PROGRESS REPORT ON A MATHEMATICAL MODEL OF A PARALLELEPIPED RESERVOIR WITH NO PENETRATING WELLBORE AND MIXED BOUNDARY CONDITIONS

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The object of this work is to develop a mathematical model, as suggested by H. J. Ramey [1], for simulating unsteady flow in geothermal reservoirs containing a single phase fluid.

The reservoir is a homogeneous, isotropic parallelepiped with no-flux or constant pressure walls, according to the most probable geological conditions.

At present our model considers a non-penetrating wellbore as it will first be applied to Travale geothermal field where many data are available for a productive well with no penetration at all.

The computer program is easily modified, however, to suit other well conditions such as partial penetration or fractured wells. The finite radius of our well was simulated computing the pressure drop at a given distance from the point sink.

A parallelepiped with no-flux walls and top, constant pressure at the bottom, was obtained from Green's instantaneous source functions VII (x), VII (y), IX (z) according to the nomenclature used in ref. [2].

The solution for constant rate q is:

$$\Delta p(M,t) = \frac{q}{\phi c\, x_e y_e z_e} \int_0^t \left[1 + 2\sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2\eta(t-\tau)}{x_e^2}} \cdot \cos\left(n\pi\frac{x_w}{x_e}\right) \cdot \cos\left(n\pi\frac{x}{x_e}\right)\right] \cdot \tag{1}$$

$$\cdot \left[1 + 2\sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2\eta(t-\tau)}{y_e^2}} \cdot \cos\left(n\pi\frac{y_w}{y_e}\right) \cos\left(n\pi\frac{y}{y_e}\right)\right] \cdot$$

$$\cdot \left[2\sum_{n=1}^{\infty} e^{-\frac{(2n-1)^2\pi^2\eta(t-\tau)}{4 z_e^2}} \cdot \cos\left(\frac{2n-1}{2}\pi\frac{z_w}{z_e}\right) \cdot \cos\left(\frac{2n-1}{2}\pi\frac{z}{z_e}\right)\right] d\tau$$

If we define the dimensionless groups as follows:

$$P_D = \frac{2\pi z_e k \Delta p}{q \mu} \quad , \quad t_{DA} = \frac{\eta t}{x_e y_e} \quad , \quad \tau_{DA} = \frac{\eta \tau}{x_e y_e} \tag{2}$$

eq. (1) gives:

$$p_D = 2\pi \int_0^{t_{DA}} \left[1 + 2\sum_{n=1}^{\infty} e^{-n^2\pi^2 (t_{DA}-\tau_{DA}) \frac{y_e}{x_e}} . \cos\left(n\pi \frac{x_w}{x_e}\right) . \cos\left(n\pi \frac{x}{x_e}\right)\right] .$$

$$. \left[1 + 2\sum_{n=1}^{\infty} e^{-n^2\pi^2 (t_{DA}-\tau_{DA}) \frac{x_e}{y_e}} . \cos\left(n\pi \frac{y_w}{y_e}\right) . \cos\left(n\pi \frac{y}{y_e}\right)\right] .$$

$$. \left[2\sum_{n=1}^{\infty} e^{-(2n-1)^2\pi^2 (t_{DA}-\tau_{DA}) \frac{x_e y_e}{4 z_e^2}} . \cos\left(\frac{2n-1}{2}\pi \frac{z_w}{z_e}\right) . \cos\left(\frac{2n-1}{2}\pi \frac{z}{z_e}\right)\right] d\tau_{DA}$$

Substituting $t_{DA} - \tau_{DA} = \theta_{DA}$ implies:

$d\tau_{DA} = - d\theta_{DA}$ and gives:

$$p_D = 2\pi \int^{t_{DA}} \left[1 + 2\sum^{\infty} \quad {}_{DA} \frac{y_e}{x_e} . \cos\left(n\pi \frac{x_w}{x_e}\right) . \cos\left(n\pi \frac{x}{x_e}\right)\right] \tag{3}$$

$$\left[1 + 2\sum \quad {}_{DA} \frac{x_e}{y_e} . \cos(n\pi \quad\quad n\pi \frac{y}{} \quad \right]$$

$$n - \frac{1}{2})^2 \pi^2 \theta_{DA} \frac{x_e y_e}{z_e^2} . \cos\left(\frac{2n-1}{} \pi \frac{z_w}{}\right) . \cos\left(\frac{2n-1}{} \pi z \quad \right] d\theta_D$$

We carried out summations for several values of t_{DA} along with numerical integration.

We had some computing difficulty in evaluating the series with very small values of t_{DA} as they do not rapidly converge. We tested our method by solving two-dimensional problems whose solutions were already known by the superposition in space of exponential integrals [3].

The great advantage of this method is that we can change boundary conditions simply by changing a FORTRAN subroutine. The program was first used to produce Horner graphs for comparing the theoretical and experimental plots, with a view to selecting the appropriate model.

Figures 1, 2, 3 and 4 show the Horner plots for a cube and a square parallelepiped both having a constant pressure or closed bottom. These theoretical Horner plots do not give the straight line section with slope of 1.15 found in two-dimensional models. This means that the procedure commonly used for evaluating the kh product cannot be directly applied when the hypothesis of this model are appropriate.

REFERENCES

[1] H. J. Ramey, Written communication (1975).

[2] A. Gringarten, H. J. Ramey, "The use of source and Green's functions in solving unsteady-flow problems in reservoirs," Soc. Pet. Eng. J. 285-296 (Oct. 1973); Trans., AIME, vol. 255.

