THE USE OF GENERAL SENSITIVITY THEORY TO ANALYZE THE GEOTHERMAL RESERVOIR MODEL'S SENSITIVITY TO THE PERMEABILITY FUNCTIONS

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Sensitivity theory is concerned with studying how a model depends upon its parameters, constants or functions. Models of real physical processes, such as flow in a geothermal reservoir, are implemented via numerical simulations. The results of the model u are available only for certain values of the parameters p_o. Because the exact dependence of u on p is not known, sensitivity studies are typically done by random or quasi-organized searches of the parameter space p, and computer output is generated for sets of points p_o for the parameters.

The mapping u (p) and its derivatives are of great interest. The derivatives are defined to be sensitivity functions. For constant parameters. the sensitivity functions are $\frac{\partial u_i}{\partial p_j}$ and for parametric functions $u'_p \cdot \delta p$ where $u'_p \cdot$ is the Frechet derivative of $u(p)$, and δp is a vector of perturbation functions.

I have developed a general sensitivity theory (1,2,3) which allows the formulation of an auxiliary or dual model from which the sensitivity functions can be calculated directly. Thus, the state and sensitivity functions can be generated for "likely" values of p, and the sensitivity functions indicate how u will change for changes in p in the neighborhood of p_o.

In the following sections I will summarize general sensitivity theory and its usefulness by presenting two examples from reservoir modeling.

General Sensitivity Theory

The derivation of the general sensitivity theory is based upon the use *of* the implicit function theorem for operators on Banach spaces. I view the model as

$$N(u,p) = 0 \tag{1}$$

Then the implicit function theorem asserts the existence of $u(p)$ such that

$$N(u(p), p) = 0 \tag{2}$$

We now differentiate (2) in the sense of Frechet to get

$$N'_u \cdot u'_p \cdot + N'_p \cdot) \, \delta p = 0 \tag{3}$$

In the case of constant parameters (3) becomes

$$N'_u \cdot \left\{\frac{\partial u_i}{\partial p_j}\right\} + N'_p = 0 \tag{4}$$

We will exhibit specific examples in the next section.

Flow in Anisotropic Porous Media

For the model I take the following set of equations (4)

$$\nabla P + \mu \underset{\approx}{D} \cdot \underset{\sim}{V} = 0 \tag{5}$$

$$A \cdot \underset{\sim}{V} = 0 \tag{6}$$

Where V is the superficial velocity. $\underset{\sim}{D}$ is the dispersion tensor; $\underset{\approx}{D} = \underset{\approx}{K}^{-1}$, and $\underset{\approx}{K}$ is the permeability tensor. Equations (5) and (6) define a model $\underset{\sim}{R}(\underset{\sim}{u}, \underset{\sim}{p})$ where $\underset{\sim}{u} = (V_1, V_2, V_3, P)$ and $P = (D_{11}, D_{12}, D_{13}, D_{22}, D_{23}, D_{33})$.

Computing derivatives R'_u and R'_p

$$R'_u \cdot = \left[\begin{array}{ccc|c} & \mu \underset{\approx}{D} \cdot & & \nabla \\ & & & \nabla \\ & & & \nabla \\ \hline \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} & 0 \end{array}\right]$$

$$R'_p \cdot = \begin{bmatrix} V_1 & V_2 & V_3 & 0 & 3 & 0 \\ 0 & & 0 & V_2 & V_3 & 0 \\ 0 & 0 & & 0 & V_2 & V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $\phi = u'_p \cdot \delta p$

Then we have 24 sensitivity equations in the form

$$R'_u \cdot \underset{\sim}{\phi} + R'_p \cdot \delta p = 0$$

where δp are the perturbations in the dispersion or permeability functions, and $\underset{\sim}{\phi} = 0$ on the boundary of the domain.

Because R is linear in u, the sensitivity and the model are governed by the same basic operator.

A Nonlinear Example

For a nonlinear example, I turn to a model of Peaceman and Rachford [5].

$$\frac{\partial}{\partial x}\left[\frac{k_x}{\mu}\left(\frac{\partial p}{\partial x} - \delta\frac{\partial z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\frac{k_y}{\mu}\left(\frac{\partial p}{\partial y} - \delta\frac{\partial z}{\partial y}\right)\right] = \quad x,y) \tag{7}$$

$$\frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c}{\partial y}\right) - u_x \frac{\partial c}{\partial x} - u_y \frac{\partial c}{\partial y} = \phi \frac{\partial c}{\partial t} \tag{8}$$

k_x, k_y, ϕ are functions of x and y only.

Nonlinearities are introduced via

$$\mu = f(p,c) \qquad \gamma = f(c) \tag{9}$$

In addition

$$D_x = D + \frac{\alpha_x}{\phi}|u_x|$$

$$D_y = D + \frac{\alpha_y}{\phi}|u_y|$$

The state is given by $u = (p,c)$ and the parameters of $\underset{\sim}{p} = (k_x, k_y)$

I will not write out R'_u and R'_p. However, it is of interest to note that because of the nonlinear coupling, σc can affect **p** even if **up** is zero; and conversely.

Summary and Conclusions

I have briefly sketched sensitivity theory and its application to two problems. A key point to recognize is that the sensitivity model is the complement of the physical model. A powerful result is that regions in ρ of unusual behavior for u can be found more easily from studying the behavior of the sensitivities than from studying u alone. This observation is true even in cases where $u(\rho)$ is known.

I expect the techniques of general sensitivity analysis to be useful in formulating and using geothermal reservoir simulators under conditions of large parameter uncertainty.

References

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