To see a rock in a grain of sand

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Permeability is perhaps one of the most important yet elusive reservoir properties. Unlike porosity, it correlates poorly with the elastic properties of rocks and, as a result, cannot be mapped remotely. Rather, it has to be directly measured in the lab on core plugs extracted from a well. In such lab experiments, a pressure gradient is applied to a fluid-filled rock sample, the fluid's volume flux is recorded, and the absolute permeability is calculated from Darcy's law by relating the flux to the pressure gradient.

Physical experiments may be augmented or even partially replaced by numerical experiments provided that a numerical simulation accurately mimics the physical process. Successful numerical "measurements" of permeability have been reported by Bakke and Oren (1997), Keehm et al. (2001), Knackstedt et al. (2004), and others. It appears that the absolute (and even relative) permeability can be recovered with reasonable accuracy from 3D micro-images of the pore space where the geometrical features of the pore-space structure relevant to fluid flow are carefully resolved and digitally represented in the computer. Such images are available from high-resolution X-ray tomography, also known as CT-scanning. CT-scan produces multiple closely spaced 2D slice-images of a rock that, when stacked together, represent the 3D volume. However, high-resolution scanning devices are still prohibitively expensive and the scanning time is too long to be practically useful in massive numerical experimentation.

An alternative is a statistical reconstruction of 3D volume from a 2D slice. A flat thin section, where the pore space appears in a single color because it is impregnated with dyed epoxy and is easily distinguishable from the grains (Figure 1), can be used for this purpose. Thin sections are relatively easy and cheap to prepare either from core plugs or cuttings.

If a thin section is available, it is image-processed to distinguish pores from grains. The statistical properties of this 2D pore space, such as porosity and the autocorrelation length, are calculated from the image. Finally, it is assumed that a 3D pore space has the same statistical properties as the thin section, and a geostatistical algorithm (e.g., the sequential indicator simulation) is used to create a 3D pore space realization with approximately the same porosity and the autocorrelation length. Keehm (2003) and others have presented a number of examples where the absolute permeability computed on such statistically reconstructed 3D volumes matches the physically measured permeability.

At the heart of the digital permeability approach is a computer simulation of fluid flow through a 3D pore space. As in the physical experiment, permeability is calculated from Darcy's law by relating the modeled flux to the numerically applied pressure gradient. A method suitable for such numerical experimentation is the Lattice-Boltzmann algorithm. It can accurately reproduce fluid flow and pressure field at the grain scale in 3D within a geometrically complex pore space and without a need for any idealization of the pore space and the representation of real pores by idealized shapes, such as are used in traditional network modeling.

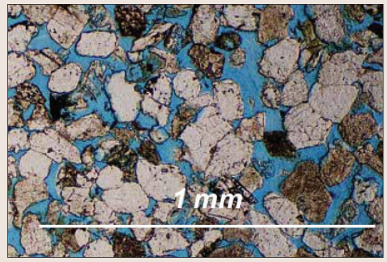

Figure 1. A sandstone thin section. The pore space is filled with epoxy and appears blue.

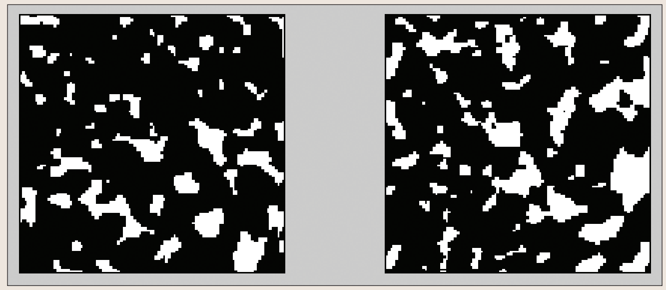

Figure 2. Two black-and-white thin sections (about 2 mm apart) of the same sandstone sample. The size of a thin section is 4 × 4 mm. The pores are white while the grains are black. The surface fraction of pores in the thin section on the left is 0.17 while it is 0.23 in the thin section on the right. The physically measured porosity of the 3D rock sample is about 0.20.

Heterogeneity and statistical representation. Even in high-quality reservoir sand, the characteristics of the pore space may vary at a millimeter scale. Consider, for example, two thin sections from the same sandstone sample taken less than 2 mm apart (Figure 2). The porosity (the ratio of the area occupied by pores to the total area of the thin section) is 0.17 for the thin section on the left while it is 0.23 for the thin section on the right.