[3] H. J. Ramey, A. Kumar, M. S. Gulati, "Gas well test analysis under water-drive conditions," American Gas Association, Arlington, Virginia (1973).

NOMENCLATURE

c = compressibility

k = permeability

M = point where pressure is measured

p_D = dimensionless pressure drop

Δp = pressure drop

q = flow rate

t = time

t_{DA} = area-based dimensionless time

x, y = horizontal coordinates

z = vertical coordinate

η = hydraulic diffusivity

θ_{DA}, τ_{DA} = dimensionless variables of integration

μ = viscosity

τ = variable of integration

ϕ = porosity

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e = reservoir dimensions

w = source or sink location

VII (x), VII (y) = Basic instantaneous source functions, for an infinite plane source in an infinite slab reservoir with prescribed flux at the boundary, applied to the x and y coordinates.

IX (z) = Basic instantaneous source function for an infinite plane source in an infinite slab reservoir with prescribed flux at $z=0$ and prescribed pressure at $z=z_e$.

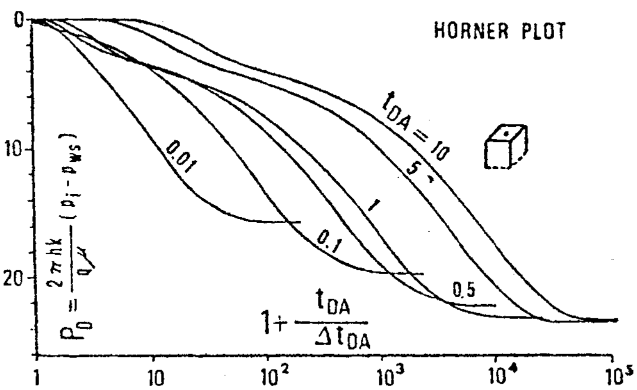


Fig.1 — Closed cube *with* costant pressure at the bottom

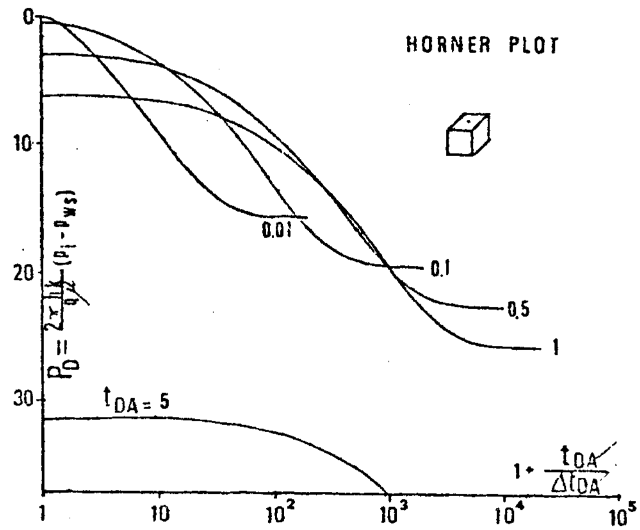
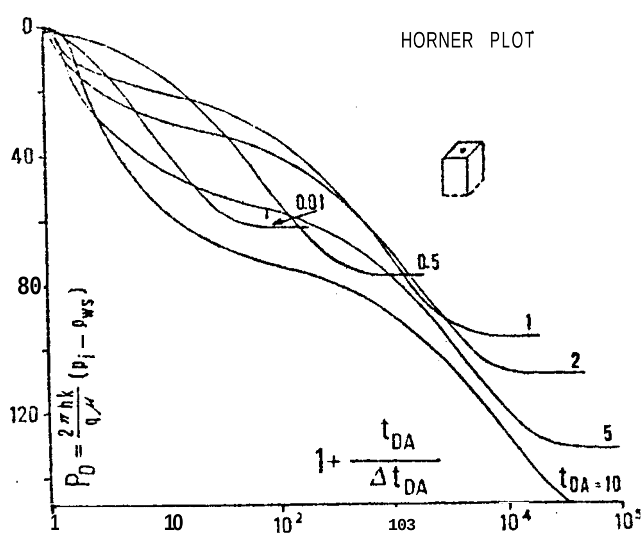


fig .2 — Closed cube



Fip.3 --Closed parallelepiped with costant pressure at the bottom

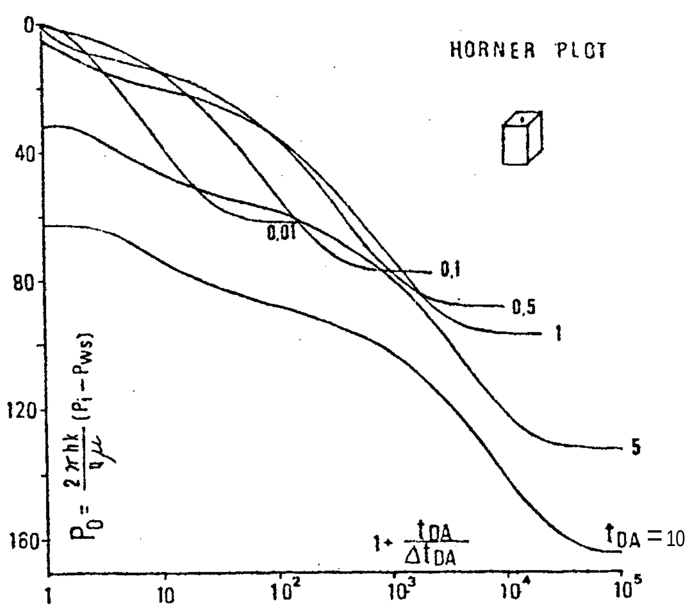


Fig.4 — Closed parallelepiped