
m
X

Figure 3