Such strong heterogeneity implies that only large digital samples should be used in numerical simulations in order for these samples to be statistically representative of physical core plugs whose size is on the order of 1 cm. Ideally, it would be desirable to directly reproduce the physical experiment in the computer, with all spatial dimensions preserved. However, this task may be technically impractical if not impossible. Indeed, the length (or width) of a standard core plug may include hundreds of grains. In order for a grain to be accurately represented in the computer, it has to contain about 10 computational nodes. The resulting number of computational nodes in a plug is 10^9. A computational task of this size can only be handled by a supercomputer and will still require hours if not days to implement. A task that can be handled by standard hardware and in real time should include no more than 10^6 nodes.

Practical application. Two potential practical applications of computational rock physics are: (a) quantifying the effect of deposition and diagenesis on permeability and (b) obtaining permeability from small fragments of rock, such as cuttings, either at the drill site or in storage.

In the first application, digital images of the pore space of a single sample or a limited number of samples can be numerically altered in the computer to, e.g., deposit diagenetic cement, dissolve minerals, or deposit small shale particles. By so doing, geologically plausible variations in the depositional environment as well as diagenesis can be represented in the computer without having direct access to a multitude of physical samples. The porosity and permeability of the altered samples can be calculated to assess how these properties may

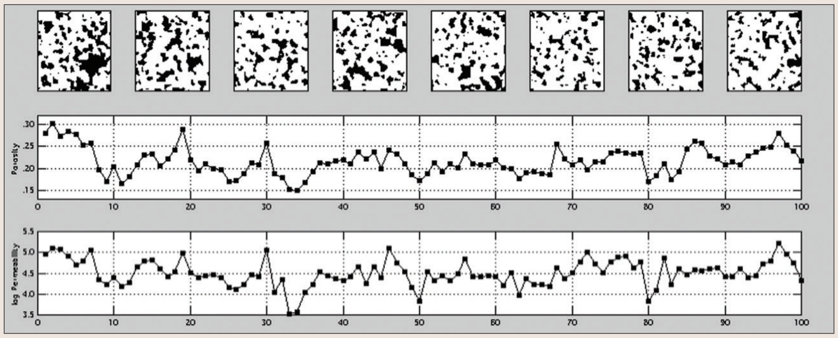

Figure 3. Hibernia sandstone sample. Top: Eight consecutive digital 4-mm thin sections sliced 0.4 mm apart. The pores are black while the grains are white. Middle: Porosity calculated for 100 thin sections sliced 0.04 mm apart, including the thin sections shown in the top frame. Bottom: Logarithm of normalized permeability calculated for these 100 thin sections in Darcy/mm^2. The absolute permeability is normalized by the grain size (0.68 mm for this sample) squared.

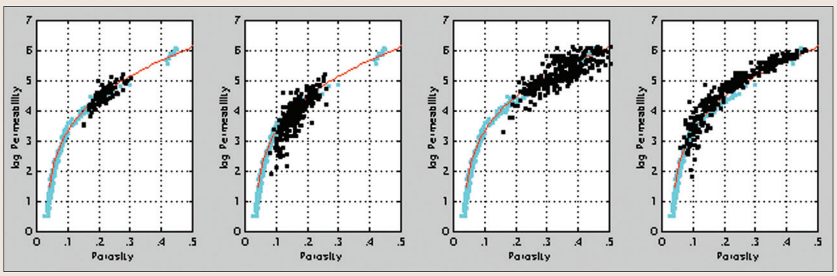

Figure 4. Logarithm of permeability normalized by the grain size squared versus porosity. The background data plotted in cyan come from Fontainebleau sandstone (zero to 0.3 porosity range) and Ottawa sand (0.4 to 0.5 porosity range). The red curve is from the Kozeny-Carman equation. Black symbols are for permeability calculated from statistically subrepresentative digital thin sections, as described in the text. From left to right: Hibernia sandstone; three aeolian samples; three high-porosity samples; and four Finney packs.

vary laterally and vertically in the field. This approach places a virtual rock physics lab at the fingertips of a practitioner.

In the second application, real-time permeability estimates can be obtained from cuttings. Also, a cutting library can be created to build a field-scale permeability map that is useful in further development of a mature basin.

In both applications, numerical experimentation is the key and, therefore, the speed of computation is essential. A single numerical experiment should not take longer than a few minutes and, ideally, should be reduced to a few seconds. This requirement puts a limitation on the size of a digital sample. It should be represented in the computer by less than 10^6 computational nodes, which translates into a linear size of less than 100 nodes, or just a few grains across. Also, if we choose to rely on the thin sections from cuttings, the data will be reduced to 2D rock images no larger than a few grains across.

The question now is whether the permeability calculated from small 2D images is practically useful at all since they are likely to be statistically subrepresentative of a larger rock mass.

Numerical experiments on clean sandstones. To address this question, high-resolution CT scans of several clean sandstone samples were obtained. These 3D images were sliced in the computer to prepare small digital 2D slices to imitate the preparation of physical thin sections from cuttings. The 2D slices were used to statistically reconstruct the 3D pore space and calculate the absolute permeability of this pore space by numerically simulating fluid flow by means of the Lattice-Boltzmann algorithm. This procedure was first applied to moderately well-sorted high-energy fluvial sandstone from the Hibernia offshore field, which is east of St. John's, Newfoundland, Canada.

A 4-mm cube of the 3D image of this sample was selected and sliced in the computer at even 0.04 mm intervals to prepare 100 parallel digital thin sections. Figure 3 shows eight examples from the 100 2D images. The variability in the pore space appears even within a small volume of rock. The porosity and permeability calculated for each of these 100 thin sections reflect this apparent variability. The porosity varies between 0.15 and 0.30 while the physically measured porosity is 0.2. The calculated permeability normalized by the grain size squared varies between 3.3 and 160 darcy/mm^2 while the measured normalized permeability is 13 darcy/mm^2. The large spread in the calculated values is due to the 4-mm thin sections being subrepresentative of the macroscopic sample. Apparently, the physically measured porosity and permeability cannot be recovered from such small rock fragments.

The normalized permeability was plotted against the porosity utilizing the Hibernia data. Unexpectedly, statistically subrepresentative values formed a trend (Figure 4). To investigate the meaning of this trend, we extended our database to include:

- Two aeolian sandstone samples from Wyoming, U.S.
- A Fontainebleau aeolian sandstone sample from France
- Three high-porosity samples, two of them from artificially packed California (U.S.) beach sand and one from a turbidite sand from the North Sea (Europe)
- Synthetic packs of identical spherical particles known as the Finney packs

For each natural or artificial rock, the numerical experiment conducted on the Hibernia sandstone was duplicated; the permeability calculated from the statistically subrepresentative 2D slices was normalized by the grain size squared (different for each physical sample), and was plotted versus the porosity for each sample. The results are superimposed on the background trends (shown in Figure 4) formed by the standard Fontainebleau sandstone data set and sorted Ottawa sand (typical sand used in construction). Also plotted in the background is the permeability-porosity curve given by the classical Kozeny-Carman equation.

We observe that although the porosity and permeability values, as calculated from statistically subrepresentative rock fragments, exhibit a large statistical spread, they are related to each other and when plotted, form a distinctive trend. This trend appears to be physically meaningful because it is similar to a trend produced by physically measured porosity and permeability from a large number of macroscopic sandstone samples (Fontainebleau and Ottawa). The examples shown here imply that we can deduce a permeability-porosity relationship for a rock from just a few grains of sand.

Conclusion and practical implications. Numerical simulation of laboratory experiments on rocks, or digital rock physics, is an emerging field that may eventually benefit the petroleum industry. In order for numerical experimentation to find its way into the mainstream, it has to be practical and easily repeatable—i.e., implemented on standard hardware and in real time. This condition reduces the size of a digital sample to just a few grains across. It is also likely that many digital images will be produced from such small fragments of rock. Will the results be meaningful for a larger rock volume?

The answer is that small fragments of medium-to-high porosity sandstones, such as cuttings, that are not statistically representative of a larger sample cannot be used to numerically calculate the exact porosity and permeability of the sample. However, by using a significant number of such small fragments, it may be possible to establish a site-specific permeability-porosity trend which can be used to estimate the absolute permeability from independent porosity data, obtained in the well or inferred from seismic.

Suggested reading. "3D pore-scale modeling of sandstones and flow simulations in the pore networks" by Bakke and Oren (*SPE Journal*, 1997). "Computational rock physics at the pore scale: Transport properties and diagenesis in realistic pore geometries" by Keehm et al. (*TLE*, 2001). Computational Rock Physics: Transport Properties in Porous Media and Applications by Keehm (PhD dissertation, Stanford University, 2003). "Digital core laboratory: Properties of reservoir core derived from 3D images" by Knackstedt et al. (*SPE Proceedings #87009*, 2004). TLE

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